Dynamic behaviour of square and triangular offshore tension leg platforms under regular wave loads

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Abstract

Among compliant platforms, the tension leg platform (TLP) is a hybrid structure. With respect to the horizontal degrees of freedom, it is compliant and behaves like a floating structure, whereas with respect to the vertical degrees of freedom, it is stiff and resembles a fixed structure and is not allowed to float freely. The greatest potential for reducing costs of a TLP in the short term is to go through previously applied design approaches, to simplify the design and reduce the conservatism that so far has been incorporated in the TLP design to accommodate for the unproven nature of this type of platform. Dynamic analysis of a triangular model TLP to regular waves is presented, considering the coupling between surge, sway, heave, roll, pitch and yaw degrees of freedom. The analysis considers various nonlinearities produced due to change in the tether tension and nonlinear hydrodynamic drag force. The wave forces on the elements of the pontoon structure are calculated using Airy's wave theory and Morison's equation, ignoring the diffraction effects. The nonlinear equation of motion is solved in the time domain using Newmark's beta integration scheme. Numerical studies are conducted to compare the coupled response of a triangular TLP with that of a square TLP and the effects of different parameters that influence the response are then investigated.
1. Introduction

Drilling of oil wells in deeper sea is continuing with striking advances, reaching a water depth of more than 1000 m. These water depths are associated with larger hydrodynamic effects and total base moment, finally resulting in more material. Offshore platforms are usually divided into two general categories, fixed and compliant. Fixed types extend to the seabed and remain in place by their weight or by piles driven into the soil. Compliant-type platforms are more responsive to external effects and their movements are controlled by mooring systems. The increase in cost of fixed offshore structures with depth of water encouraged the development of compliant-type structures. The key idea behind their installation is the minimization of the resistance of the structure to environmental loads by making the structure flexible. The tension leg platform (TLP) is a hybrid structure that, with respect to the horizontal degrees of freedom, is compliant and behaves like a floating structure, whereas with respect to the vertical degrees of freedom it is stiff and resembles a fixed structure and is not allowed to float freely.

2. Features of the TLP

In general, the TLP is similar to other column-stabilized moored platforms with the one exception that its buoyancy exceeds its weight and thus the vertical equilibrium of the platform requires taut moorings connecting the upper structure to a foundation on the seabed. The tension in the tether is created by ballasting the structure at site, connecting the vertical mooring system and then deballasting the structure. The TLP has six degrees of freedom: surge, sway and heave as displacement in the x, y and z axes, and rotations about these axes resulting in roll, pitch and yaw motions, respectively, as shown in Fig. 1 for a square TLP. Tethers present at each corner connect the hull and the foundation, allowing the platform to move in the horizontal plane (surge, sway and yaw) but restricting its motion in the vertical plane (heave, pitch and roll). The extra buoyancy over the platform weight ensures that the tethers are always kept in tension. The cost curves for offshore structures will rise more rapidly than the TLP in deep-water reservoirs, because for a TLP, only the cost of the mooring system and its installation increases as the water depth increases.

The TLP is essentially advantageous for the following reasons:

- It attracts a lesser impact of the wave loading due to its compliant nature and hence can operate even in rough sea.
- The natural frequencies in the main or soft degrees of freedom (surge, sway and yaw) are well below the wave frequencies, thus avoiding the occurrence of resonance and reducing the horizontal motion and hence loading on the tether platform system.
- It is less expensive than the bottom-supported structures, especially in deeper sea.
- It can be easily dismantled, installed and transported according to site require-
ments. The change in the water depth essentially requires a change in the tether length.
• It is much safer in a seismically active zone compared with any other fixed platform.
• Because of the restrained vertical motion of the TLP, it is quite convenient to monitor and maintain the risers, oil wells and tethers.
• A particularly attractive feature of the TLP is the ability to shift any resonance outside the frequency region of the active wave energy.

Most of the literature available is on conventional square (four-legged) TLPs. Paul-ling and Horton (1970) reported a method of predicting the platform motions and tether forces due to regular waves using a linearized hydrodynamic synthesis tech-
nique. Each member was assumed to be cylindrical in shape with cross-sectional
dimensions small in comparison to both the length of the cylinder and the wave-
length. The hydrodynamic interactions between adjacent or intersecting members
were neglected. The drag term was linearized and the free-surface effect was neg-
lected. The results agreed well with experimental model results. The motions and
tensions due to regular waves were shown to vary in a linear fashion with wave
amplitude. Angelides et al. (1982) considered the influence of hull geometry, force
coefficients, water depth, pre-tension and tether stiffness on the dynamic responses
of the TLP. The floating part of the TLP was modeled as a rigid body with six
degrees of freedom. The tethers were represented by linear axial springs. Wave forces
were evaluated using a modified Morison equation on the displaced position of the
structure considering the effect of the free sea surface variation. Faltinsen et al.
(1982) developed a comprehensive theoretical model for the behaviour of a TLP and
verified using the model test programme. The basic outline of the model included:
(i) the velocity potential solution for first- and second-order hydrodynamics, except
for the slender members which were modelled with Morison's equation; (ii) Mor-
son's theory and Newman's approximation to calculate drift forces' (iii) the large-
deflection three-dimensional finite element theory with forces from Morison's equa-
tion which was used for the tethers, (iv) the short-crestedness of waves, and (v)
the wind and current. The origin for the Mathieu-type instabilities was the presence
of a constant plus a time-dependent restoring force for surge, sway and yaw. The
amplitudes of oscillations due to the Mathieu-type instabilities depended on the
damping in the system and the relative importance of the time-dependent restoring
term compared to the constant restoring term. Lyons et al. (1983) presented compar-
sions between the results of hydrodynamic analyses and two sets of large-scale model
test results for the wave-induced motion responses of TLPs. The results of analyses
and tests showed good agreement for surge motions although discrepancies were
observed for the tether tension responses at certain wave frequencies. Linear wave
theory was used and hydrodynamic interference between members was neglected.
The nonlinear damping was linearized by assuming an effective linear damping,
which would dissipate the same amount of energy at resonance as the nonlinear
damping. Teigen (1983) presented the response of a TLP in both long-crested and
short-crested waves through model tests. It was concluded that the low-frequency
part of the horizontal response looked enlarged in tests carried out in long-crested
seas, compared to tests carried out in short-crested seas, irrespective of the actual
shape of the directional distribution. Morgan and Malae (1983) investigated the
dynamic response of TLPs using a deterministic analysis. The analysis was based
on coupled nonlinear stiffness coefficients and closed-form inertia and drag-forcing
functions using the Morison equation. The time histories of motions were presented
for regular wave excitations. The nonlinear effects considered in the analysis were
stiffness nonlinearity arising from coupling of various degrees of freedom, large
structural displacements and hydrodynamic drag force nonlinearity arising from the
square of the velocity terms. It was reported that stiffness coupling could significa-
nantly affect the behaviour of the structure and the strongest coupling found to exist between
heave and surge or sway. Spanos and Agarwal (1984) used a single degree-of-free-
dom model of a TLP and calculated wave forces at the structure’s displaced position using the Morison equation. It was shown that by numerically integrating the equation of motion, the calculation of wave forces, on the displaced position of the structure, introduces a steady offset component in the structural response for either deterministically or stochastically described wave fields. The formulation did not involve any velocity-squared type of terms, and yet an offset component was found to be present. Mekha et al. (1994) studied the nonlinear effect of evaluating the wave forces on a TLP up to the wave-free surface. Several approximate methods were evaluated for regular and irregular wave forces, with and without current, and compared to Stokes’ second-order wave theory. The tethers were treated as massless springs providing axial and lateral stiffness at their connection with the hull. The following approximate methods were used to evaluate the wave kinematics from the mean water level to the wave free surface: hyperbolic extrapolation, linear extrapolation, stretching methods and uniform extrapolation. For a TLP subject to regular waves, the surge amplitude turns out not to be affected by the method chosen. However, the surge mean drift was very sensitive to the method used. Heave amplitude and mean offset were both affected by the method selected but were not significantly different from calculating the response to the mean water level only. The pitch response at its natural frequency was amplified at the free water surface, particularly for irregular waves, and was affected by the method selected. Lee (1994) presented the analytical solution of the coupling problem of a 2D tension leg structure interacting with a monochromatic linear wave train. Fluid-induced drags, including form drag and inertia drag, on linearly elastic tension legs had been considered in the study. The nonlinear form drag was then replaced by a linear drag according to Lorentz’s hypothesis of equivalent work. Analytical solutions showed that the inertia drag on tension legs was negligible compared to that due to the evanescent waves caused by the wave-structure interaction. However, the form drag on the legs altered the structural motion and, consequently, the wave field, especially when wave periods were close to the structure’s resonant frequency. Hahn (1994) reported the effects of wave stretching on realistic representations of the wave forces that act on offshore structures. The structures considered were modelled as linear, cantilever, stick-like systems. The lateral responses of such systems to wave forces, computed from water particle kinematics calculated by using the standard and stretching approaches, were examined. The results showed that the effects of stretching on the governing wave forces and the resulting structural responses were small, indicating that they could be ignored in design practice. It was also shown that the action of stretching could not materially influence the governing excitation and the corresponding structural response. Duggal and Niedzwecki (1995) presented results from a large-scale experimental study of the interaction of regular and random waves with a long, flexible cylinder, exhibiting the dynamic characteristics of a TLP riser or tether in approximately 1000 m of water depth. Regular wave conditions were chosen to provide a large range of Keulegan-Carpenter numbers. Classification of the transverse response in regular waves showed similarities with results obtained by previous investigators with oscillating flow on rigid cylinders. For high Keulegan-Carpenter numbers, the
response became more irregular, with response at harmonics of the incident wave frequency and at several natural frequencies of the cylinder.

The greatest potential for reducing costs of a TLP in the short term is to go thoroughly through previously applied design approaches, to simplify the design and reduce the conservatism that so far have been incorporated in the TLP design to accommodate for the unproven nature of this type of platform. According to Natvig and Vogel (1995), focus on design of future TLPs should be on the aspects of the platform geometry that affects tether loading and on the tether system itself. Their experience with a four-legged TLP has shown that the indeterminate tether system implies some very heavy cost items. The new concept of a three-legged TLP, which will be statically determinate, will not require complicated devices and the foundations can be placed with larger tolerances without affecting tether behaviour. The main aspect of three-legged TLP is that all tethers share approximately the same loads despite weather directions. With the near-equal load sharing of the three-legged TLP, the maximum load level in one group is less, thus requiring less tether cross-section material than that of a four-legged TLP. Studies indicate that 12 tethers are feasible for a three-legged TLP whilst 16 would be required for a four-legged equivalent TLP. This is thus an important area for savings since tethers are important cost items. Munkejord (1996) presented a conceptual analysis of the triangular TLP behaviour and then compared the results with data from model tests. The objective was to verify maximum tether tension, maximum platform offset, minimum air gap and tether fatigue. Aker and Saga Petroleum developed the concept of a triangular TLP, which has enabled significant savings in main steel for both hull and deck due to fewer main element intersections and effective force distributions. Munkejord (1996) summarized the design features for the triangular TLP of Aker as a statically determinate system with effective distribution of dynamic loads and fixed-length tethers. No design cases where TLP sustained a maximum storm with one tether missing were reported. No tether tension measurements required day-to-day operation and increased tolerances for the position of the foundation and increased draught and heel tolerances. No numerical study was reported on the triangular TLP.

In view of the non-availability of any numerical study on the response behaviour of the triangular TLP, the present study deals with the investigation of the dynamic response of offshore TLPs under regular sea waves in the presence of current. Diffraction effects and second-order wave forces have been neglected and the evaluation of hydrodynamic forces is carried out using the modified Morison's equation with water particle kinematics using Airy's linear wave theory. The scope of the work is set to compare the structural response of a triangular-shaped TLP under regular waves in various structural degrees of freedom with that of a four-legged TLP to evaluate the viability of the former.

3. Development of a triangular TLP model

The side of the triangular TLP is kept the same as the side of the four-legged (square) TLP, thus forming an equilateral triangle in plan (Fig. 2). The diameter of
the columns, pontoons and the tether length are kept the same for both TLPs. The equivalence considered is in terms of natural time periods of the two models. For a four-legged TLP, the equation of equilibrium between the buoyancy, dead weight and the tether tension is given by:

\[ F_B = 4(r_0)_{\text{sqaure}} + T \]

where

- \( F_B \) is the buoyant force,
- \( T_0 \) is the initial pre-tension in each tether, and
- \( W \) is the weight of the platform.
For a triangular TLP,
\[ F_B = 3(T_0)_{\text{triangular}} + W \]  (2)

To form the basis of comparison of the newly proposed triangular TLP model with that of the four-legged (square) TLP, two cases were taken for consideration:

Case I. Total \( T_0 \) was kept the same for both models and hence the initial pre-tension in the triangular TLP will be more than that of the four-legged (square) TLP. Total weight was kept the same for both the models.

Case II. Initial pre-tension per tether was kept the same for both the models and the total \( T_0 \) of the triangular TLP was less than that of the four-legged (square) TLP. \( F_B \) was kept the same and hence the weight of the triangular TLP model was more than that of the four-legged (square) TLP.

4. Assumptions and structural idealization

The platform and the tethers are treated as a single system and the analysis is carried out for the six degrees of freedom under different environmental loads. The following assumptions were made in the analysis.

1. Initial pre-tension in all tethers is equal and remains unaltered over time. It is quite large in comparison to the changes that occur during the lifetime of the TLP. However, the total pre-tension changes with the motion of the TLP.
2. Wave forces are estimated at the instantaneous equilibrium position of the platform by Morison's equation using Airy's linear wave theory. The wave diffraction effects have been neglected.
3. Wave force coefficients, \( C_d \) and \( C_m \), are the same for the pontoons and the columns and are independent of frequencies as well as constant over the water depth.
4. Change in pre-tension is calculated at each time step, and writing the equation of equilibrium at that time step modifies the elements of the stiffness matrix.
5. The platform is considered as a rigid body having six degrees of freedom.
6. The platform has been considered symmetrical along the surge axis. Directionality of wave approach to the structure has been ignored in the analysis and only a uni-directional wave train is considered.
7. The damping matrix has been assumed to be mass and stiffness proportional, based on the initial values.

5. Stiffness matrix of the triangular configuration TLP

The coefficients, \( K_{ij} \), of the stiffness matrix of the triangular TLP are derived as the reaction in the degree of freedom \( i \) due to unit displacement in the degree of
freedom \( j \), keeping all other degrees of freedom restrained. The degrees of freedom surge (1), sway (2), heave (3), roll (4), pitch (5) and yaw (6) are shown in Fig. 3. The plan at the hull of the triangular TLP is shown in Fig. 4. Fig. 5 shows the schematic elevation of the triangular TLP. The coefficients of the stiffness matrix have nonlinear terms due to the cosine, sine, square root and squared terms of the displacements. Furthermore, the tether tension changes due to the motion of the TLP in different degrees of freedom makes the stiffness matrix response-dependent. The coefficients of the stiffness matrix \([K]\) of a triangular TLP are:
Fig. 5. Schematic elevation of the triangular configuration tension leg platform.

\[
[K] = \begin{bmatrix}
K_{11} & 0 & 0 & 0 & 0 & 0 \\
0 & K_{22} & 0 & 0 & 0 & 0 \\
K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\
0 & K_{42} & 0 & K_{44} & 0 & 0 \\
K_{51} & 0 & 0 & 0 & K_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & K_{66}
\end{bmatrix}
\]
5.1. *Surge (1) direction* (Fig. 6)

By giving an arbitrary displacement $x_1$ in the surge direction, the increase in the initial pre-tension, in each leg, is given by:

$$\Delta T_1 = \left( \sqrt{x_1^2 + l^2} - l \right) AE/l$$

where

$A$ is the cross-sectional area of the tether,

$E$ is Young’s Modulus of the tether,

$\Delta T_1$ is the increase in the initial pre-tension due to the arbitrary displacement given in the surge degree of freedom,

$l$ is the length of the tether, and

$x_1$ is the arbitrary displacement in the surge degree of freedom.

Equilibrium of forces in the surge direction gives
\[ K_{11} x_1 = 3(T_0 + t \pm T_1) s w \cdot y_x \] (5)

where

\[ T_0 \] is the initial pre-tension in the tether, and
\[ g_x \] is the angle between the initial and the displaced position of the tether for unit displacement given in the surge direction.

\[ \sin \gamma = \frac{x_1}{\sqrt{x_1^2 + l^2}} \] (6)

Putting Eq. (6) in Eq. (5), we get:

\[ K^{11} = \frac{3(r_0 + A r_1)}{\sqrt{x_1^2 + l^2}} \] (7)

\[ K_{22} = 0, \] as no force develops in the sway direction when arbitrary displacement is given in the surge direction.

Equilibrium of forces in the heave direction gives:

\[ K_{33} x_1 = 3T_0 \cos \gamma + 3A r_1 \cos \gamma - 3J_0 \] (8)

where

\[ \cos \gamma = \frac{l}{\sqrt{x_1^2 + l^2}} \] (9)

Rearranging terms, we get:

\[ K_{ij} = \frac{(3T_0(\cos \gamma - 1) + 3A r_1 \cos \gamma) x_1}{l} \] (10)

\[ K_{44} = 0, \] as no moment develops along the roll direction when arbitrary displacement is given in the surge direction.

Summation of moments along the pitch direction gives:

\[ K_{55} x_1 = -K_{11} h \quad \text{or} \quad K_{33} = -K_{44} h \] (11)

where \( h \) is the distance of the centre of gravity (CG) from the base of the pontoon.

The force \( K_{11} x_1 \) acts at the bottom of hull and gives rise to a moment along pitch direction which is considered at CG. The negative sign occurs due to counterclockwise moment \( K_{11} h \).

\[ K_{66} = 0, \] as no moment develops along the yaw direction when arbitrary displacement is given in the surge direction.

5.2. Sway (2) direction (Fig. 7)

By giving an arbitrary displacement \( x_2 \) in the sway direction, the increase in the initial pre-tension in each leg is given by:
where 

\[ \Delta T_2 = \left( \sqrt{x_2^2 + l^2} - l \right) AE/l \]  

Equilibrium of forces in the sway direction gives:

\[ K_{22}x_2 = 3(T_0 + T_2) \sin y, \]  

where \( g_y \) is the angle between the initial and displaced position of the tether for unit displacement given in the sway direction.

\[ \sin y = -\frac{x_2}{\sqrt{x_2^2 + l^2}} \]
Putting Eq. (14) in Eq. (13), we get:

\[ K_{22} = \frac{3(T_0 + \Delta T_2)}{\sqrt{x_2^2 + l^2}} \]  

(15)

Equilibrium of forces in the heave direction gives:

\[ K_{32}x_2 = 3T_0 \cos \gamma + 3A \gamma_0 \cos \gamma \]  

(16)

where

\[ \cos \gamma = \frac{l}{\sqrt{x_2^2 + l^2}} \]  

(17)

Rearranging terms, we get:

\[ K_{32} = \frac{3T_0 (\cos \gamma - 1) + 3AJ_2 \cos \gamma}{x_2} \]  

(18)

Summation of moments along the roll direction gives:

\[ K_{42}x_2 = K_{22}x_2, \quad \text{or} \quad K_{42} = -K_{22}h \]  

(19)

The force \( K_{22}x_2 \) acts at the bottom of hull and gives rise to a moment along the roll direction which is considered at the CG. The negative sign occurs due to the counterclockwise moment \( K_{22}x_2h \).\[ K_{52} = 0, \quad \text{as no moment develops along the pitch direction when an arbitrary displacement is given in the sway direction; and} \]

\[ K_{62} = 0, \quad \text{as no moment develops along the yaw direction when an arbitrary displacement is given in the sway direction.} \]

5.3. Heave (3) direction

\[ K_{33} = 0, \quad \text{as no force develops in the surge direction when an arbitrary displacement is given in the heave direction, and} \]

\[ K_{23} = 0, \quad \text{as no force develops in the sway direction when an arbitrary displacement is given in the heave direction.} \]

Equilibrium of forces in the heave direction gives:

\[ K_{33} = \frac{AE_x}{3} + \frac{3}{4} \]  

(20)

where

\[ x_3 \] is the displacement in the heave direction,
\[ g \] is the acceleration due to gravity,
p is the mass density of water,
$D_c$ is the diameter of the column,
$K_{43c}=0$, as no moment develops along the roll direction when an arbitrary displacement is given in the heave direction,
$K_{53}=0$, as no moment develops along the pitch direction when an arbitrary displacement is given in the heave direction, and
$K_{63}=0$, as no moment develops along the yaw direction when an arbitrary displacement is given in the heave direction.

5.4. Roll (4) direction (Fig. 8)

$K_{14}=0$, as no force develops in the surge direction when an arbitrary displacement is given in the roll direction, and
$K_{24}=0$, as no force develops in the sway direction when an arbitrary displacement is given in the roll direction.

By giving an arbitrary rotation $q_4$ in the roll degree of freedom, the change in the initial pretension, in each leg, is given by:

![Fig. 8. Displacement in roll degree of freedom.](image)
\[ AEP \]
\[ A_{r4} = r^c o s \theta_4 (0_{-4}) = A_{r4} \]

Equilibrium of forces in the heave direction gives:

\[ K_{s_{-4}} = A T , \text{ or } K_{s_{-4}} = A \rho P \cos q_{-4} \] (22)

where

- \( q_{-4} \) is the arbitrary rotation given in the roll degree of freedom,
- \( A_{r4} \) is the increase in the initial pre-tension in the tether due to the arbitrary rotation given in the roll degree of freedom,
- \( A_{r4} \) is the increase in the initial pre-tension in the farther tether due to the arbitrary rotation given in the roll degree of freedom,
- \( e_{-4} \) is the perpendicular distance of the new centre of buoyancy from the axis passing through the \( CG = h \sin q_{-4} \),
- \( P_l \) is the dimension in plan, measured perpendicular to the wave direction,
- \( S_1 = \frac{P_{-l}}{2} e_{-4} \),
- \( S_2 = \frac{P_{-l}}{2} e_{-4} \), and
- \( F_c = \frac{3}{4} \pi \rho D_c^2 g \)

Summation of moments along the roll direction gives:

\[ \theta_{-4} = F_b e_{-4} + (r_o + A r_{-4}) (^c - e_{-4}) + (r_o + A r_{-4}) e_{-4} - (r_o + A r_{-4}) (^2 + e_{-4}) \] (23)

or

\[ K_{s_{-4}} \theta_{-4} = (F_b + T_o + \Delta T_{-4} \) \]

\[ \]

\( K_{s_{-4}} = 0 \), as no moment develops along the pitch direction when an arbitrary displacement is given in the roll direction, and

\( K_{s_{-4}} = 0 \), as no moment develops along the yaw direction when an arbitrary displacement is given in the roll direction.

5.5. Pitch (5) direction (Fig. 9)

\( K_{15} = 0 \), as no force develops in the surge direction when an arbitrary displacement is given in the pitch direction, and

\( K_{25} = 0 \), as no force develops in the sway direction when an arbitrary displacement is given in the pitch direction.
By giving an arbitrary rotation $q_5$ in the pitch degree of freedom, the change in the initial pre-tension, in each leg, is given by:

$$
\Delta T_5 = \^p_b \cos \theta_s \left( \theta_s - 2 \frac{A E}{l} \right) \cos \theta_s
$$

(25)

Force in the heave direction is given by:

$$
K_{35} e_5 = AT_5 \quad \text{or} \quad K_{35} = 0
$$

(26)

where

- $T_5$ is the increase in the initial pre-tension of the tether due to an arbitrary rotation given in the pitch degree of freedom,
- $P_b$ is the dimension in plan, measured along the wave direction,
- $\theta_s$ is an arbitrary rotation given in the pitch degree of freedom,

$$
S_1 = \frac{P_b + e_5}{3},
$$

$$
e_5 = h \sin q_5,
$$

$$
S_2 = P_b - S_1,
$$
\( F_b = 3prD2cg, \) and
\( K_{d3} = 0, \) as no moment develops along the roll direction when an arbitrary displacement is given in the pitch direction.

Summation of moments along the pitch direction gives:
\[
A_{55}0 = F_b e_5 + (7'o + A7'_5)2(5_1 - e_5) - (7'o + A7'_5)(5_2 + e_5)
\]
or
\[
K_{55}0 = F_b e_5
\]
and
\[
sin q_5
\]
\( K_{65} = 0, \) as no moment develops along the yaw direction when an arbitrary displacement is given in the pitch direction.

5.6. Yaw (6) direction (Fig. 10)

\( K_{16} = 0, \) as no force develops in the surge direction when an arbitrary displacement is given in the yaw direction, and
\( K_{26} = 0, \) as no force develops in the sway direction when an arbitrary displacement is given in the yaw direction.

By giving an arbitrary rotation \( q_6 \) in the yaw degree of freedom, we get:
\[
s^2 = \left( \frac{P_1}{2} \right)^2 + \left( \frac{P_6}{3} \right)^2
\]
\[
l_1 = \sqrt{P + \theta_0^2(2s^2)}
\]
where \( q_6 = \) arbitrary rotation in your degree-of-freedom.

The change in the initial pre-tension, in each leg, is given by:
\[
M_6 = \frac{F}{j i h - t}
\]
Force in the heave direction is given by:
\[
\hat{3606} = 37_0^\text{\hat{3606}} - 1_1 + 3A7_7^\text{\hat{3606}}
\]
Fig. 10. Displacement in yaw degree of freedom.

\( K_{46} = 0 \), as no moment develops along the roll direction when an arbitrary displacement is given in the yaw direction, and \( K_{56} = 0 \), as no moment develops along the pitch direction when an arbitrary displacement is given in the yaw direction.

Summation of moments along the yaw direction gives:

\[
K_{66} \Delta \theta_6 = 3 \frac{(T_0 + \Delta T_0)(2a^2)}{I_1} \Delta \theta_6
\]

(34)

The stiffness matrix shows:

- the presence of off-diagonal terms, which reflects the coupling effect between the various degrees of freedom;
that the coefficients depend on the change in the tension of the tethers, which is affecting the buoyancy of the system. Hence, the matrix is response dependent.

Hence, the \( [K] \) is not constant for all time instants but the coefficients are replaced by a new value computed at each time instant depending upon the response value at that time instant. The stiffness matrix of the four-legged square TLP is taken as suggested by Morgan and Malaeb (1983) and is not presented here.

6. Mass matrix, \([M]\)

Structural mass is assumed to be lumped at each degree of freedom. Hence, it is diagonal in nature and is constant. The added mass, \( M_a \), due to the water surrounding the structural members and arising from the modified Morison equation has been considered up to the mean sea level (MSL) only. The fluctuating component of added mass due to the variable submergence of the structure in water is considered in the force vector depending upon whether the sea surface elevation is above (or) below the MSL.

\[
[M] = \begin{bmatrix}
M_{11} + M_{a11} & 0 & 0 & 0 & 0 & 0 \\
0 & M_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & M_{33} + M_{a33} & 0 & 0 & 0 \\
0 & 0 & 0 & M_{44} & 0 & 0 \\
M_{a51} & 0 & M_{a53} & 0 & M_{55} & 0 \\
0 & 0 & 0 & 0 & M_{66} & 0
\end{bmatrix}
\]  

(35)

where \( M_{11} = M_{22} = M_{33} = M \).  

\( M \) is the total mass of the entire structure,  
\( M_{44} \) is the total mass moment of inertia about the \( x \) axis = \( M r_x^2 \),  
\( M_{55} \) is the total mass moment of inertia about the \( y \) axis = \( M r_y^2 \),  
\( M_{66} \) is the total mass moment of inertia about the \( z \) axis = \( M r_z^2 \),  
\( r_x \) is the radius of gyration about the \( x \) axis,  
\( r_y \) is the radius of gyration about the \( y \) axis, and  
\( r_z \) is the radius of gyration about the \( z \) axis.

The added mass terms of Eq. (35) will be

\[
M_{a11} = 0.257 r \xi C_m - 1 [p x_{\text{surge}}] \\
M_{a33} = 0.25 n D^2 [C_m - 1] p x_{\text{heave}}
\]

\( M_{a51} \) is the added mass moment of inertia in the pitch degree of freedom due to hydrodynamic force in the surge direction. \( M_{a53} \) is the added mass in the pitch degree of freedom due to hydrodynamic force in the heave direction. The presence of off-
diagonal terms in the mass matrix indicates a contribution in the added mass due to the hydrodynamic loading. The loading will be attracted only in the surge, heave and pitch degrees of freedom due to the unidirectional wave acting in the surge direction on a symmetric configuration of the platform about the \( x \) and \( z \) axes).

7. Damping matrix, \([C]\)

Assuming \([C]\) to be proportional to \([K]\) and \([M]\), the elements of \([C]\) are determined by the equation given below, using the orthogonal properties of \([M]\) and \([K]\):

\[
\langle \dot{D} \rangle = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \gamma = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

(36)

4 is taken as 0.05. This matrix is calculated based on the initial values of \([K]\) and \([M]\) only.

8. Hydrodynamic force vector, \([F(t)]\)

Water particle kinematics are evaluated using Airy’s linear wave theory. This description assumes the wave form whose wave height, \( H \), is small in comparison to its wave length, \( L \), and water depth, \( d \). Knowing the water particle kinematics, the hydrodynamic force vector is calculated in each degree of freedom. According to Morison’s equation, the intensity of wave force per unit length on the structure is given as:

\[
f(x, y, t) = 0.5 \rho_w C_D (\dot{u} - \dot{x} + \dot{u}_c) |\dot{u} - \dot{x} + \dot{u}_c| + 0.25 \pi D^2 \rho_w C_m |\ddot{x} + \ddot{u}_c| + 0.25 \pi D^2 [C_m - 1] \rho_w \gamma
\]

(37)

where

- \( i_{c} \) is the current velocity,
- \( \dot{u} \) is the horizontal water particle velocity,
- \( \dot{x} \) is the horizontal structural velocity,
- \( D \) is the diameter of the column,
- \( \ddot{x} \) is the horizontal structural acceleration, and
- \( \ddot{u} \) is the horizontal water particle acceleration.

The last term in Eq. (37) is the added mass term and a positive sign is used when the water surface is below the MSL and a negative sign is used when water surface is above the MSL. The contribution of added mass up to the MSL has already been considered along with structural mass.

The force vector \([F(t)]\) is given as:
The hydrodynamic force attracted by the members in the surge, sway and heave degrees of freedom are computed and designated as $F_{11}$, $F_{21}$ and $F_{31}$, respectively. The moment of these forces about the $x$, $y$ and $z$ axes are designated as $F_{41}$, $F_{51}$ and $F_{61}$, respectively.

9. Equation of motion

The equation of motion of the triangular TLP under a regular wave is given as:

$$[M]\{\dot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{F(t)\}$$

(39)

where

$\{X\}$ is the structural displacement vector,

$\{\dot{X}\}$ is the structural velocity vector, and

$\{\ddot{X}\}$ is the structural acceleration vector.

As the force vector is response dependent and fluctuations in added mass are included at every time instant, the force vector is changing at every time step. Hence, the solution procedure becomes iterative and is solved by using Newmark's $\beta$ time integration procedure.

10. Solution of the equation of motion in the time domain

The equation of motion, having time dependent components, can be solved by using Newmark's $\beta$ time integration procedure. This procedure incorporates changes (a) in stiffness coefficients which varies with tether tension, (b) in added mass varying with sea surface fluctuations, (c) in evaluation of wave forces at the instantaneous position of the displaced structure, (d) due to set-down effects of the structure. Wave loading constitutes the primary loading on offshore structures. The dynamic behaviour of these structures is, therefore, of design interest. When the dynamic response predominates, the behaviour under wave loading becomes nonlinear because the drag component of the wave load, according to Morison's equation, varies with the square of the velocity of the water particle relative to the structure. At each step, the force vector is updated to take into account the change in the tether
tension. The algorithm based on Newmark’s β method for solving the equation of motion is given below:

Step 1. The stiffness matrix \([K]\), the damping matrix \([C]\), the mass matrix \([M]\), the initial displacement vector \({X_0}\), and the initial velocity vector \({\dot{X}}_0\) are given as the known input data.

Step 2. The force vector \({F(t)}\) is calculated as in Section 8.

Step 3. The initial acceleration vector is then calculated.

Step 4. \(\dot{K}\) is evaluated.

Step 5. For each time step, the \(F(t), X, \dot{X}\) and \(\ddot{X}\) are evaluated at \((M-A?)\).

Step 6. The values of \(X, \dot{X}, \ddot{X}\), which are calculated at the time step \(M-A?\), are used to evaluate \(F_{t+At}\) such that convergence is achieved to the accuracy of 0.01%, before going to the next time step, otherwise iteration is carried out. Since the \(K\) of the TLP is response dependent, the new \(K\) is generated and that is used from Step 4 onwards.

11. Numerical studies and discussion

The numerical studies are conducted to compare the natural periods of the proposed triangular TLP with that of the four-legged TLP so as to have the natural time periods in similar ranges. Table 1 shows the geometric properties of the four-legged TLP models taken for comparison. Table 2 shows the natural periods of the four-legged TLP and Table 3 shows the natural periods of the triangular TLP models. On comparing these values, it is seen that the natural periods of the triangular TLP of case D-I of Table 3 are closer to the natural periods of the respective four-legged TLP (case D) given in Table 2 and hence case D-I of Table 3 triangular TLP is considered for further analysis.

<table>
<thead>
<tr>
<th>Description</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Case D</th>
<th>Case E</th>
<th>Case F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (kN)</td>
<td>351 600</td>
<td>321 000</td>
<td>209 500</td>
<td>330 000</td>
<td>330 000</td>
<td>370 000</td>
</tr>
<tr>
<td>(F_B) (kN)</td>
<td>521 600</td>
<td>470 440</td>
<td>334 000</td>
<td>0625 500</td>
<td>520 000</td>
<td>625 500</td>
</tr>
<tr>
<td>(T_0) (kN)</td>
<td>170 000</td>
<td>149 440</td>
<td>124 500</td>
<td>255 500</td>
<td>190 000</td>
<td>255 500</td>
</tr>
<tr>
<td>Tether length, (l) (m)</td>
<td>568</td>
<td>485</td>
<td>471</td>
<td>269</td>
<td>568</td>
<td>1166</td>
</tr>
<tr>
<td>Water depth, (d) (m)</td>
<td>600</td>
<td>527.8</td>
<td>500</td>
<td>300</td>
<td>600</td>
<td>1200</td>
</tr>
<tr>
<td>CG above keel (m)</td>
<td>28.44</td>
<td>38.32</td>
<td>26.6</td>
<td>27.47</td>
<td>28.5</td>
<td>30.31</td>
</tr>
<tr>
<td>(AE/l) (kN/m)</td>
<td>84 000</td>
<td>57 623</td>
<td>58 060</td>
<td>34 000</td>
<td>82 000</td>
<td>400 0820</td>
</tr>
<tr>
<td>Plan dimension (m)</td>
<td>70</td>
<td>99.4</td>
<td>92.5</td>
<td>75.66</td>
<td>78.5</td>
<td>83.5</td>
</tr>
<tr>
<td>(D) and (D_c) (m)</td>
<td>17</td>
<td>14.14</td>
<td>14.2</td>
<td>16.39</td>
<td>17</td>
<td>18.8</td>
</tr>
<tr>
<td>(r_x) (m)</td>
<td>35.1</td>
<td>55.93</td>
<td>29.15</td>
<td>35.1</td>
<td>35.1</td>
<td>35.1</td>
</tr>
<tr>
<td>(r_y) (m)</td>
<td>35.1</td>
<td>52.7</td>
<td>29.15</td>
<td>35.1</td>
<td>35.1</td>
<td>35.1</td>
</tr>
<tr>
<td>(r_z) (m)</td>
<td>35.1</td>
<td>67.15</td>
<td>32.1</td>
<td>42.4</td>
<td>42.4</td>
<td>42.4</td>
</tr>
</tbody>
</table>
Table 2
Natural wave periods (in seconds) of the four-legged tension leg platforms

<table>
<thead>
<tr>
<th>Case</th>
<th>Surge</th>
<th>Sway</th>
<th>Heave</th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>107</td>
<td>107</td>
<td>2.2</td>
<td>2.4</td>
<td>2.4</td>
<td>87</td>
</tr>
<tr>
<td>B</td>
<td>97</td>
<td>97</td>
<td>2.91</td>
<td>2.98</td>
<td>3.14</td>
<td>85</td>
</tr>
<tr>
<td>C</td>
<td>92.5</td>
<td>92.5</td>
<td>2</td>
<td>2.2</td>
<td>2.2</td>
<td>71</td>
</tr>
<tr>
<td>D</td>
<td>85</td>
<td>85</td>
<td>1.8</td>
<td>2.3</td>
<td>2.3</td>
<td>74</td>
</tr>
<tr>
<td>E</td>
<td>107</td>
<td>107</td>
<td>2.2</td>
<td>2.4</td>
<td>2.4</td>
<td>87</td>
</tr>
<tr>
<td>F</td>
<td>140</td>
<td>140</td>
<td>3.5</td>
<td>3.5</td>
<td>2.5</td>
<td>116</td>
</tr>
</tbody>
</table>

The stiffness matrix has been considered as coupled (Morgan and Malaeb, 1983).

Table 3
Natural wave periods (in seconds) of the equivalent triangular tension leg platforms

<table>
<thead>
<tr>
<th>Case</th>
<th>Surge</th>
<th>Sway</th>
<th>Heave</th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
</tr>
</thead>
<tbody>
<tr>
<td>A I</td>
<td>98</td>
<td>98</td>
<td>1.92</td>
<td>2.11</td>
<td>2.11</td>
<td>81</td>
</tr>
<tr>
<td>A II</td>
<td>118.5</td>
<td>118.5</td>
<td>1.98</td>
<td>2.19</td>
<td>2.19</td>
<td>98</td>
</tr>
<tr>
<td>B I</td>
<td>84</td>
<td>84</td>
<td>2.34</td>
<td>2.35</td>
<td>2.51</td>
<td>79</td>
</tr>
<tr>
<td>B II</td>
<td>100</td>
<td>100</td>
<td>2.41</td>
<td>2.41</td>
<td>2.59</td>
<td>95</td>
</tr>
<tr>
<td>C I</td>
<td>85</td>
<td>85</td>
<td>1.92</td>
<td>1.96</td>
<td>1.96</td>
<td>68</td>
</tr>
<tr>
<td>C II</td>
<td>102</td>
<td>102</td>
<td>2.01</td>
<td>2.07</td>
<td>2.07</td>
<td>83</td>
</tr>
<tr>
<td>D I</td>
<td>87.2</td>
<td>87.2</td>
<td>1.96</td>
<td>2.155</td>
<td>2.155</td>
<td>77.5</td>
</tr>
<tr>
<td>D II</td>
<td>94</td>
<td>94</td>
<td>1.75</td>
<td>2.18</td>
<td>2.18</td>
<td>83</td>
</tr>
<tr>
<td>E I</td>
<td>97</td>
<td>97</td>
<td>1.92</td>
<td>2.06</td>
<td>2.06</td>
<td>80.5</td>
</tr>
<tr>
<td>E II</td>
<td>114.2</td>
<td>114.2</td>
<td>2.01</td>
<td>2.1</td>
<td>2.1</td>
<td>97</td>
</tr>
<tr>
<td>F I</td>
<td>132</td>
<td>132</td>
<td>3.11</td>
<td>3.11</td>
<td>3.12</td>
<td>107</td>
</tr>
<tr>
<td>F II</td>
<td>158</td>
<td>158</td>
<td>3.85</td>
<td>3.85</td>
<td>3.87</td>
<td>121</td>
</tr>
</tbody>
</table>

I: Total $T_0$ is kept the same.
II: $T_0$ per tether is kept the same.
The stiffness matrix has been considered as coupled.

11.1. Response of a four-legged TLP and the triangular TLP

The triangular TLP selected (case D-I of Table 3) is analysed under regular wave loading and its structural response is compared with that of a four-legged TLP (case D of Table 2). Table 4 gives the geometric properties of both these models. The wave heights and wave periods are selected closer to the natural periods of these TLPs so as to study the near-resonating structural response of these structures, if any. Table 5 gives the hydrodynamic data considered for wave force evaluation.

11.2. Coupled surge response

The maximum and minimum values of the coupled surge response of the four-legged and the triangular TLPs are shown in Table 6. It is seen from the maximum positive values that, for the same wave period of 10 s and with the wave height of
Table 4
Properties of the TLP models taken for comparison of response

<table>
<thead>
<tr>
<th>Description</th>
<th>Four-legged TLP (case D)</th>
<th>Triangular TLP (case D-I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (kN)</td>
<td>330 (330)</td>
<td>330 (330)</td>
</tr>
<tr>
<td>$P_n$ (kN)</td>
<td>465 500</td>
<td>465 500</td>
</tr>
<tr>
<td>$T_0$ (kN)</td>
<td>135 500</td>
<td>135 500</td>
</tr>
<tr>
<td>Tether length, l (m)</td>
<td>269</td>
<td>269</td>
</tr>
<tr>
<td>CG above keel (m)</td>
<td>27.47</td>
<td>27.47</td>
</tr>
<tr>
<td>$AE/l$ (kN/m)</td>
<td>34 0</td>
<td>34 0</td>
</tr>
<tr>
<td>Plan dimension (m)</td>
<td>75.66</td>
<td>75.66</td>
</tr>
<tr>
<td>$D$ and $D_e$ (m)</td>
<td>16.39</td>
<td>16.39</td>
</tr>
<tr>
<td>$r_x$ (m)</td>
<td>35.1</td>
<td>35.1</td>
</tr>
<tr>
<td>$r_z$ (m)</td>
<td>42.4</td>
<td>42.4</td>
</tr>
</tbody>
</table>

Water depth=300 m.

Table 5
Hydrodynamic data considered for force evaluation

<table>
<thead>
<tr>
<th>Description</th>
<th>Wave height, $H$ (m)</th>
<th>Wave period, $P$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Case II</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Case III</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>Case IV</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>Case V</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>Case VI</td>
<td>12</td>
<td>15</td>
</tr>
</tbody>
</table>

Water depth, $d=300$ m; $C_d=1.0$; $C_m=2.0$.

Table 6
Coupled surge response (in metres) of the tension leg platform models

<table>
<thead>
<tr>
<th>$H$-$P$ combination</th>
<th>Maximum positive value</th>
<th>Maximum negative value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Four-legged</td>
<td>Triangular</td>
</tr>
<tr>
<td>I</td>
<td>1.65</td>
<td>1.40</td>
</tr>
<tr>
<td>II</td>
<td>1.82</td>
<td>1.43</td>
</tr>
<tr>
<td>III</td>
<td>2.023</td>
<td>1.58</td>
</tr>
<tr>
<td>IV</td>
<td>1.75</td>
<td>1.42</td>
</tr>
<tr>
<td>V</td>
<td>2.34</td>
<td>1.845</td>
</tr>
<tr>
<td>VI</td>
<td>2.91</td>
<td>2.295</td>
</tr>
</tbody>
</table>
8 m, the triangular TLP shows a 15% lower response than that of the four-legged TLP; for the 10-m wave height it is 21% lower, and for the 12-m wave height it is 22% lower than that of the four-legged TLP. For the wave period of 15 s and with wave height of 8 m, the triangular TLP shows 18.8% lower response than that of the four-legged TLP; for the 10-m wave height it is 21% lower, and for the 12-m wave height it is 21.1% lower than that of the four-legged TLP.

However, for the same wave period of 10 s, by increasing the wave height from 8 to 10 m (i.e. an increase of 25%), the maximum positive surge response of the four-legged TLP increases by 10.3%, whereas in the triangular TLP, it increases by 2.1%. On increasing the wave height from 8 to 12 m (i.e an increase of 50%) for the same wave period of 10 s, the coupled surge response of the four-legged TLP increases by 22.6%, whereas for the triangular TLP, it increases by 12.8%. For the wave period of 15 s, for an increase of 25% (from 8 to 10 m) in the wave height, the response increases by 33.7% for the four-legged TLP and by 29.9% for the triangular TLP, and by increasing the wave height by 50% (from 8 to 12 m), the response increases by 66% for the four-legged TLP and by 61.6% for the triangular TLP. The time histories of the coupled surge response have been plotted in Figs. 11-16. The regular wave of the 10-s wave period causes a more positive (surge) offset than the 15-s wave period waves. This could be because the lower wave period is closer to the natural periods of the TLP causing a higher response.

It is seen that the coupled surge response of a triangular TLP is less than that of a four-legged TLP, taken to be an equivalent model for comparing the responses.

![Fig. 11. Coupled surge response of triangular and square TLPs: wave height 8 m; wave period 10 s.](image)
Fig. 12. Coupled surge response of triangular and square TLPs: wave height 10 m; wave period 10 s.

Fig. 13. Coupled surge response of triangular and square TLPs: wave height 12 m; wave period 10 s.
Fig. 14. Coupled surge response of triangular and square TLPs: wave height 8 m; wave period 15 s.

Fig. 15. Coupled surge response of triangular and square TLPs: wave height 10 m; wave period 15 s.
11.3. Coupled heave response

The maximum positive and minimum negative values of the coupled heave response of the four-legged and the triangular TLPs are shown in Table 7. It is seen from the maximum positive values that for the same wave period of 10 s and with the wave height of 8 m, the triangular TLP shows a 24.8% lower response than that of the four-legged TLP; for the 10-m wave height it is 24.2% lower, and for the 12-m wave height it is 29.7% lower than that of four-legged TLP. For the wave period

<table>
<thead>
<tr>
<th>H-P combinations</th>
<th>Maximum positive value</th>
<th>Maximum negative value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Four-legged</td>
<td>Triangular</td>
</tr>
<tr>
<td>I</td>
<td>1.956</td>
<td>1.47</td>
</tr>
<tr>
<td>II</td>
<td>2.315</td>
<td>1.755</td>
</tr>
<tr>
<td>III</td>
<td>2.64</td>
<td>1.857</td>
</tr>
<tr>
<td>IV</td>
<td>2.2</td>
<td>1.674</td>
</tr>
<tr>
<td>V</td>
<td>2.66</td>
<td>1.93</td>
</tr>
<tr>
<td>VI</td>
<td>3.165</td>
<td>2.311</td>
</tr>
</tbody>
</table>
of 15 s and with wave height of 8 m, the triangular TLP shows 23.9% lower response than that of the four-legged TLP; for the 10-m wave height it is 27.4% lower, and for the 12-m wave height it is 27% lower than that of the four-legged TLP.

However, for the same wave period of 10 s, by increasing the wave height from 8 to 10 m (i.e. increasing by 25%), the response of the four-legged TLP increases by 18.3%, whereas in the triangular TLP, it increases by 19.4%. On increasing the wave height from 8 to 12 m (i.e. increasing by 50%) for the same wave period of 10 s, the coupled heave response of the four-legged TLP increases by 35%, whereas for the triangular TLP, it increases by 26.3%. For the wave period of 15 s, for an increase of 25% (from 8 to 10 m) in the wave height, the response increases by 20.9% for the four-legged TLP and by 15.3% for the triangular TLP and by increasing the wave height by 50% (from 8 to 12 m), the response increases by 43.9% for the four-legged TLP and by 38% for the triangular TLP. The time histories of the coupled heave response have been plotted in Figs. 17-22. The heave response is more in the four-legged (square) TLP than in the triangular TLP for waves of 15 s and 10 s periods. The heave response appears to have a mean value of nearly zero.

It is seen that the coupled heave response of a triangular TLP is less than that of a four-legged TLP, taken to be an equivalent model for comparing the responses.

11.4. Coupled pitch response

The maximum positive and minimum negative values of coupled pitch response of the four-legged and the triangular TLPs are shown in Table 8. It is seen from the

![Fig. 17. Coupled heave response of triangular and square TLPs: wave height 8 m; wave period 10 s.](image)
Fig. 18. Coupled heave response of triangular and square TLPs: wave height 10 m; wave period 10 s.

Fig. 19. Coupled heave response of triangular and square TLPs: wave height 12 m; wave period 10 s.
Fig. 20. Coupled heave response of triangular and square TLPs: wave height 8 m; wave period 15 s.

Fig. 21. Coupled heave response of triangular and square TLPs: wave height 10 m; wave period 15 s.
maximum positive values that for the same wave period of 10 s and with wave height of 8 m, the triangular TLP shows a 15% higher response than that of the four-legged TLP; for the 10-m wave height it is 4% higher, and for the 12-m wave height it is 2.3% higher than that of the four-legged TLP. For the wave period of 15 s and with wave height of 8 m, the triangular TLP shows 12.1% higher response than that of the four-legged TLP; for the 10-m wave height it is 3.4% higher, and for the 12-m wave height it is 5.5% higher than that of the four-legged TLP.

However, for the same wave period of 10 s, by increasing the wave height from 8 to 10 m (i.e. increasing by 25%), the maximum positive pitch response in the four-
legged TLP increases by 26.7%, whereas in the triangular TLP, it increases by 14.5%. On increasing the wave height from 8 to 12 m (i.e. increasing by 50%) for the same wave period of 10 s, the coupled pitch response of the four-legged TLP increases by 43.3%, whereas for the triangular TLP, it increases by 27.5%. For the wave period of 15 s, for an increase of 25% (from 8 to 10 m) in the wave height, the maximum positive coupled pitch response increases by 33.3% for the four-legged TLP and by 23% for the triangular TLP and by increasing the wave height by 50% (from 8 to 12 m), the response increases by 75% for the four-legged TLP and by 64.9% for the triangular TLP.

It is seen that the coupled pitch response of a triangular TLP is higher than that of a four-legged TLP, taken to be an equivalent model for comparing the responses.

12. Conclusions

The triangular TLP exhibits a lower response in the surge and heave degrees of freedom than that of the four-legged (square) TLP considered for comparing the response, under a regular wave. However, the triangular TLP attracts more forces in the pitch degree of freedom and the response in this degree of freedom is more than that of the four-legged (square) TLP. The pontoons, which are neither perpendicular nor parallel to the unidirectional wave, attract forces in the surge direction, which results in the moment about the sway axis (pitch direction). Hence, the triangular TLP, which has inclined (in plan) pontoons, attracts more force in the pitch and, therefore, its response in this degree of freedom is more than that of the four-legged (square) TLP, in which all the pontoons are either perpendicular or parallel to the unidirectional wave.

References


