Cross-section deformation of tubular composite shafts subjected to static loading conditions

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Abstract

Theoretical and experimental studies have been carried out on deflection and cross-section deformation of tubular composite shafts subjected to point static loading. Out of plane displacement, which is absent for metallic shaft/beams, is observed. Experimental result for (±45°)2 shaft compares qualitatively with that obtained theoretically.

1. Introduction

Generally composite shafts are thin tubular shafts, for such, studying cross-section deformation is important. Formulations based on beam theories fail to predict shaft cross-section deformation. However, formulations based on shell elements are most suitable for studying shaft cross-section deformation, but they are computationally costlier compared to beam formulations. Recently, with the advances in computer technology, memory and computational time become minor problems. This facilitates the use of shell finite elements in several applications. In this study, Ahmed type degenerated isoparametric shell element which takes into account transverse shear deformation, geometric nonlinearity, dynamic behavior, arbitrary lamination scheme and lamina properties is adopted. Theoretical and experimental results have been compared.

2. Analysis

Ahmed type 'degenerated shell element', which has become popular in recent years (Huang, 1989; Richardet et al., 2000; Gubran, 2000), is considered. This is an isoparametric shell element based on an independent rotation and translation displacement interpolation. In this element, the Mindlin-type theory is
employed. The three dimensional stress and strain conditions are degenerated to shell behavior by adopting assumptions representing a typical shell behavior. These assumptions are, first the normal to the middle surface remain straight after deformation but not necessary normal to the mid-surface and second, stresses normal to the mid-surface are negligible.

Referring to Fig. 1, quantities \( u, v \) and \( w \) are the nodal displacements along the global coordinates \( x, y \) and \( z \) respectively. The shape functions \( N_k \) are expressed in terms of the curvilinear coordinate \( \xi, \eta, \zeta \) which are known as natural coordination system. Local coordination system \( x_0^0, y_0^0, z_0^0 \) are used to define local stresses and strains at any point within the shell element. The local coordination system varies through the shell and is used to define the direction cosine matrix which is used for transformation from local to global coordinate system.

The vectors \( v_1^0, v_2^0 \) and \( v_3^0 \) are defined in \( x', y', \) and \( z^0 \) directions and can be given by,

\[
\begin{align*}
\mathbf{v}_1^0 & = \xi \times \mathbf{R} = \begin{bmatrix} \frac{\partial x}{\partial \xi} \\ \frac{\partial y}{\partial \xi} \\ \frac{\partial z}{\partial \xi} \end{bmatrix} \times \begin{bmatrix} \alpha x + \alpha y + \alpha z \\ \alpha y + \alpha \eta + \alpha \zeta \\ \alpha z + \alpha \eta + \alpha \zeta \end{bmatrix} \\
\mathbf{v}_2^0 & = \zeta \times \mathbf{R} = \begin{bmatrix} \frac{\partial x}{\partial \zeta} \\ \frac{\partial y}{\partial \zeta} \\ \frac{\partial z}{\partial \zeta} \end{bmatrix} \times \begin{bmatrix} \alpha x + \alpha y + \alpha z \\ \alpha y + \alpha \eta + \alpha \zeta \\ \alpha z + \alpha \eta + \alpha \zeta \end{bmatrix} \\
\mathbf{v}_3^0 & = \eta \times \mathbf{R} = \begin{bmatrix} \frac{\partial x}{\partial \eta} \\ \frac{\partial y}{\partial \eta} \\ \frac{\partial z}{\partial \eta} \end{bmatrix} \times \begin{bmatrix} \alpha x + \alpha y + \alpha z \\ \alpha y + \alpha \eta + \alpha \zeta \\ \alpha z + \alpha \eta + \alpha \zeta \end{bmatrix}
\end{align*}
\]

The direction cosines \( h \) can be given by \( h = [v_1, v_2, v_3] \) where \( v_1, v_2, v_3 \) are unit vectors along \( x', y', z^0 \) axes and normalized from \( v_1, v_2, v_3 \) respectively. The nodal Cartesian coordinate system is the local Cartesian coordinate system associated with each nodal point of the shell element at its origin at the shell mid-surface. The vector \( \mathbf{v} \) is constructed for the nodal coordinates of the top and bottom surfaces at node \( k \), so that,

\[
\mathbf{v}_j = \Delta r_j / (|\Delta r|^2 + |\Delta r|^2 + |\mathbf{A}|^2)^{1/2}
\]

where \( \mathbf{A} = x_k \mathbf{e}_x - x_k \mathbf{e}_x \) (\( i = 1, 2, 3 \)).

The vector \( \mathbf{v} \) is perpendicular to \( \mathbf{v}_j \) and parallel to the global \( x-z \) plane so that,

\[
\mathbf{v}_f = \mathbf{j} \times |\mathbf{j} \times \mathbf{v}_j|
\]
or if $v^k_3$ is in the y-direction

$$v^*_j = v^*_j \times \hat{i}/|v^*_j \times \hat{i}|$$

(6)

where $\hat{i}$ and $\hat{j}$ are the unit vectors along the x, y directions respectively. The vector $\hat{\iota}$ is normal to the plane defined by $\hat{i}$ and $\hat{j}$ as

$$\iota_k = v^\iota \times v^\iota|v^\iota \times v^\iota|$$

(7)

The superscripts refer to node number.

The vector $v^\iota_k$ defines the direction of the normal at node $k$ which is not necessarily perpendicular to the mid-surface at $k$. Vectors $v^\iota_k$ and $v^\iota_l$ define the rotations $a^\iota_k$ and $a^\iota_l$ respectively.

The element geometry can be defined in terms of the global coordinates of pairs of points on the top and bottom surface at each node (as shown in Fig. 1) as,

$$x_i = \sum_{k=1}^n N^k(\xi, \eta) \left[ 1 + \frac{\xi}{2} x_{i_{\text{top}}} + \frac{1 - \xi}{2} x_{i_{\text{bot}}} \right]$$

(8)

where $x_i$ is the Cartesian coordinate of any point in the element ($x_1 = x, x_2 = y, x_3 = z$); $x_{i_k}$ is the Cartesian coordinate of nodal point $k$; $f$ is the distance from the middle surface and $N^k\partial \eta$; $g^\iota$ is the two dimensional interpolation function corresponding to node $k$ which can be taken as, $TV^n = f k d l \partial f b d 2 \partial d 1 = f 2 \left( 1 - \frac{f}{2} \right)$.

At any point $k$ along the thickness, the displacement field $u^i_k$, can be expressed in global coordinate (Huang, 1989; Chao and Reddy, 1984) as,

$$u^i_k = u^0_k + \frac{f}{2} h^k (v^k_1 x^i_1 - v^k_2 x^i_2)$$

Alternatively,

$$u_i = \sum_{i=1}^n N^\iota V j d^k \quad (i = 1, 2, 3)$$

(9)

where

$$d^k \iota \iota, u^k_1, u^k_2, u^k_3, k^i_{\iota}, o^\iota = X$$

(10)

Here $u^k_{0i}$ is the displacement of Mi nodal point, $a_1, a_2$ are the rotations about $\iota$ and $\iota$ respectively, $h^k$ is the thickness of the shell in f direction at node $k$. Upon finite element discretisation, the following equations are obtained:

$$[K^I] \{d_I\} = \{F_I\}$$

where $[K^\iota I]$ is the stiffness matrix linking nodes $i$ and $j$ and $f^I g$ is the applied load matrix.

3. Results and discussion

This paper presents the study carried out on two shafts. The first one, has been studied theoretically, is of single ply (i.e. $45^\circ$). However, the second has been studied theoretically and experimentally, is of four plies (i.e. $\pm 45^\circ$). Both of the shafts are of mean radius of 0.05 m, thickness of 0.005 m and a span length of 1 m. Shafts material properties are given in Table 1. As shown in Fig. 2, a point transverse load of 100 N is applied at the shafts mid-span. Theoretically, the analysis has been carried out using finite shell element.
Table 1
Shaft material properties

<table>
<thead>
<tr>
<th>Major elastic modulus, ( E_1 ) (GPa)</th>
<th>Minor elastic modulus, ( E_2 ) (GPa)</th>
<th>Major Poisson’s ratio, ( v_1 )</th>
<th>Shear modulus, ( G_{12} = G_{13} ) (GPa)</th>
<th>Density, ( P ) (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>10</td>
<td>0.25</td>
<td>7</td>
<td>1500</td>
</tr>
</tbody>
</table>

mid-span section

Fig. 2. Shaft loading conditions (a) in-plane deflection and (b) out of plane deflection.

model of 6 x 4 mesh (as shown in Fig. 3) i.e., six elements in the circumferential direction and four elements along the shaft axial length. A total of 24 elements have been used with nine nodes per element and five degrees of freedom per node. A total of 108 nodes with 12 nodes per each circumferential direction are used. In Figs. 4 and 5 abscissa represents node number. The in-plane, longitudinal and the out of plane displacements along \( x, y \) and \( z \) respectively are represented along the ordinate. It is to be observed that nodes (1-12) are distributed circumferentially at the same axial position along the shaft length (i.e., Section 1). Similarly nodes \( 13-24; 25-36; \ldots; 97-108 \) are respectively at sections 2, 3, \ldots, 9. Ideally (i.e., for no deformation of the shaft cross-section) different nodes located at the same axial position have to have the

![Fig. 3. Shaft finite element mesh.](image)
same displacement which will be represented by horizontal line in Figs. 4 and 5. However, results presented in Figs. 4 and 5 for both shafts show that different nodes, located at the same axial position, have different displacements which indicate deformation of the shaft cross-section. This deformation, for the present case,
is maximum at the shaft mid-span. The distortion of the cross-section at the mid-span for the (45°) and (±45°)2 shafts are shown in Figs. 4a and 5a respectively. Results also show that the in-plane deflection of (±45°)2 shaft is less compared to that of single ply (45°), however, cross-section deformation is more for (±45°)2 shaft.

Another important observation for the (45°) ply angle (Fig. 4b), is that while the applied load is in the x-y plane (in-plane) the shaft does deflect in the y-z plane (out of plane) also. The out of plane deflection can not be predicted by classical beam theory. Out of plane deflection is small compared to in-plane deflection and depends mainly on the degree of coupling present in different configurations. It is absent for 0° and 90° ply angles and minimum for angled ply configurations.

4. Experimental study

A composite shaft of (±45°), fibre angle and an average thickness of 0.005 m, mean radius of 0.05 m and a span length of 1 m made of carbon fibres, Hercules 929-6H(6K) and epoxy resin Ciba Geigy HT-972 is used. Two mild steel flanges are fixed at both ends of the shaft to mount it on the bearings. A rigid mild steel frame is fabricated for fixing the probes against the four points of measurements, the frame is bolted to a heavy rigid base. As shown in Fig. 6, the shaft is mounted on self-aligning ball bearings bolted to the C-shaped mild steel channel which in turn is bolted to the heavy steel base. The static load was applied by putting weights at the upper surface of the shaft through weight attachment. Proximity transducers are used to measure shaft deflection in all the four points around the shaft cross-section simultaneously. The signal from the proximity probes goes through the proximitors and vibration monitors to a digital oscilloscopes. Initially prior to the measurements, the signal is fed to two channel digital vector filter to ensure that the initial gap between each probe and the shaft surface is within the linear range of the probe calibration. Later, the signals from different probes are fed to different channels of digital oscilloscopes as shown in Fig. 6. The static deflection of four points along the shaft cross-section located at the shaft mid-span under...
static loading is measured. The distribution of these points is such that, two points (180° apart) are located in the plane of bending. Two other points (180° apart) are located in a plane perpendicular to the plane of bending. The load is applied in increments through weight mounting attachment and the corresponding voltage signal is recorded from the digital oscilloscope. This completes one set of readings. In order to avoid variation in results due to any bow of the shaft or any defects present in the shaft (like delamination, or any manufacturing defects), the shaft is rotated 90° so that different points of measurement get interchanged and the above procedure is repeated. The sample result presented in Fig. 7 shows that the displacement of upper point is more than the displacement of the lower point. This gives an indication of the deformation of the shaft cross-section.

The deformation of the shaft cross-section obtained theoretically and experimentally shows the same behavior of greater displacement of the upper points compared to that of the lower points and the outward displacement of points at the sides of the shaft cross-section. However, numerically the displacements obtained experimentally are more. This can be explained as follows, in theoretical modeling, the shaft is assumed to be supported on rigid supports, i.e., zero deflection of the shaft ends. However experimentally, the shaft is supported on self-aligning bearings. Bearing stiffness is not known and it has not been included in theoretical modeling. There is also possibilities of some clearance in the bearings and some defects in the shaft. These factors could give rise to greater deflection as measured experimentally. It is therefore difficult
to compare theoretical and experimental results for shaft deflection. However, to have some basis for comparison between theoretical and experimental results, the differences between the displacements of the upper (U55) and lower (L49) points have been compared and presented in Fig. 8. It is clear from this figure, that the results match well with maximum difference of 10-12% between theoretical and experimental values. This establishes that the estimation of shaft distortion by theoretical FEM and experiments match reasonably well.

5. Conclusion

Theoretical and experimental studies have been carried out on deflection and cross-section deformation for composite tubular shaft of $(\pm45^\circ)_2$. The shaft is mounted on self-aligning bearings and a static point load is applied at the shaft mid-span. Eddy current probes are used to measure the displacements of points located at the top and bottom on the shaft cross-section located at the mid-span. Shaft deflection and cross-section deformation obtained theoretically and experimentally have been compared. The followings are the main conclusions:

1. Out of plane displacement is clearly observed for 45° ply angle shaft, however, this displacement is small compared to that of in-plane displacement. Out of plane displacement is absent for 0° and 90° ply angles and minimum for shafts with angled ply configuration (like, i.e. $\pm45^\circ$).2.
2. Deflection of angled ply shaft (i.e. $\pm45^\circ$)2 is less compared to single ply shaft (i.e. 45°), however, shaft cross-section deformation is more in the first case.
3. Theoretical and experimental results for shaft cross-section deformation compared well with each other. The maximum difference is found to be about 12%.

References