Multiobjective load dispatch by fuzzy logic based searching weightage pattern

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Abstract

A multiobjective thermal power dispatch problem minimizes number of objectives viz cost and emission together while allocating the electricity demand among the committed generating units subject to physical and technological constraints. Such problems are solved to generate non-inferior solutions using weighting method or ε-constraint method. Afterwards the decision maker is provided with a set of simple but effective tools to choose the best alternative among non-inferior solutions. The generation of non-inferior solution requires an enormous amount of computation time when the number of objectives is more than two. In the paper, the multiobjective problem has been solved using a weighted technique. The Evolutionary optimization technique has been employed in which the 'preferred' weightage pattern has been searched to get the 'best' optimal solution in non-inferior domain. Decision making theories attempt to deal with the vagueness or fuzziness inherent in subjective or impressive determination of goals. So fuzzy set theory has been exploited to decide the 'preferred' optimal operating point. The non-inferior solution that attains maximum satisfaction level from the membership functions of the participating objectives has been adjudged the 'best' solution. The proposed method requires few search moves to get the optimal operating point in the non-inferior domain for any number of goals. The validity of the proposed method has been demonstrated on a 25 nodes IEEE system comprising five generators.

Keywords: Multiobjective optimization; Fuzzy set; Decision making; Membership function; Evolutionary optimization technique

1. Introduction

The methodology, which simultaneously satisfies multiple contradictory criteria/goals, is called multiobjective optimization. The consideration of many objectives accomplishes three major improvements in the problem solving. First, multiobjective programming promotes a more appropriate rate for the decision making process. Second, a wider range of alternative is usually identified when a multiobjective methodology is employed. Third, perception of a problem and modeling becomes more realistic for analysts if many objectives are considered.

In the past, many approaches and methods have been proposed to solve multiobjective problems [1]. These methods are broadly grouped under two major titles: 'non-interactive' and 'interactive' methods. In non-interactive methods, a global preference function of objective is identified and optimized with respect to the constraints. On the other hand, in the interactive methods, interacting with the decision maker identifies a local preference function or trade-off among objectives, and the solution process proceeds gradually towards the globally satisfactory solution. The interactive method can be characterized by the following procedure:

1) Find a non-inferior solution.
2) Interact with decision maker to obtain his reaction response to the solution, and
3) Repeat steps 1 and 2 until satisfaction is reached or until some other termination criterion is met [1].

The interactive methods are most often solved to find non-inferior solutions. Qualitatively, a non-inferior solution of a multiobjective problem is one where any
improvement of one objective function can be achieved only at the expense of another. The most widely used methods of generating such non-inferior solutions are the o-constraint method, the weighted minimax method and shifted minimax method [2].

Apart from heat, power utilities using fossil fuels as a primary energy source, produce particulates and gaseous pollutants. The particulates and the gaseous pollutants such as CO2, SOX and NOX cause detrimental effect on human beings. Pollution control agencies restrict the amount of emission of pollutants depending upon their relative harmfulness to human beings. So multiple criteria must be considered simultaneously to attain a more meaningful, practical, optimal schedule of operation. Nanda et al. [3] have proposed a goal programming technique to solve the economic emission load dispatch problem for thermal generating units running with natural gas and fuel oil. Yokoyama et al. [4] have proposed an efficient algorithm to obtain the optimal power flow in power system operation and planning phases by solving a multiobjective optimization problem. Minimum influence on the environment was one of the objectives in deciding the optimal system operation. Karmanshahi et al. [5] presented a decision-making methodology to determine the optimal generation dispatch and environmental marginal cost for power system operation with multiple conflicting objectives. Kubokawa et al. [6] have proposed an efficient solution methodology for a class of multiobjective optimal power flow problem, which makes use of a heuristic search method. An optimal solution has been found in the proposed heuristic search method based on local information arbitrarily from a given set of objectives. Wong et al. [8] proposed a bi-criterion global optimization approach to determine the most appropriate generation dispatch solution taking into account fuel cost, environmental costs and security requirements of power networks. Dhillon et al. [7] have solved a stochastic economic emission load dispatch in which non-inferior solution has been generated by weighted minimax technique and the fuzzy set theory has been used for decision-making. In another attempt, non-inferior surface of the multiobjective thermal power dispatch problem has been generated using o-constraint method by Dhillon and Kothari [9]. The surrogate worth trade off method and utility approach have been utilized to choose the best solution.

The intent of the paper is to solve multiobjective thermal power dispatch problem having four objectives viz: the economic index and impact on environment due to NOX, SO2 and CO2 gaseous pollutants emission taken as individual objectives. Initially the multiobjective optimization problem is converted into single objective optimization problem using a weighted method. Fuzzy decision making theories attempt to deal with the vagueness or fuzziness inherent in subjective or imprecise determinations of preferences, constraints and goals. So fuzzy methodology has been put up for solving a decision-making problem involving a multiplicity of objectives and selection criteria for 'best' compromised solution. Since an enormous amount of computing time is required in the determination of the complete non-inferior solution surface. To reduce the computation time, evolutionary optimization technique is proposed to search for the optimal weight pattern in the non-inferior domain. In the proposed method, a hypercube of weight combination is formed around an initial search point. To continue the iterative process, another hypercube is formed around the relatively better 'preferred' point as compared to the previous one. It is repeated until the solution criteria for 'best' compromised solution is met. Being the imprecise nature of the decision maker's (DMS's) judgement, it is assumed that the DM has fuzzy goals for each of the objective functions. The fuzzy goals are quantified by defining their corresponding membership functions. So, the 'best' compromised solution is one, which provides the maximum satisfaction level from the membership function of the participating goals/objectives. The effectiveness of the proposed method is tested on IEEE 25-node system comprising five generators.

2. Multiobjective problem formulation

In the multiobjective problem formulation four important non-commensurable objectives in an electrical thermal power system are considered. These are economy and environmental impact because of NOX, SO2 and CO2 pollutants.

2.1. Economy objective

The fuel cost of a thermal unit is regarded as an essential criterion for economic feasibility. The fuel cost curve is assumed to be approximated by a quadratic function of generator power output, as $P_{Gi}$ as [15]:

$$F_i = \sum_{i=1}^{N} (a_i P_{Gi}^2 + b_i P_{Gi} + c_i) \ \text{$/h}$$(1)

where $a_i$, $b_i$ and $c_i$ are cost coefficients and $N$ is number of generators.

2.2. Environmental objectives

The emission curves can be directly related to the cost curve through the emission rate per M Kcal, which is a constant factor for a given type and grade of fuel. Therefore, the amount of NOX emission is given as a quadratic function of generator output, $P_{Gi}$:
\[ F_2 = \sum_{i=1}^{N} (d_1i P_{Gi}^2 + e_1i P_{Gi} + f_1i) \text{ kg/h} \]

where \( d_1i, e_1i \) and \( f_1i \) are NOx emission coefficients [8].

Similarly, the amount of SO2 emission is given as a quadratic function of generator output, \( PGI \):

\[ F_3 = \sum_{i=1}^{N} (d_2i P_{Gi}^2 + e_2i P_{Gi} + f_2i) \text{ kg/h} \]

where \( d_2i, e_2i \) and \( f_2i \) are SO2 emission coefficients [8].

The amount of CO2 emission is also represented as quadratic function of generator output, \( PGI \):

\[ F_4 = \sum_{i=1}^{N} (d_3i P_{Gi}^2 + e_3i P_{Gi} + f_3i) \text{ ton/h} \]

where \( d_3i, e_3i \) and \( f_3i \) are CO2 emission coefficients [8].

### 2.3. Constraints

To ensure a real and reactive power balance, the following equality constraints are imposed:

\[ \sum_{i=1}^{M} P_{Di} - \sum_{i=1}^{M} P_{Gi} + P_L = 0 \quad (5) \]
\[ \sum_{i=1}^{M} Q_{Di} - \sum_{i=1}^{M} Q_{Gi} + Q_L = 0 \quad (6) \]

where \( P_{Di} \) and \( Q_{Di} \) are real power and reactive power demands, respectively at \( i \)-th bus. \( P_{Gi} \) and \( Q_{Gi} \) are real power and reactive power generation, respectively at \( i \)-th bus. \( P_L \) and \( Q_L \) are real and reactive losses, respectively at \( i \)-th bus. \( M \) is total number of buses.

The inequality constraints imposed on generator output are:

\[ P_{Gi}^{imin} \leq P_{Gi} \leq P_{Gi}^{imax}, \quad i = 1, 2, \ldots, N \quad (7) \]
\[ Q_{Gi}^{imin} \leq Q_{Gi} \leq Q_{Gi}^{imax}, \quad i = 1, 2, \ldots, N \quad (8) \]

where \( P_{Gi}^{imin} \) and \( P_{Gi}^{imax} \) are lower and upper limits of active power generation, respectively. \( Q_{Gi}^{imin} \) and \( Q_{Gi}^{imax} \) are lower and upper limits of reactive power generation, respectively.

#### 2.3.1. Power transmission losses

The real and reactive power transmission losses, \( P_L \) and \( Q_L \) are given by following equations [13]:

\[ P_L = \sum_{i=1}^{M} \sum_{j=1}^{M} \left[ A_{ij} (P_{Pi} P_{Pj} + Q_{Pi} Q_{Pj}) + B_{ij} (Q_{Pi} P_{Pj} - P_{Pi} Q_{Pj}) \right] \quad (9) \]
\[ Q_L = \sum_{i=1}^{M} \sum_{j=1}^{M} \left[ C_{ij} (P_{Pi} P_{Pj} + Q_{Pi} Q_{Pj}) + D_{ij} (Q_{Pi} P_{Pj} - P_{Pi} Q_{Pj}) \right] \quad (10) \]

where \( P_{i} = P_{Gi} - P_{Di} \)
\( Q_{i} = Q_{Gi} - Q_{Di} \)

\[ A_{ij} = \frac{R_{ij}}{V_i V_j} \cos(\delta_i - S_j) \]
\[ B_{ij} = \frac{1}{V_i V_j} \sin(\delta_i - S_j) \]
\[ C_{ij} = \frac{X_{ij}}{V_i V_j} \cos(\delta_i - S_j) \]
\[ D_{ij} = \frac{X_{ij}}{V_i V_j} \sin(\delta_i - S_j) \]

\( \delta_i \) and \( \delta_j \) are load angles at \( i \)-th and \( j \)-th buses, respectively. \( V_i \) and \( V_j \) are voltage magnitude at \( i \)-th and \( j \)-th buses, respectively. \( R_{ij} \) is the real component of impedance bus matrix. \( X_{ij} \) is the reactive component of impedance bus matrix.

Aggregating Eqs. (1)–(10) the multiobjective optimization problem is defined as:

Minimize \([F_1(P_{Gi}), F_2(P_{Ga}), F_3(P_{Ga}), F_4(P_{Ga})]^{T}\) \quad (15a)

Subject to:

\[ \sum_{i=1}^{M} P_{Di} - \sum_{i=1}^{M} P_{Gi} + P_L = 0 \quad (15b) \]
\[ \sum_{i=1}^{M} Q_{Di} - \sum_{i=1}^{M} Q_{Gi} + Q_L = 0 \quad (15c) \]
\[ P_{Gi}^{imin} \leq P_{Gi} \leq P_{Gi}^{imax}, \quad i = 1, 2, \ldots, N \quad (15d) \]
\[ Q_{Gi}^{imin} \leq Q_{Gi} \leq Q_{Gi}^{imax}, \quad i = 1, 2, \ldots, N \quad (15e) \]

where \( F_1(P_{Ga}), F_2(P_{Ga}), F_3(P_{Ga}) \) and \( F_4(P_{Ga}) \) are the objective functions to be minimized over the set of admissible decision vector, \( P_{Gi} \) and \( Q_{Gi} \) ; \( i = 1, 2, \ldots, N \).

### 3. Weighting method

To generate the non-inferior solution, the multiobjective problem is converted into a scalar optimization problem and is given below:

Minimize \( \sum_{k=1}^{N} w_k r_k \) \quad (16a)

Subject to:

\[ \sum_{i=1}^{M} P_{Di} - \sum_{i=1}^{M} P_{Gi} + P_L = 0 \quad (16b) \]
\[ \sum_{i=1}^{M} Q_{Di} - \sum_{i=1}^{M} Q_{Gi} + Q_L = 0 \quad (16c) \]
\[ P_{Gi}^{imin} \leq P_{Gi} \leq P_{Gi}^{imax}, \quad i = 1, 2, \ldots, N \quad (16d) \]
\[ Q_{Gi}^{imin} \leq Q_{Gi} \leq Q_{Gi}^{imax}, \quad i = 1, 2, \ldots, N \quad (16e) \]
\[ \sum_{k=1}^{N} w_k = 1.0, \quad w_k \geq 0 \] \quad (16f)

where \( w_k \) are the levels of the weighting coefficients. \( L \) is the number of objectives.

This approach yields meaningful results to the decision maker (DM) when solved many times for different
value of $w_k; k = 1, 2, \ldots, L$. The values of the scalar optimization problem the Lagrangian function is defined as:

$$L(P_G; Q_G; l_p; l_Q) = \sum_{i=1}^{N} \frac{W_i}{F_i} + \lambda_G \left( \sum_{i=1}^{M} Q_{Gi} - \sum_{i=1}^{L} Q_{DL} - Q_{L} \right) + \lambda_P \left( \sum_{i=1}^{N} P_{Gi} - \sum_{i=1}^{M} P_{Di} - P_L \right)$$

(17)

where $L_p$ and $L_Q$ are the Lagrangian multipliers.

The necessary conditions to minimize the unconstrained Lagrangian function are

$$\frac{\partial L}{\partial P_i} = - \lambda_P \frac{\partial P_i}{\partial Q_{Gi}} + \lambda_Q \left( 1 - \frac{\partial Q_L}{\partial Q_{Gi}} \right) = 0 \quad i = 1, 2, \ldots, N$$

(18a)

$$\frac{\partial L}{\partial Q_{Gi}} = - \lambda_P \left( \frac{\partial P_i}{\partial Q_{Gi}} \right) + \lambda_Q \left( 1 - \frac{\partial Q_L}{\partial Q_{Gi}} \right) = 0 \quad i = 1, 2, \ldots, N$$

(18b)

The Newton-Raphson algorithm has been applied to obtain the solution, for the weight combinations generated during search moves.

4. Decision making

Considering the imprecise nature of the DM’s judgement, it is natural to assume that the DM may have fuzzy or imprecise goals for each objective function. The fuzzy sets are defined by equations called membership functions. These functions represent the degree of membership in certain fuzzy sets using values from 0 to 1 [10]. The membership value 0 indicates incompatibility with the sets, while 1 means full compatibility. By taking account of the minimum and maximum values of each objective function together with the rate of increase of membership satisfaction, the DM must determine the membership function $m(F_i)$ in a subjective manner.

$$m(F_i) = \begin{cases} 1; & F_i < \min_{i \in \text{set}} F_i, \text{set} = \{i \in \text{set} \} \\ \frac{F_i - \min_{i \in \text{set}} F_i}{F_{\text{max}} - \min_{i \in \text{set}} F_i}; & F_{\text{max}} \leq F_i \leq \max_{i \in \text{set}} F_i \\ 0; & \max_{i \in \text{set}} F_i < F_{\text{max}} \end{cases}$$

(19)

where $F_{\text{min}}$ and $F_{\text{max}}$ are the minimum and maximum values of $i$th objective function in which the solution is expected.

The value of the membership function indicates, how much (in scale from 0 to 1) a solution is satisfying the $F_i$ objective. The decision regarding the 'best' solution is made by the selection of minmax of membership function as defined below [11]:

$$\text{Max}_{j} \text{Min}_{k} [m(F_j)^k; j = 1, 2, \ldots, L; k = 1, 2, \ldots, 2^{L-1} + 1]$$

(20)

where $2^{L-1}$ is number of corner points of an $(L-1)$-dimensional hypercube.

5. Evolutionary search

Evolutionary optimization method [12] is proposed to search the optimal weight combination. In this method, $(2^{L-1} + 1)$ weight combinations are simulated at $2^{L-1}$ corner points of an $(L-1)$-dimensional hypercube centred on initial point $w_i (2^{L-1} + 1)$ non-inferior solutions are generated and membership functions are obtained using Eq. (19). The 'best' or 'preferred' non-inferior solution is identified using Eq. (20). To continue the iteration process, another hypercube is formed around the 'preferred' point.

The weights are generated as given below.

$$w_i = w_i + g_i; \quad i = 2, 3, \ldots, L \quad \text{and} \quad j = 1, 2, \ldots, 2^{L-1}$$

(21)

$$w_i = 1 - \sum_{i=2}^{L} w_i; \quad 7 = 1, 2, \ldots, 2^{L-1}$$

(22)

where $g$ is the distance of the corners of the hypercube from the point around which hypercube is generated. The $g$ matrix has been generated from possible combinations of binary bits. '0' bit is replaced by $-y$ and '1' bit is replaced by $+y$.

6. Algorithm

To implement the evolutionary search, stepwise procedure is outlined as below:

1) Input the data.
2) Find the minimum and maximum values of objectives $F_{\text{min}}$ and $F_{\text{max}}$; $i = 1, 2, \ldots, L$.
3) Set the initial centre $w_i$ of the hypercube; $i = 2, 3, \ldots, L$.
4) Set initial maximum value of membership function $m^p = 0$.
5) Initialize iteration counter, $r = 0$.
6) Increment the iteration counter, $r = r+1$. 
Table 1
Fuel cost ($/h) equations

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<tr>
<th>$F_1$</th>
<th>$F_1^+$</th>
<th>$P_{G1}$ + 8.4205</th>
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Table 2
NOX emission (kg/h) equations

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Table 3
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Table 4
CO$_2$ emission (kg/h) equations

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Table 5
Generation of weights at hypercube corners (two objectives)

<table>
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<tr>
<th>Hypercube corners</th>
<th>Possible combinations of one Binary point</th>
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<th>Possible generated weights at hypercube corners</th>
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<td>$w_1^c$</td>
<td>$w_1^c$</td>
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Table 6
Variations in weight search (two objectives)

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<th>$F_4$ ($$/h)$</th>
<th>$F_5$ ($$/h)$</th>
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Fig. 1. Generation of weights.
7. Test system and results

The validity of the proposed method is illustrated on a 25-bus, 35-lines IEEE system [14] comprising five-generator system [8]. The fuel cost, NO_x emission, SO_2 emission and CO_2 emission equations are given in Tables 1-4.

7.1. Minimum and maximum values of objectives

Minimum values of the objectives \(F_i\), \(i = 1, 2, \ldots, L\) are obtained by giving full weightage to one of the objectives and neglecting the others. When the given weight value is 1.0, it means that full weightage is given to the objective and when the weightage is zero, the objective is neglected. Owing to the conflicting nature of the objectives, \(F_2\) and \(F_3\) objectives will have maximum values when \(F_1\) objective is minimum.

7.2. Determination of optimal or 'best' solution

7.2.1. Case 1 (two objectives)

Here two objectives, cost and NO_x emission are considered which have weightages, \(w_1\) and \(w_2\), respectively. The minimum and maximum values of these objectives are given below.

\[
in = 4948.729 \quad \$=h, \quad F_{1\max} = 5170.30 \quad \$=h
\]

7.3. Generation of weights

Hypercube is formed around weight, \(w_2\) only. The one binary bit can be represented in 2^1 (two) possible different combinations. These two binary bit combinations are the corners of hypercube away from the centre of hypercube. The distance, \(g\) of hyper cube corners from its centre point \(w_2\) is generated from binary digits is given in Table 5. Fig. 1 and Table 5 show the generation of weights at the hypercube corners.

The values of cost and NO_x emission in each iteration of search are tabulated in Table 6. The membership functions, \(m(F_i)\) are also given in Table 6. Minimum

\[
F_{1}^{a} = 705.9319 \quad \text{kg}=h, \quad F_{1\max}^{X} - 8:0034 \quad \text{kg}=h
\]

![Fig. 2. Generation of weights.](image-url)
Table 8
Variation in weight search. (Three objectives)

<table>
<thead>
<tr>
<th>k</th>
<th>w^1</th>
<th>wk</th>
<th>wk</th>
<th>(p(F_k)^k)</th>
<th>(n(F_k)^k)</th>
<th>(p^{\text{min}})</th>
<th>(M^o)</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>0.50</td>
<td>0.26</td>
<td>0.24</td>
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<td>0.9861320</td>
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<td>0.2831344</td>
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<td>0.2952730</td>
</tr>
<tr>
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<td>0.3074356</td>
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<td>0.3208923</td>
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<td>0.3208923</td>
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<td>0.3634350</td>
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<tr>
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<td>0.34</td>
<td>0.16</td>
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<td>0.9670584</td>
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<td>0.4440799</td>
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<td>0.9378780</td>
<td>0.4512184</td>
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<tr>
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<td>0.09</td>
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<tr>
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<td>0.44</td>
<td>0.06</td>
<td>0.8929812</td>
<td>0.5998783</td>
<td>0.8717294</td>
<td>0.5998783</td>
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<tr>
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<td>0.45</td>
<td>0.05</td>
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<td>0.04</td>
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<td>0.7128852</td>
<td>0.7379513</td>
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<td>0.01</td>
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<td>0.739513</td>
<td>0.7128852</td>
<td>0.739513</td>
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Table 9
Variation of cost and emission due to NOX and SOX (three objectives)

<table>
<thead>
<tr>
<th>k</th>
<th>(k1) ($/h)</th>
<th>(Fk2) (kg/h)</th>
<th>(3k) (kg/h)</th>
<th>(k)</th>
<th>(1k) ($/h)</th>
<th>(F2k) (kg/h)</th>
<th>(F3k) (kg/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4975.613</td>
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<td>4973.763</td>
<td>886.832</td>
<td>3158.554</td>
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<td>4975.558</td>
<td>941.780</td>
<td>3037.942</td>
<td>14</td>
<td>4973.483</td>
<td>880.032</td>
<td>3182.645</td>
</tr>
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<td>4975.491</td>
<td>937.864</td>
<td>3043.234</td>
<td>15</td>
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<td>3211.360</td>
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<td>826.8136</td>
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<td>910.2518</td>
<td>3092.977</td>
<td>21</td>
<td>4972.237</td>
<td>815.0629</td>
<td>3450.074</td>
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<td>904.8778</td>
<td>3105.866</td>
<td>22</td>
<td>4972.113</td>
<td>802.3043</td>
<td>3490.074</td>
</tr>
<tr>
<td>11</td>
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<td>899.2078</td>
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<td>4971.987</td>
<td>792.0036</td>
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<td>24</td>
<td>4971.860</td>
<td>781.8036</td>
<td>3570.074</td>
</tr>
</tbody>
</table>

Table 10
Generation of weights at hypercube corners (three objectives)

| Hypercube corners | Possible combinations of two Binary point | Distance of hypercube corners from centre Possible generated weights at the hypercube corners |
|---|---|---|---|
| 1 | 0, 0 | \(-y\) \(+y\) | \(w_{2}\) \(-y\) \(+y\) |
| 2 | 0, 1 | \(-y\) \(+y\) | \(w_{2}\) \(+y\) \(-y\) |
| 3 | 1, 0 | \(+y\) \(-y\) | \(w_{2}\) \(+y\) \(-y\) |
| 4 | 1, 1 | \(+y\) \(+y\) | \(w_{2}\) \(+y\) \(-y\) |
membership function values from two objectives, \( m_k \) are also given in Table 6. The 'best' weights: \( w_1 = 0.45 \), \( w_2 = 0.55 \) are obtained after six iterations. The 'best' power schedule and voltage profile corresponding to 'best' weight combination is shown in Table 7.

### 7.2.2. Case 2 (three objectives)

Three objectives, cost, NOX emission and SO2 emission are considered which have weightages \( w_1 \), \( w_2 \) and \( w_3 \), respectively. The minimum and maximum values of these objectives are given below.

\[
F_{1\text{TM}} = 4948.729 \text{ $=} h, \quad F_{2\text{max}} = 5170.30 \text{ $=} h
\]

\[
F_{2\text{TM}} = 705.9319 \text{ kg=} h, \quad F_{3\text{TM}} = 1034.931 \text{ kg=} h
\]

\[
F_{3\text{max}} = 2989.757 \text{ kg=} h, \quad F_{2\text{max}} = 6123.19 \text{ kg=} h
\]

Hypercube is formed around weights \( w_2 \) and \( w_3 \). If two binary bits are combined then \( 2^2 \) (four) different possible combinations can be obtained. These four binary bit combinations are the corners of hypercube away from the centre point of hypercube. The distance \( g \) is generated from binary digits is given in Table 10. Fig. 2 and Table 10 show the generation of weights at the hypercube corners.

The values of cost, NOX emission and SO2 emission in each iteration of search are shown in Table 9. The membership functions, \( m(F_k) \) are given in Table 8. Minimum membership function values from three objectives, \( m_k \) are also given in Table 8. The 'best'
weights: $14^e=0.52$, $w_2=0.46$, $w_3=0.02$ are obtained after 24 iterations. The 'best' power schedule and voltage profile corresponding to preferred weight combination is tabulated in Table 11.

### 7.2.3. Case 3 (four objectives)

If there are four objectives, i.e. cost, NO\(_X\) emission, SO\(_2\) emission and CO\(_2\) emission are considered which have weightages $w_1$, $w_2$, $w_3$ and $w_4$, respectively. The minimum and maximum values of these objectives are given below:

- Cost: $\text{in} = 4948.729 \quad \$ / \text{h}$
- NO\(_X\) emission: $F_2^{\text{TM}^n} = 705.9319 \quad \text{kg} / \text{h}$
- SO\(_2\) emission: $F_3^n = 2989.757 \quad \text{kg} / \text{h}$
- CO\(_2\) emission: $F_4^{\text{TM}^n} = 8.600655 \quad \text{Ton} / \text{h}$

Considering each of the emission pollutants objective NO\(_X\), SO\(_2\) and CO\(_2\) represent the binary bit. The three binary bits can be represented in $2^3$ (eight) possible different combinations. These eight binary bit combinations are the corners of hypercube away from the point around which hypercube is generated. The distance $g$ is generated from binary digits is given in Table 12. Fig. 3 and Table 12 show the generation of weights at the hyper cube corners.

The values of cost and emission due to pollutants NO\(_X\), SO\(_2\), and CO\(_2\) in each iteration are tabulated in Table 13. The membership functions, $m(F_i)^k$ are given in Table 14. Minimum membership function value from four objectives, $\mu^f$ are also given in Table 14. The 'best' weights: $w_1 = 0.399$, $w_2 = 0.287$, $w_3 = 0.027$, $w_4 = 0.287$ are obtained after 15 iterations. The best power

---

### Table 13

<table>
<thead>
<tr>
<th>Iteration</th>
<th>$k_1$ ($$/h)</th>
<th>$k_2$ (kg/h)</th>
<th>$k_3$ (kg/h)</th>
<th>$k_4$ (Ton/h)</th>
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</thead>
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<td>827.8791</td>
<td>3540.081</td>
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schedule and voltage profile corresponding to 'best' weight combination is shown in Table 15.

8. Conclusion

In the multiobjective framework it is realized that cost and emission are conflicting objectives and subject to mutual interface. The solution set of the formulated problem is non-inferior due to contradictions among objectives taken and has been obtained through weighting method. The number of non-inferior solutions of the problem increases exponentially with the number of objectives, e.g. if normalized weights are varied by 0.1, then there is need of 11 non-inferior solutions in the case of two objectives, 60 non-inferior solutions in the case of three objectives, 238 non-inferior solutions in the case of four objectives, to interact with the decision maker to obtain his/her reaction/response to the solution. Similarly, if the normalized weights are varied by 0.01, then
there is need of 101 non-inferior solutions in the case of
two objectives, 5119 non-inferior solutions in case of
three objectives. To reduce the computational burden
and reduce the complexity to select the 'best' solution,
the multiobjective problem has been solved by searching
the optimal weightage pattern of objectives with evolu-
tionary optimization technique. The non-inferior
solution that attains maximum satisfaction level from the
membership functions of the participating objectives,
has been adjudged the 'best' solution. It can be observed
from Tables 6 and 8 that the proposed method needs 18
(number of iterations x 3-hypercube corners including centre)
non-inferior solutions to be evaluated in the case
of two objectives and 120 (24-number of iterations x 5-
hypercube corners including centre) non-inferior solutions
to be evaluated in the case of three objectives
respectively, to achieve the 'best' solution by searching
weights up to the second significant digit. Where as the
solution methodology used in [7] requires 101 non-
inferior solutions to be evaluated in case of two
objectives and 5119 non-inferior solutions to be evaluated
in case of three objectives, respectively. The proposed
method requires few search moves to get the
optimal operating point in the non-inferior domain for
any number of goals. Moreover, to find the 'best'
solution, the proposed method takes 0.60 s and the
method used in [7] takes 3.07 s on Pentium-III, 850
MHz; personal computer. It has been explored that
further refinement to choose 'g', the distance of the
corners of the hypercube from the point around which
hypercube is generated, could further improve the
convergence of proposed method as number of itera-
tions increase by decreasing the value of 'g'. Ensuing
paper includes the further refinement to choose 'g'.

Appendix A

To generate the non-inferior solution, Newton Raphson
method has been applied and Eq. (17) is solved. The
step-wise procedure is outlined as below:

1) Input the required data, obtain \( Y_{BUS} \) and \( Z_{BUS} \)
(= \( Y_{BUS}^{-1} \)).
2) Compute \( P_{Gi} \), \( Q_{Gi} \); from loss less case; \( i = 1, 2, \ldots, N \), \( l_{p} \), optimal cost \( F \) and put \( A_{Q} = 0 \).
3) Set iteration counter, \( IT = 0 \).
4) Increment the iteration counter, \( IT = IT + 1 \).
5) If (\( IT > IT_{max} \)), then go to step 17.
6) Store optimal cost \( ^{\circ}F \).
7) Compute, \( P_{i} = P_{Gi} \cdot P_{w} \); \( i = 1, 2, \ldots, M \). \( Q_{i} = Q_{Gi} \cdot Q_{DU} \)
\( 1 = 1, 2, \ldots, M \), for PQ buses. \( P_{Gi} = Q_{Gi} = 0 \), for non-generating
buses.
8) Run the decoupled load flow to obtain real and
reactive powers, \( P_{i} \), \( Q_{i} \) and voltage magnitude and
angles, \( V_{i} \), \( d_{i} \) at each bus.

References

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