Nonlinear coupled dynamic response of offshore Spar platforms under regular sea waves

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Abstract

Oil and gas exploration of large reservoirs in deep water has accelerated the need to explore structures suitable for these depths, which operate more economically in deep water. A Spar platform is one such compliant offshore floating structure used for deep water applications for the drilling, production, processing, storage and offloading of ocean deposits. The Spar is modeled as a rigid body with 6 degrees-of-freedom, connected to the sea floor by multi-component catenary mooring lines, which are attached to the Spar at the fairleads. The response-dependent stiffness matrix consists of three parts: (a) the hydrostatics provide restoring force in heave, roll and pitch; (b) the mooring lines provide the restoring force which are represented here by nonlinear horizontal; and (c) vertical springs. An unidirectional regular wave model is used for computing the incident wave kinematics by Airy’s wave theory and force by Morison’s equation. The response analysis is performed in time domain to solve the dynamic behavior of a moored Spar platform as an integrated system using the iterative incremental Newmark’s Beta approach. Numerical studies are conducted for several regular waves.

Keywords: Wave structure interaction; Offshore structural dynamics; Spar platform; Multicomponent catenary mooring with horizontal and vertical spring
1. Introduction

As offshore oil and gas exploration is pushed into deeper and deeper water, many innovative floating offshore structures are being proposed for cost savings. To reduce wave induced motion, the natural frequency of these newly proposed offshore structures are designed to be far away from the peak frequency of the force power spectra. Spar platforms being one such compliant offshore floating structure used for deep water applications for the drilling, production, processing, storage and offloading of ocean deposits. It is being considered the next generation of deep water offshore structures by many oil companies. It consists of a vertical cylinder, which floats vertically in the water. The structure floats so deep in the water that the wave action at the surface is dampened by the counter balance effect of the structure weight. Fin like structures called strakes, attached in a helical fashion around the exterior of the cylinder, act to break the water flow against the structure, further enhancing the stability. Station keeping is provided by lateral, multicomponent catenary anchor lines attached to the hull near its center of pitch for low dynamic loading. The analysis, design and operation of Spar platform turn out to be a difficult job, primarily because of the uncertainties associated with the specification of the environmental loads. The role of various nonlinearities has been a topic of significant interest in the area of numerical Spar response prediction. In deep water, minor changes in some design parameters may have sensitive effects on the buoy behavior and therefore, on its operational characteristics. The present generation of Spar has the following features.

1. It can be operated at depths of upto 3000 m from full drilling and production to production only.
2. It can have large range of topside payloads.
3. Rigid steel production risers are supported in the center well by separate buoyancy cans.
4. It is always stable because the center of buoyancy (CB) is above the center of gravity (CG).
5. It has favorable motions compared to other floating structures.
6. It can have steel or concrete hull.
7. It has minimum hull-deck interface.
8. Oil can be stored at low marginal cost.
9. It has sea keeping characteristics superior to all other mobile drilling units.
10. It can be used as a mobile drilling rig.
11. The mooring system is easy to install, operate and relocate.
12. The risers, which normally take a breathing in the wave zone from high waves on semi-submersible, drilling units would be protected inside the Spar. Sea motion inside the Spar's center well would be minimal.

The concept of Spar as an offshore structure is not new. Spar buoy type of structures have been built before. For example, a floating instrument platform (FLIP) was built in 1961 to perform oceanographic research (Fisher and Spiess, 1963), the Brent
Spar was built by Royal Dutch Shell as a storage and offloading platform in the North sea at intermediate water depth (Bax and de Werk, 1974, Van Santen and de Werk, 1976, Glanville et al., 1997). The use of Spar platforms as a production platform is relatively recent (Converse and Bridges, 1997).

Lowd et al. (1971) discussed the design and model basin test results with respect to a floating and portable production processing facility, which will improve the initial cash flow pattern from certain marine fields, if interim production facilities, storage and a sales outlet can be devised quickly. The semi-submerged production unit was a Spar buoy combining the stability, portability and minimum size of a vertical cylindrical vessel with the utility of a complete production processing unit mounted therein. The unit design provides transport to the location on the deck of presently available vessels and for launching procedures on the final location position without derrick barge assistance.

Triantafyllou et al. (1982) used extensible catenary equations to derive the force-excursion relationships of the mooring line at the guy attachment point.

Dutta (1984) used a catenary equation to solve the multicomponent mooring system for guyed towers taking into account the line extensibility. The inherent nonlinear equation is solved using the secant method for fast convergence. Different relationships for the different cases of clump weights lying on the sea bed or lift up were reported and it was assumed that the initial lengths of guy line and anchor line are inclusive of elongation due to the initial line tension. The study was carried for lumped and distributed clump weights, and it was concluded that less excursion and more cable force at the attachment point with lumped clump weights leads to an underestimation of the fundamental period that may lead to an overestimation of the structural response for higher periods.

Verma (1990) has presented the mathematical formulations for a multicomponent mooring line using both a small strain elastic catenary approach and the finite element method. The parametric study related to the properties of various cable components and the sloping sea bed are carried out in the case of clump-mooring line undergoing small strains. In the finite element method, an efficient solution procedure is proposed in which the compatible starting configuration is chosen in a straightforward and simple manner. The effects of discretization on the results are investigated. A flexible mooring line constituting expansive rope and steel wire rope is analyzed by both the finite element and catenary approach in order to understand the limitations of the elastic catenary approach in the analysis of deeper-water mooring systems.

Glanville et al. (1991) gave the details of the concept, construction and installation of the Spar platform. He concluded that the Spar platform allows flexibility in selection of well systems and drilling strategies, including early production or predrilling programs.

D’Souza et al. (1992) presented important design considerations for deepwater mooring systems for permanently moored floating platforms.

Mekha et al. (1995) modeled the Spar platform with 3 degrees-of-freedom, i.e. surge, heave and pitch. The inertia forces were calculated using a constant inertia coefficient, C_m, as in the standard Morison's equation or using a frequency dependent C_m coefficient based on the diffraction theory. The drag forces were computed using
the nonlinear terms of Morison’s equation in both cases. The analysis was performed in the time domain. The result showed that using frequency dependent or constant inertia coefficient, $C_m$, produces similar results since most of the wave energy is concentrated over the range of frequencies where the value of $C_m$ is 2.0 for the Spar size used in the literature.

Mekha et al. (1996) used the same model (Mekha et al., 1995) with the frequency dependent, $C_m$, coefficient based on diffraction theory. Different nonlinear modifications to Morison's equation were induced to account any diffraction effects. The results obtained for a variety of sea state conditions were compared to the experimental data.

Halkyard (1996) reviewed the status of several Spar platform concepts emphasizing the design aspect of these platforms.

Cao and Zhang (1996) discussed an efficient methodology to predict slow drift response of slack moored slender offshore structures due to ocean waves using a hybrid wave model. The hybrid wave model considers the wave interactions in an irregular wave field up to second order of wave steepness and is able to accurately predict incident wave kinematics, including the contributions from nonlinear difference frequency interactions. A unique feature of this approach is that a measured wave elevation time series can be used as input and the structure responses to measured incident waves can be deterministically obtained.

Ran and Kim (1996) studied the nonlinear response characteristics of a tethered/moored Spar platform in regular and irregular waves. A time-domain coupled nonlinear motion analysis computer program was developed to solve both the static and dynamic behaviors of a moored compliant platform as an integrated system. In particular, an efficient global-coordinate based dynamic finite element program was developed to simulate the nonlinear tether/mooring responses. Using this program, the coupled dynamic analysis results are obtained and they are compared with uncoupled analysis results to see the effects of tethers and mooring lines on hull motions and vice versa.

Ran et al. (1996) studied the response characteristics of a large slack-moored floating Spar in regular waves, bichromatic waves and unidirectional irregular waves with or without sheared currents by experimental and numerical methods. It was observed that the low-frequency surge and pitch responses were in general greater than the wave-frequency response, and the slowly varying responses were appreciably reduced in the presence of currents. The total response amplitudes were found to be practically acceptable in the survival condition characterized by a 100-year storm sea.

Jha et al. (1997) compared the analytically predicted motions of a floating Spar buoy platform with the results of wave tank experiments considering, surge and pitch motions only. Base-case predictions combine nonlinear diffraction loads and a linear, multi-degree-of-freedom model of the Spar stiffness and damping characteristics, refined models and the effect of wave-drift damping, and of viscous forces as well. Consistent choices of damping and wave input were considered in some detail.

Fischer and Gopalkrishnan (1998) presented the importance of heave characteristics of Spar platforms that have been gleaned from wave basin model tests, numeri-
cal simulations and combination of the two. The heave performance of small Spars, e.g. mini Spars has been examined and found to be potentially problematic.

Chitrapu et al. (1998) studied the nonlinear response of a Spar platform under different environmental conditions such as regular, bi-chromatic, random waves and current using a time domain simulation model. The model can consider several non-linear effects. Hydrodynamic forces and moments were computed using the Morison's equation. It appears that Morison's equation combined with accurate prediction of wave particle kinematics and force calculations in the displaced position of the platform give reliable prediction of platform response both in the wave-frequency and low-frequency ranges.

Ran et al. (1999) discussed the nonlinear coupled response of a moored spar in random waves with and without co-linear current in both time and frequency domain. The first and second order wave forces, added mass, radiation damping and wave drift damping were calculated from a hydrodynamic software package called WINTCOL. The total wave force time series (or spectra) were then generated in the time (or frequency) domain based on a two-term volterra series method. The mooring dynamics were solved using the software package WINPOST, that is based on a generalized coordinate based FEM. The mooring lines were attached to the platform through linear and rotational springs and dampers. Various boundary conditions can be modeled using proper spring and damping values. In the time domain analysis, the nonlinear drag forces on the hull and mooring lines were applied at the instantaneous position. In the frequency domain analysis, nonlinear drag forces were stochastically linearized and solutions were obtained by an iterative procedure.

Datta et al. (1999) described recent comparisons of numerical predictions of motions and loads to typical truss Spar model test results. The purpose of this comparison was to calibrate hydrodynamic coefficients which, were to be used for the design of a new truss Spar platform for Amoco.

Chitrapu et al. (1999) discussed the motion response of a large diameter Spar platform in long crested and random directional waves and current using a time-domain simulation model. Several nonlinearities such as free surface force calculation, displaced position force computation, nonlinearities in the equations of motion and the effect of wave-current interaction were considered in determining the motion response. The effect of wave directionality on the predicted surge and pitch response of the Spar platform has been studied. It was seen that both wave-current interaction and directional spread of wave energy have significant effect on the predicted response.

Chen et al. (1999) presented two existing numerical schemes, which were dynamically linked to compute the nonlinear response of a slack moored slender body to steep irregular ocean waves. The combined program known as COUPLE, allows for the nonlinear wave force estimated at the instantaneous position of the body and the dynamic interactions between the mooring system and body. The motions of a slack moored Spar and tensions in its mooring lines using the program COUPLE are predicted and compared with the corresponding laboratory measurements and predictions based on the static mooring system. The comparisons indicate that COUPLE is reliable and accurate in simulating the dynamic interactions between a mooring sys-
tem and a slender-body structure. Current laboratory test simulates a slack mooring system by taut mooring lines and springs. This kind of simulation may not truly model the dynamic effect of mooring system on a moored structure in the real sea. Hence, the numerical simulation based on COUPLE can be a suitable tool to compensate these laboratory tests, especially in deep water.

Kim et al. (2001) numerically carried out the nonlinear hull/mooring coupled dynamic analysis of a truss Spar in waves with collinear steady winds and currents in time domain and the results were compared with 1:61 scale experiments as well as uncoupled analysis. The numerical results showed that dynamic effects were very important for the present mooring design. The motion and tension spectra of uncoupled analysis with a linear massless spring or nonlinear massless spring were also compared with those obtained from fully coupled analysis to assess the importance of hull/mooring coupling and mooring-line damping.

2. Structural model

The Spar is modeled as a rigid cylinder with 6 degrees-of-freedom (i.e. three displacement degrees-of-freedom, i.e. surge, sway and heave along the X, Y and Z axes and three rotational degrees-of-freedom, i.e. roll, pitch and yaw about the X, Y and Z axes) at its CG. The Spar is assumed to be closed at its keel. The stability and stiffness is provided by a number of the mooring lines attached near the CG for low dynamic positioning of the Spar platforms. When the platform deflects the movement will take place in a plane of symmetry of the mooring system, the resultant horizontal force will also occur in this plane and the behavior of the mooring will be two-dimensional. It is the force and displacement (excursion) at this attachment point that is of fundamental importance for the overall analysis of the platforms. It is assumed that the Spar is connected to the sea floor by four multi component catenary mooring lines placed perpendicular to each other, which are attached to the Spar at the fairleads. The development of Spar platform model for dynamic analysis involves the formulation of a nonlinear stiffness matrix considering mooring line tension fluctuations due to variable buoyancy and other nonlinearities. The model considers the coupled behavior of a Spar platform for various degrees-of-freedom. Fig. 1. shows a typical offshore Spar platform.

3. Assumptions and structural idealization

The platform and the mooring lines are treated as a single system and the analysis is carried out for the 6 degrees-of-freedom under the environmental loads. The following assumptions have been made in the analysis:

1. Initial pretension in all mooring lines is equal. However, total pretension changes with the motion of the Spar Platform.
2. Wave forces are estimated at the instantaneous equilibrium position of the Spar
platform by Morison's equation using Airy's linear wave theory. The wave dif-
fractio effects have been neglected.
3. Integration of fluid inertia and drag forces are carried out up to the actual level of
submergence according to the stretching modifications considered in the analysis.
4. Wave force coefficients, $C_d$ and $C_m$ are independent of frequencies as well as
constant over the water depth.
5. Current velocity has not been considered and also the interaction of wave and
current has been ignored, Wind forces have been neglected.
6. Change in pretension in mooring line is calculated at each time step, and the
equation of equilibrium at that time step modifies the elements of the stiffness
matrix.
7. The platform is considered as a rigid body having 6 degrees-of-freedom.
8. Platform has been considered symmetrical along surge axis. Directionality of wave
approach to the structure has been ignored in the analysis and only uni-directional
wave train is considered.
9. The damping matrix has been assumed to be mass and stiffness proportional,
based on the initial values.

4. Catenary mooring line analysis

Some of the assumptions made for the analysis of catenary mooring line are:
1. The sea floor (having negligible slope) offers a rigid and frictionless support to
the mooring line, which is lying on it.
2. All the components of the mooring line move very slowly inside the water, so that
the generated drag forces on the line due to the motion can be treated as negligible.
3. The change in the line geometry and thereby in the line force due to direct fluid
loading caused from waves and/or currents is insignificant.
4. Initial length of the mooring line and anchor line is inclusive of elongation due
to the initial line force.
5. The clump weight segment is inextensible.
6. Anchor point does not move in any direction.
7. Both horizontal and vertical excursion of the catenary mooring line is considered.

The force-excursion relationship is nonlinear and requires an iterative solution.
Equation of a catenary was used for evaluation of force-excursion relationship of a
catenary mooring line. The horizontal projection $X$ and vertical projection $Y$ of any
segment hanging freely under its own weight $w$ per unit length as shown in Fig. 2
can be expressed [considering horizontal force ($H_t$), top slope ($q_t$), length ($S$) and
weight ($W$)] as:

\[ \text{(a)} \]

\[ \text{Clump weight} W_{cl} , S_{cl} \]

\[ \text{Anchor line} W_a , E_a , A_a \]

\[ s_1 , s_2 \]

\[ X = X_h \]

\[ X_c \]

\[ \text{Mooring line} W_c , E_c , A_c \]

\[ S_c \]

\[ Y = h \]

\[ Y_f \]

\[ H_b \]

\[ \theta_b \]

\[ \theta_t \]

\[ H_t \]

\[ S_w \]

\[ S \]

\[ W \]

\[ (b) \]

\[ \text{Fig. 2. Multicomponent mooring line. (a) Initial configuration with different sectional properties. (b) Free body diagram of uniform mooring line suspended freely between two points not in the same elevation.} \]
\[ Y = \left( H \right) \left[ \cosh \{ \sinh^{-1}(\tan(0)) \} - \cosh \{ \sinh^{-1}(\tan(\theta)) \} \right] \] (1)

\[ X = \left( H \right) \left[ \sinh^{-1}(\tan(\theta)) - \sinh^{-1}(\tan(\phi)) \right] \] (2)

\[ \tan(\phi) = \frac{y}{x} \] (3)

\[ V_t = H \tan(\phi) \]

When for any segment the bottom slope \( \phi \) is zero, eqs. (1) and (2) reduces to:

\[ Y = \left( H \right) \left[ \cosh \{ \sinh^{-1}(\tan(\theta)) \} - 1 \right] \] (4)

\[ X = \left( H \right) \left[ \sinh^{-1}(\tan(\theta)) \right] \] (5)

If \( Y, W, \phi \), are known then

\[ \tan(\phi) = \sinh \left[ \cosh^{-1} \left( \cosh \left[ \sinh^{-1} \tan(\theta) \right] \right) \right] \] (6)

\[ S = \frac{H (\tan(\theta) - \tan(\phi))}{W} \] (7)

and \( X \) can be evaluated by eq. (2).

The extension of any segment under increased line tension can be approximately evaluated as follows. Let the initial average line tension be \( T_o \) when the segment length is \( S_o \). For increased average line tension \( T \), the stretched length becomes:

\[ S = S_o \left[ \frac{1 + (T - T_o)}{EA} \right] \] (8)

where \( E \) and \( A \) are Young’s modulus and the effective area of the segment, respectively. \( T \) and \( T_o \) are the arithmetic means of the line tensions at two ends. As the total weight of any segment \( W \) remains the same

\[ W = \frac{S W_o}{y^2}, \] (9)

where \( W \) is the modified unit weight due to stretching and \( W_o \) is the unit weight of the unstretched segment.

5. Analysis of the mooring line with distributed clump weight for horizontal excursion

The analysis of multi component catenary mooring line with distributed clump weight for horizontal excursion for a single mooring line and for the entire mooring
system has been given in Agarwal and Jain (2002). The force-excursion relationship for vertical excursion is given here.

6. Analysis of the mooring line with distributed clump weight for vertical excursion

In the present work, $H_o$, $q_o$, $h$, zero bottom slope and the elastic and physical properties of the segments, as shown in Fig. 3a, are chosen as the known parameters. The unknowns which are to be evaluated are $S_c$, $S_h$ and then $Y_c$, $Y_h$, $X_c$ and $X_h$.

6.1. Initial configuration

The following configuration steps are used to find the unknowns given above.

Step 1. Calculate $V_o$ from the known values of $H_o$ and $q_o$.
Step 2. Find the slope at the junction of the clump weight and mooring line, then find vertical force at the junction $V_j$ (which will be equal to $S_hW_{cl}$ as the bottom slope is equal to zero). Using the known values of the horizontal force, use eq. (4) to find $Y_h$.
Step 3. Find $S_c = (V_o - S_hW_{cl})/W_c$ and then find $Y_c$ using eq. (1).
Step 4. Add up $Y_c$ and $Y_h$ and compare with $h$. If the difference is less than a specified limit, go to the next step. Otherwise change $V_j$ appropriately and repeat the procedure from step 2. Note: For the first iteration the change of $V_j$ can be taken as $\pm 1\%$ depending upon the sign of error. For the subsequent iterations the following equation is to be used to get a new value of $V_j$.

![Fig. 3. Configuration of the mooring line for increased horizontal force, $H$ (Condition 1). (a) Initial configuration, (b) final configuration.](image-url)
(10)

\[ (V_j)_{k+1} = (V_j)_k - \left[ \frac{e_k((V_j)_{k-1}-(V_j)_k)}{e_{k-1} - e_k} \right] \]

where \( k \) is the number of the last iteration, \( (V_j)_k \) is the vertical force at the junction of the mooring line and clump weight and \( e_k \) is the difference between the vertical projection of the hanging length of the mooring line calculated in the \( kth \) iteration and the mooring level (\( h \)).

Step 5. Find \( X_c \) and \( X_h \) from eqs. (2) and (5), respectively.

Step 6. Find initial total hanging length \( S_i = S_c + S_h \) and its horizontal projection \( X_i = X_c + X_h \).

7. Evaluation of force-excursion (vertical) relationship for a single mooring line

The vertical force \( (V_o) \) is changed which allows direct checking of the condition regarding the lifting off of the clump weight. The corresponding horizontal force is found iteratively, ending with the determination of a new configuration including the excursion of the attachment point. The procedures for the two alternative states of lifting-off of the clump weight are given below. Note that \( W_c = S_c W_c \) being the weight of the mooring line and \( W_{cl} = S_{cl} W_{cl} \) is the total weight of the clump weight.

Find the initial tension in the mooring line.

7.1. Condition 1 (when \( V \leq W_C + W_{cl} \))

Step 1. Increase \( V_o \) by \( AV \) [Fig. 3(b)].
Step 2. Find the vertical and horizontal force at the junction of the clump weight and mooring line. Find the new average line tension \( T \) in the mooring line.
Step 3. Find the stretched length of the mooring line \( S_{cn} \) and hence modified \( W_c \) using eqs. (8) and (9), respectively.
Step 4. Find the horizontal projections of the mooring line (\( X_{cn} \)) and clump weight (\( X_{hn} \)) with the help of eqs. (2) and (5), respectively.
Step 5. Find the elastic stretch of anchor line \( e_a \),

\[ e_a = S_a (H - H_o) \]

\[ E_a A_o \]  

where \( E_a \) is Young’s modulus of the anchor line, \( A_o \) is the effective area of the anchor line, \( H_o \) is the initial horizontal force at the top, \( H \) is the final horizontal force at the top, and \( S_a \) is the length of the anchor line.

Step 6. Add up \( (X_{cn} + X_{hn} + (S^c - S^h) + S_{hn}) \) and compare it with initial configuration of the total horizontal projection \( X \). If the difference \( e \) is less than tolerable limit (1 cm) go to the next step. Otherwise, change \( H \) appropriately and repeat the procedure from step 2.

Step 7. Find the new hanging length of the clump weight \( Shn \) from the zero bottom
slope condition and add it with the stretched mooring line length $S_{cn}$ to get the new length $S_f$.

Step 8. Find $Y_{cn}$ and $Y_{hn}$ from eqs. (1) and (4), respectively, and add them to get the vertical projection $Y_f$.

Step 9. Evaluate vertical excursion (d)

$$S = Y_f - Y_i$$

where $Y_i$ is the initial vertical projection of the mooring line, $Y_f$ is the final stretched vertical projection of the mooring line.

Step 10. Repeat steps 1-9 for the increased values of $V_o$ till $V = W_c + W_{cl}$.

7.2. Condition 2 (when $V > W_c + W_{cl}$)

Step 1. Increase $V'_o$ by AF [Fig. 4(b)].

Step 2. Find the vertical and horizontal force at the junction of the clump weight and mooring line. For the value of ($V = F'_o + V$) find the vertical force, $V_a$ at the junction of the anchor line and the clump weight as shown in Fig. 4(c). Find the new average line tension in the mooring line and the anchor line and hence the modified $W_c$ and $W_a$ [considering the system configuration when the clump weight is just lifted as shown in Fig. 4(a) as the initial configuration].

Step 3. Find the hanging length of anchor line $S_{ah}$ using eq. (7) with $q_b = 0$. Add up $S_{cn}$ and $S_{cl}$ to get $S_f$ (which includes the mooring line stretch).

Step 4. Find $X_{cn}$ and $X_{cln}$ using eq. (2) and $X_{ah}$ from eq. (5).

Step 5. Find the elastic stretch of anchor line $e_a'$

Fig. 4. Configuration of the mooring line for increased horizontal force, $H$ (Condition 2). (a) Configuration when the far end of the clump just lifts off. (b) Final configuration. (c) Free body diagram of the anchor line.
\[
S \left( H - H' \right) = \frac{S_H}{2E_\alpha^A} \quad (1')
\]

where \( H' = \left( H + \frac{iH^2 + Va2}{2} \right) \), \( V_a = V - W_{o'} - W_{cl} \), \( H_o \) is the initial horizontal force when \( V = W_{o'} + W_{cl} \), \( H' \) is the increased horizontal force due to increased vertical force at the top, and \( W_o \) and \( W_{cl} \) are the total weights of the mooring line and clump weight, respectively.

Step 6. Add up \( \{X_{cn} + X_{cln} + X_{ah} + (S_m - S^0)\} \) and compare it with initial configuration of the total horizontal projection \( X \). If the difference \( e \) is less than tolerable limit (1 cm) go to the next step. Otherwise, change \( H \) appropriately and repeat the procedure from step 2.

Step 7. Find \( Y_{cn} \) and \( Y_{cln} \) using eq. (1) and \( Y_{ah} \) from eq. (4). Add them to get the vertical projection \( Y_v \).

Step 8. Find the excursion from

\[
S = Y_i - Y_C \quad (14)
\]

where \( Y_i \) is the initial stretched vertical projection when the clump weight just lift's off and \( Y_f \) is the final stretched vertical projection after the lifting of the clump weight.

Step 9. Repeat steps 1-8 for increasing values of \( V_{o'} \), till the tension equals the permissible value (approximately half of the breaking strength of the mooring line material). In the above \( S_{ah} \) and \( Y_{ah} \) are found by treating the anchor line as a freely hanging mooring line and making use of the modified unit weight for the stretched segment. If \( S_{ah} \) is found more than the total (stretch) anchor line length calculate \( \tan(q_h) = \frac{V_a - W_{ah}}{H} \), \( W_a \) being the total weight of anchor line) and recalculate \( S_{ah} \). \( Y_{ah} \) is evaluated using eq. (1).

The behavior of the mooring system will be planer if the tower excursion takes place in a plane of symmetry of the mooring system. For an excursion of \( d \) at the attachment point the resultant horizontal force is given by

\[
H(d) = 2 \left( \frac{H}{\sin q_j} - O_j \right) \quad (15)
\]

where \( p \) is the total number of the mooring lines, \( (q_j) \) is the angle between the \( j \)th mooring line and the direction of excursion. \( dj \) is the excursion for the \( j \)th mooring line and \( (H_j(D_j)) \) is the associated horizontal force, with \( dj = d \). The resultant vertical force at the mooring attachment point will be

\[
V(8) = 2 \left( V_j(d_j) \right) \quad (16)
\]

In this study the mooring line is modeled as a nonlinear horizontal and vertical spring located at the fairleads along the Spar center with no hydrodynamic forces applied on them. The stiffness matrix representing the horizontal spring (hs) is given below.
The stiffness matrix representing the vertical spring (vs) is

\[
[K^{\text{vs}}] = \begin{bmatrix}
K_{v1} & 0 & 0 & K_{v3} & 0 \\
0 & K_{v2} & 0 & K_{v4} & 0 \\
K_{v5} & K_{v6} & 0 & K_{v7} & 0 \\
0 & K_{v8} & 0 & A & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]  

(17)

The equation of motion of spar platform under regular waves is given below:

\[
[M][\dot{X}] + [C][\dot{X}] + [K][X] = \{F(t)\}
\]

(19)

where \(X\) is the structural displacement vector, \(\dot{X}\) is the structural velocity vector, \(\ddot{X}\) is the structural acceleration vector, \(M = MS_{\text{par}} + M_{\text{added mass}}\), \(K = K_{\text{hydrostatic}} + K_{\text{horiz spring}} + K_{\text{vert spring}}\), \(C\) is the structural damping matrix and \(F(t)\) is the hydrodynamic forcing vector.

The mass matrix represents the total mass of the Spar including the mass of the soft tanks, hard tanks, deck, ballast and the entrapped water. The added mass matrix is obtained by integrating the added mass term of Morison's equation along the submerged draft of the Spar. Mass is taken to be constant and it is assumed that the masses are lumped at the CG. The structural damping matrix is taken to be constant and is dependent on mass and initial stiffness of the structure. The elements of \([C]\) are determined by eq. (20), using the orthogonal properties of \([M]\) and \([K]\), where, 4 is the structural damping ratio, \(\theta\) is the modal matrix, \(\omega_i\) is natural frequency and \(m_i\) is the generalized mass

\[
O^CJO = [2^\text{ft}, f]_{ij}.
\]

(20)

The stiffness matrix consists of three parts: the restoring hydrostatic force and the stiffness due to mooring lines (horizontal and vertical springs). The coefficients, \(K_{ij}\)
of the stiffness matrix of the Spar platform are derived as the force in degree-of-freedom $i$ due to unit displacement in the degree-of-freedom $j$, keeping all other degrees-of-freedom restrained. The coefficients of the stiffness matrix have nonlinear terms. Furthermore, the mooring line tension changes due to the motion of the Spar platform in different degrees-of-freedom which makes the stiffness matrix response dependent. Fig. 5 shows the degrees-of-freedom of the Spar platform at its CG.

The hydrostatic stiffness matrix is calculated based on the initial configuration of the Spar. The stiffness coefficients are given in the following equations

$$[K^{hy}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{hy33} & 0 & 0 \\ 0 & 0 & 1550 & 0 \\ 0 & 0 & 0 & K_{hy55} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$ \hspace{1cm} (21)

where

$$K_{33}^{hy} = \frac{n}{4} \gamma_o D^2$$ \hspace{1cm} (22)

$$K_{44}^{hy} = K_{33}^{hy} h_1 \frac{n}{64} \gamma_o D^4$$ \hspace{1cm} (23)

$$K_{55}^{hy} = K_{33}^{hy} h_1 \frac{\pi}{64} \gamma_o D^4$$ \hspace{1cm} (24)

![Fig. 5. Degrees-of-freedom at CG of spar.](image-url)
and \( h_i = S_{cb} - S_{cg} \). \( D \) is the diameter of the Spar, \( S_{cg}, S_{cb} \) are the distance from the keel of the Spar to its CG and CB, respectively, \( H_d \) is the draft of the Spar, and \( g_w \) is the weight density of water.

**9. Hydrodynamic forcing vector**

Ocean surface waves refer generally to the moving succession of irregular humps and hollows of the ocean surface. They are generated primarily by the drag of the wind on the water surface and hence are the greatest at any offshore site, when storm conditions exist there. For analyzing the offshore structures, it is customary to analyze the effects of the surface waves on the structures either by use of a single design wave chosen to represent the extreme storm conditions in the area of interest or by using the statistical representation of waves during extreme conditions. In either case, it is necessary to relate the surface wave data to the water velocity, acceleration and pressure beneath the waves. An unidirectional wave modal is used for computing the incident wave kinematics. The kinematics of the water particles has been calculated by Airy’s wave theory. The sea surface elevation, \( \eta(x, t) \) is given as

\[
h(x, t) = \frac{1}{2} H \cos(Ax - \omega t).
\]

The horizontal and vertical water particle velocities are given as

\[
u = \frac{coH \cos'[(ky)]}{2 \sinh(kh)} \cos(kx - \omega t) \nonumber
\]

\[
u = \frac{wH \sinh(ky)}{2 \sinh(kh)} \nonumber
\]

where \( k \) and \( \omega \) denotes the wave number and the wave frequency, respectively.

\[
k = \frac{2 \pi}{L}
\]

and

\[
\omega = \frac{2 \pi}{P}
\]

where \( P \) is the wave period, \( x \) is the displacement of Spar, \( t \) is the time instant at which the water particle kinematics are evaluated, \( L \) is the wave length, \( H \) is the wave height, and \( h \) is the water depth.

The acceleration of the water particle in horizontal and vertical directions are given as

\[
a = \frac{\omega^2 \cosh(ky) \sin(fo - t_1)}{2 \sinh(kh)} \nonumber
\]

\[
a = \frac{\omega^2 \cos(kx - \omega t)}{2 \sinh(kh)} \nonumber
\]
where $y$ is the height of the point of evaluation of water particle kinematics.

A simplified alternative proposed in this study is to predict the response of a deep-drafted offshore structure based on the slender body approximation, that is, without explicitly considering the diffraction and radiation potential due to the presence of the structure. For typical deep-water offshore structures such as Spar, the ratio of the structure dimension to spectrum-peak wave length is small. Hence, it is assumed that the wave field is virtually undisturbed by the structure and that the Morison’s equation is adequate to calculate the wave exiting forces. The wave loads on a structure are computed by integrating forces along the free surface centreline from the bottom to the instant free surface at the displaced position. The added mass is based on the initial configuration of the Spar and is added to the mass matrix. The inertia force ($F_i$) and added mass force ($F_{AM}$) per unit of length are given by

$$F_i = \frac{\pi D^2}{4} \rho_w C_m \dot{\gamma}$$

$$F_{AM} = \pi \int [C_m - 1] \rho_w \dot{X},$$

where $\rho_w$ is the mass density of the fluid, $D$ is the diameter of Spar, $C_m$ is the inertia coefficient, $\dot{X}$ is the acceleration of Spar, and $\dot{u}$ is the acceleration of the fluid.

The drag force ($F_D$), which includes the relative motion between the structure and the wave, per unit of length is given by

$$F_D = 2 \rho_w C_D \left| \dot{u} - \dot{X} \right|$$

where $C_D$ is the drag coefficient, $\dot{u}$ is the velocity of the fluid, and $\dot{X}$ is the velocity of Spar.

10. Solution of the equation of motion

In the time domain using the numerical integration technique, incorporating all the time dependent nonlinearities, stiffness coefficient depends on change of the mooring line tension with time, added mass from Morison’s equation, evaluation of wave forces at the instantaneous displaced position of the structure, the response of Spar can be evaluated. Wave loading constitutes the primary loading on offshore structures. The dynamic behavior of these structures is, therefore, of design interest. When the dynamic response predominates, the behavior under wave loading becomes nonlinear because the drag component of the wave load, according to Morison’s equation, varies with the square of the velocity of the water particle relative to the structure and at each time step, the force vector is updated to take into account the change in the mooring line tension. The equation of motion has been solved by an iterative procedure using unconditionally stable Newmark’s Beta method. The algorithm based on Newmark’s method for solving the equation of motion is given below.
Step 1. The stiffness matrix $[K]$, the damping matrix $[C]$, the mass matrix $[M]$, the initial displacement vector $[X_0]$ and the initial velocity vector $[\dot{X}_0]$ are given as the known input data.

Step 2. The force vector $[F(t)]$ is calculated.

Step 3. The initial acceleration vector is then calculated as

$$[\ddot{X}_0] = -[F(t)]^T [C] + [K] [X_0].$$

Step 4. Evaluation of constants from $a_0$ to $a_7$.

Step 5. $\hat{K} = K + a_0 M + a_1 C$.

Step 6. For each time step, the following are calculated

$$K^* = F_{t+At} + M(a_0 X_t + a_2 \dot{X}_t + a_4 j_t) + C(a_3 X_t + a_4 \dot{X}_t + a_5 \ddot{X}_t)$$

$$X_{t+At} = \hat{K}^{-1} \dot{F}_{t+At}$$

$$\dot{X}_{t+At} = a_3 X_t - a_0 X_t - a_2 \dot{X}_t - a_4 \ddot{X}_t$$

$$\ddot{X}_{t+At} = \ddot{X}_t + a_6 X_t + a_7 \ddot{X}_t.$$

Step 7. The values of $X$, $\dot{X}$, $\ddot{X}$, which are calculated at the time step $t + At$ are used to evaluate $F_{t+At}$ such that convergence is achieved in displacements to the accuracy of 0.01%, before going to the next time step, otherwise iteration is carried out. Since the $[K]$ of the Spar platform is response dependent, the new $[K]$ is generated and the difference from the old $[K]$ is used from step No. 5 onwards by taking it to $F_{t+At}$.

11. Numerical results and discussions

Fig. 6 shows the plan and schematic elevation of the Spar platform with different environmental loadings. Wind and current have not been studied herein.

The particulars for the multi component catenary mooring line are given in Table 1 unless otherwise specified.

The nonlinear mooring line behaviour is evaluated in order to study the behavior during the movement of the moored offshore Spar platforms. The fairlead point is allowed to move in the horizontal and vertical directions.

Two cases are taken for initial horizontal force of 2500 (Case A) and 2000 kN (Case B). Fig. 7 shows the force-excursion relationship of a single mooring line for horizontal excursion. Fig. 8 shows the force-excursion relationship of a single mooring line for vertical excursion.

In this paper all the 6 degree-of-freedom with three-dimensional behavior is considered with both horizontal and vertical excursion of the mooring lines whereas the literature shows that 2 or 3 degrees of freedom with only horizontal excursion of
Plan showing the position of mooring line

Fig. 6. Schematic elevation of a Spar platform.

Table 1
Data for a multicomponent catenary mooring line

<table>
<thead>
<tr>
<th>Data Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial horizontal force</td>
<td>2500 kN</td>
</tr>
<tr>
<td>Effective area of the mooring and anchor lines</td>
<td>0.0032 m²</td>
</tr>
<tr>
<td>Weight of the mooring and anchor lines</td>
<td>293.2 N/m</td>
</tr>
<tr>
<td>Weight of clump weight</td>
<td>25000 N/m</td>
</tr>
<tr>
<td>Length of clump weight</td>
<td>40 m</td>
</tr>
<tr>
<td>Length of anchor line</td>
<td>800 m</td>
</tr>
<tr>
<td>Mean sea level</td>
<td>914.4 m</td>
</tr>
<tr>
<td>Height of fairlead point</td>
<td>808.8 m</td>
</tr>
<tr>
<td>Angle of inclination at the fairlead point</td>
<td>30°</td>
</tr>
<tr>
<td>Young's modulus of the mooring and anchor lines</td>
<td>0.21E+9 kN/m²</td>
</tr>
</tbody>
</table>
the mooring line has been considered in the past studies. In all the numerical studies discussed below the effect of inclusion of vertical excursion in the stiffness matrix is compared with the effect of horizontal excursion alone.

In general, it is evident from Figs. 7 and 8 that the force-excursion relationship is nonlinear in nature. In case of horizontal excursion of the fairlead point the nonlinear effect is quite large for the horizontal force. However, this effect is small in case of the vertical force component in comparison to that of the horizontal component.
of the mooring force. When, the slope of the force-excursion curve has its peak value, around the zero displacement position of the attachment point, the mooring lines resistance to the movements of the floating structure is maximum and the mooring system behaves in the stiff mode. Increase in initial horizontal force at the attachment point makes the system taut as it decreases the length of the mooring line. The horizontal and vertical force-excursion relation of a single mooring line is greatly influenced by the initial horizontal force, inclination at the fairlead point and its level from the sea floor and also to some extent by the submerged unit weight of the clump weight.

From Fig. 7, it is observed that for positive horizontal excursion the horizontal and vertical forces at the fairlead point increases with the increase in the value of initial horizontal force but for initial horizontal force of 2500 kN its slope decreases after the excursion increases from 12 m in comparison to 2000 kN. The difference in the force is small for vertical force after 12 m for both the cases. Similarly for negative horizontal excursion the horizontal and vertical force decreases when the initial horizontal force decreases from 2500 to 2000 kN. The difference in the horizontal and vertical forces is small after —13 m.

From Fig. 8, it is observed that for positive vertical excursion the horizontal and vertical forces at the fairlead point increases with the increase in the value of initial horizontal force but for initial horizontal force of 2500 kN its slope decreases after the excursion increases from 24 m in comparison to 2000 kN. The difference in the force is small for vertical force after 40 m for both the cases. Similarly for negative vertical excursion the horizontal and vertical force decreases when the initial horizontal force decreases from 2500 to 2000 kN. The difference in the horizontal and vertical forces is small after —38 m.

The Spar platform dimensions and wave data are given in Table 2.

### 11.1. Effect of initial horizontal force

Two cases are taken for the initial horizontal force: (A) 2500 kN and (B) 2000 kN for coupled stiffness matrix considering horizontal excursion of the mooring line

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Dimensions of the Spar platform and wave data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight of the structure</td>
<td>$2.6 \times 10^6$ kN</td>
</tr>
<tr>
<td>Height of the Spar</td>
<td>216.4 m</td>
</tr>
<tr>
<td>Radius of the Spar</td>
<td>20.26 m</td>
</tr>
<tr>
<td>Distance of CG to buoyancy</td>
<td>6.67 m</td>
</tr>
<tr>
<td>Distance of CG from keel</td>
<td>92.4 m</td>
</tr>
<tr>
<td>Distance of CG to fairleads</td>
<td>0.2 m</td>
</tr>
<tr>
<td>Structural damping ratio</td>
<td>0.10, 0.05 and 0.03</td>
</tr>
<tr>
<td>Wave period</td>
<td>12.5 s</td>
</tr>
<tr>
<td>Wave height</td>
<td>7 m</td>
</tr>
<tr>
<td>Drag coefficient ($C_d$)</td>
<td>1.0 and 0.0</td>
</tr>
<tr>
<td>Inertia coefficient ($C_m$)</td>
<td>2.0 and 1.8</td>
</tr>
</tbody>
</table>
Table 3
Natural time period for a Spar platform with different initial horizontal force (s)

<table>
<thead>
<tr>
<th>Time Instant</th>
<th>Case</th>
<th>Surge</th>
<th>Sway</th>
<th>Heave</th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response at t = 0</td>
<td>A</td>
<td>215.43</td>
<td>215.43</td>
<td>28.04</td>
<td>50.84</td>
<td>50.84</td>
<td>102.97</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>254.60</td>
<td>254.60</td>
<td>28.04</td>
<td>50.84</td>
<td>50.84</td>
<td>115.13</td>
</tr>
<tr>
<td>Steady state response</td>
<td>A</td>
<td>392.23</td>
<td>215.43</td>
<td>39.79</td>
<td>50.84</td>
<td>50.84</td>
<td>102.97</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>360.41</td>
<td>254.60</td>
<td>39.79</td>
<td>50.84</td>
<td>50.84</td>
<td>115.13</td>
</tr>
<tr>
<td>Response at t = 0</td>
<td>C</td>
<td>215.43</td>
<td>215.43</td>
<td>4.20</td>
<td>50.77</td>
<td>50.77</td>
<td>102.97</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>254.60</td>
<td>254.60</td>
<td>4.69</td>
<td>50.80</td>
<td>50.80</td>
<td>115.13</td>
</tr>
<tr>
<td>Steady state response</td>
<td>C</td>
<td>386.80</td>
<td>209.25</td>
<td>43.51</td>
<td>50.77</td>
<td>50.77</td>
<td>102.97</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>355.60</td>
<td>251.09</td>
<td>43.54</td>
<td>50.80</td>
<td>50.80</td>
<td>115.13</td>
</tr>
</tbody>
</table>

and two cases are taken for the initial horizontal force: (C) 2500 kN and (D) 2000 kN for coupled stiffness matrix considering both horizontal and vertical excursion of the mooring line. Table 3 shows the natural time period for all the four cases at the time, t = 0 and at steady state response. Table 4 gives the maximum value of steady state response for 2500 and 2000 kN initial horizontal force for different cases.

In the calculation of natural frequency, only diagonal term of the stiffness matrix is effective, as the mass matrix is diagonal due to lumped mass assumption.

It is observed from Table 3 that at time t = 0 for surge degree-of-freedom, time period for case B is more than case A, since at time t = 0 the response is nearly zero and from Fig. 7, the horizontal mooring force is more in case A than case B, so case B is less stiff than case A. Whereas it is observed from steady state response values in surge degree-of-freedom that the horizontal mooring force for case B (for nearly 12 m surge response) is more in comparison to case A. So the time period for case B decreases from case A in surge degree-of-freedom at steady state. There is no change in time periods at t = 0, for cases C and D in comparison to cases A and B because there is no effect of vertical excursion in the stiffness matrix for surge. The time periods at steady state response for cases C and D are similar to that of case A and B. Cases C and D for steady state response becomes stiffer so the time period is less in comparison to steady state response time periods for case A and B.

Table 4
Response for different initial horizontal force, H_o

<table>
<thead>
<tr>
<th>H_o(kN)</th>
<th>Case</th>
<th>Maximum displacement (m)</th>
<th>Maximum rotation (radian)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Surge</td>
<td>Sway</td>
<td>Heave</td>
</tr>
<tr>
<td>2500</td>
<td>A</td>
<td>-15.769</td>
<td>0.0008</td>
</tr>
<tr>
<td>2000</td>
<td>B</td>
<td>-13.474</td>
<td>-0.0021</td>
</tr>
<tr>
<td>2500</td>
<td>C</td>
<td>-14.773</td>
<td>0.2779</td>
</tr>
<tr>
<td>2000</td>
<td>D</td>
<td>-12.625</td>
<td>0.2278</td>
</tr>
</tbody>
</table>
For sway degree-of-freedom at time $t = 0$ it is observed that time period for case B is more than case A, since at time $t = 0$, the response is nearly zero and from Fig. 7, the horizontal mooring force is more in case A than case B, so case B is less stiff than case A. There is no change in the time periods for cases A and B at steady state response, because an unidirectional wave train is considered and nearly zero response occur in sway degree-of-freedom. There is no change in time periods at $t = 0$, for cases C and D because there is no effect of vertical excursion in the stiffness matrix for sway. There is a little change in time periods at steady state response for cases C and D, because considering vertical excursion there is a small displacement in sway direction, which makes the system stiffer, so the time period reduces in comparison to at time $t = 0$.

For heave degree-of-freedom only hydrostatic force influences the stiffness matrix for cases A and B and as there is no change in the hydrostatic force so there is no change in the time period for both the cases A and B. The time period increases in cases A and B for the steady state response, because mass is increased by added mass, which makes the system more flexible in heave degree-of-freedom. Whereas, considering both horizontal and vertical excursion the stiffness matrix is modified by an additional component, which makes the system stiffer, so the time period decreases for cases C and D, when $t = 0$. At steady state response for cases C and D time period increases due to added mass, similar to cases A and B and also due to negative excursion in heave direction stiffness reduces in comparison to cases A and B, so the time period is greater in comparison to cases A and B.

For roll and pitch degree-of-freedom for cases A and B, it is observed that there is very little change in mooring stiffness because of unidirectional wave train nearly zero response in roll and little change in response of pitch degree-of-freedom is achieved. Whereas for cases C and D at time $t = 0$, additional component in the stiffness matrix makes it stiffer, so the time period is less in comparison with cases A and B. There is no change in the time period at the steady state response, because an unidirectional wave train is considered resulting in nearly zero response in roll and little change in response in the pitch degree-of-freedom.

For the yaw degree-of-freedom at time $t = 0$ it is observed that the time period for case B is more than case A, since at $t = 0$, the response is nearly zero and from Fig. 7, the horizontal mooring force is more in case A than in case B, so case B is less stiff than case A. There is no change in the time periods at the steady state response, because an unidirectional wave train is considered and a nearly zero response occurs in the yaw degree-of-freedom. There is no change in time period at $t = 0$ for cases C and D in comparison to cases A and B because there is no effect of vertical excursion in the stiffness matrix for yaw. Steady state response for cases C and D is similar to that of cases A and B.

Table 4 shows that there is a decrease of 14.5% in surge, a decrease of 23.4% in heave and a decrease of 0.5% in pitch responses when case B is considered in comparison to case A. Similarly, there is a decrease of 14.5% in surge, a decrease of 18% in heave and a decrease of 0.78% in pitch responses when case D is considered in comparison to case C. For the pitch degree-of-freedom it is observed that there
is very slight difference between cases A and C. A reduction in initial horizontal force decreases surge and heave responses significantly.

From Table 4 it is also seen that there is a decrease of 6.32% in surge, an increase of 50.51% in heave and an increase of 0.26% in pitch response, when case C is compared to case A. Similarly there is a decrease of 6.30% in surge and increase of 61.12% in heave response, when case D is compared to case B. There is no change for pitch response when case D is compared to case B. Inclusion of vertical spring in the stiffness matrix reduces the stiffness component in sway, heave, roll, pitch and yaw direction and makes it less stiff, so the displacement is more in cases C and D in comparison to cases A and B. Whereas the stiffness component in surge direction get enhanced and makes it stiffer, so the displacement in surge direction is less in cases C and D in comparison to cases A and B. Considering vertical excursion in mooring line affects surge and heave response significantly when cases A and C and cases B and D is compared. From Table 4 it is also seen that considering horizontal excursion of the mooring line there is no sway, roll and yaw responses, whereas when both horizontal and vertical excursions are considered there is slight sway response in addition to surge, heave and pitch.

With having a lower mooring system stiffness in cases B and D, the structure is more flexible and gives lower dynamic amplification of the response, although the static contribution of response being higher for lower stiffness of the structure. Cases A and C, on the other hand, gives higher response as the structure is stiff and produces comparatively more dynamic amplification of the static response which is lower than cases B and D. This is due to the nonlinear behavior of the cable force. The difference in the cable forces for cases B and D is more than the difference in the cable forces for cases A and C. So for lower initial horizontal force it become stiffer in comparison to higher initial horizontal force. This indicates that the better performance of Spar platforms can be achieved with lower stiffness of mooring system. It is necessary to understand the correct nonlinear behavior of the mooring lines as it directly effects the Spar response.

Figs. 9 and 10 gives the comparison between the steady state response time history of sway and heave of Spar platform for different initial horizontal force for cases A, B, C and D.

From Fig. 9 it is seen that considering the horizontal excursion of the mooring line sway response is almost zero for cases A and B, whereas when both horizontal and vertical excursions of the mooring line are considered then there is a slight sway response for cases C and D. The maximum values of positive response for cases C and D is 0.278 and 0.228 m, respectively, whereas the maximum values of the negative response for cases C and D is -0.091 and -0.113 m, respectively.

From Fig. 10 it is seen that considering only horizontal excursion of the mooring line the maximum values of heave response for cases A and B is -1.778 and -1.362 m, respectively, whereas when both horizontal and vertical excursions of the mooring line are considered then the maximum values of heave responses for cases C and D is -2.678 and -2.195 m, respectively. It is observed that consideration of vertical excursion in mooring line increases heave responses and also there is a set down in heave direction for the Spar platform.
Fig. 9. Steady state time history of sway response.

Fig. 10. Steady state time history of heave response.
11.2. Effect of coupling of stiffness matrix

Two cases are taken: (A) for the coupled stiffness matrix; and (B) for the uncoupled stiffness matrix considering horizontal excursion of the mooring line and two cases are taken: (C) for the coupled stiffness matrix and (D) for the uncoupled stiffness matrix considering both horizontal and vertical excursion of the mooring line. Table 5 gives the comparison between the maximum values of the steady state response for coupled and uncoupled stiffness matrix for different cases.

Table 5 shows that there is a decrease of 0.16% in surge response, 98.56% in heave response and 2.08% in pitch response, when the uncoupled stiffness matrix for case B is considered in comparison to the coupled stiffness matrix for case A. There is an increase of 6.6% in surge response, a decrease of 83.01% in heave response and 1.55% in pitch response, when the uncoupled stiffness matrix for case D is considered in comparison to the coupled stiffness matrix for case C. Coupling plays an important role on the dynamic analysis of an offshore Spar platform. It is seen that when coupled and uncoupled stiffness matrix is considered heave is affected most. It is also seen that there is a decrease of 6.32% in surge, an increase of 50.51% in heave and an increase of 0.26% in pitch response, when the coupled stiffness matrix for case C is compared in comparison to the coupled stiffness matrix for case A. There is an increase of 0.03% in surge, an increase of 1677.3% in heave and an increase of 1.06% in pitch response, when the uncoupled stiffness matrix for case D is compared with the uncoupled stiffness matrix for case B. Displacements and rotation is more in case D because the stiffness is less when compared to case B. The stiffness matrix plays the most important role in the overall response analysis because it is response dependent and is based on the nonlinear cable force. Consideration of vertical excursion gives rise to significant changes in surge and heave response, when cases A and C and cases B and D are compared.

The sway, roll and yaw response is zero for the uncoupled case as an unidirectional wave is taken, while for the coupled case the responses are almost zero, which means in these degrees-of-freedom there is no displacement and rotation. The result shows that coupling of degrees-of-freedom has a significant effect. In all further studies, coupled stiffness matrix is considered.

<table>
<thead>
<tr>
<th>Stiffness</th>
<th>Case</th>
<th>Max. displacement (m)</th>
<th>Max. rotation (radian)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Surge</td>
<td>Heave</td>
</tr>
<tr>
<td>Coupled</td>
<td>A</td>
<td>-15.769</td>
<td>-1.779</td>
</tr>
<tr>
<td>Uncoupled</td>
<td>B</td>
<td>-15.744</td>
<td>-0.0256</td>
</tr>
<tr>
<td>Coupled</td>
<td>C</td>
<td>-14.773</td>
<td>-2.678</td>
</tr>
<tr>
<td>Uncoupled</td>
<td>D</td>
<td>-15.749</td>
<td>-0.455</td>
</tr>
</tbody>
</table>
Fig. 11 gives the comparison between the steady state response time history of heave of the Spar platform for different coupled and uncoupled stiffness matrices for cases A, B, C and D.

From Fig. 11 it is seen that considering horizontal excursion of the mooring line for the uncoupled stiffness matrix for case B the heave response is almost zero. For the coupled case A the maximum value of the heave response is -1.779 m, whereas when both horizontal and vertical excursions of the mooring line are considered then there is a slight increase in displacement for uncoupled case D and the maximum value of the heave response is -0.455 m. The maximum value of the heave response for coupled case C is -2.678 m. It is observed that consideration of vertical excursion in the mooring line increases heave responses and there is a set down in the heave direction for the Spar platform.

11.3. Effect of structural damping

Three cases are taken for a damping ratio of 10% (case A), 5% (case B) and 3% (case C) considering horizontal excursion of the mooring line and three cases are taken for structural damping ratio of 10% (case D), 5% (case E) and 3% (case F) considering both horizontal and vertical excursion of the mooring line. Table 6 gives the response for 10, 5 and 3% structural damping ratios for different cases.

Table 6 shows that when only the horizontal excursion of the mooring line is considered there is no change in the surge and pitch response when 5% structural damping in case B is compared to 10% structural damping in case A. There is a
Table 6
Response for different structural damping ratios

<table>
<thead>
<tr>
<th>Structural damping (%)</th>
<th>Case</th>
<th>Maximum displacement (m)</th>
<th>Maximum rotation (radian)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Surge</td>
<td>Heave</td>
</tr>
<tr>
<td>10</td>
<td>A</td>
<td>-15.769</td>
<td>-1.710</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>-15.769</td>
<td>-1.779</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>-15.733</td>
<td>-1.845</td>
</tr>
<tr>
<td>10</td>
<td>D</td>
<td>-13.144</td>
<td>-2.734</td>
</tr>
<tr>
<td>5</td>
<td>E</td>
<td>-14.773</td>
<td>-2.678</td>
</tr>
<tr>
<td>3</td>
<td>F</td>
<td>-15.392</td>
<td>-2.662</td>
</tr>
</tbody>
</table>

decrease of 0.23% in surge, an increase of 0.26% in pitch response when 3% structural damping in case C is compared to 10% structural damping in case A. There is increases of 4.04%, 7.89% in heave response when cases B, C are compared to case A, respectively. Whereas, when both horizontal and vertical excursions of the mooring line are considered for cases D, E and F it is observed that there is increases of 12.39%, 17.10% in surge, decreases of 2.04%, 2.63% in heave when cases E and F are compared with case D, respectively. There is no change in pitch response for cases D and E, whereas there is a decrease of 0.26% in pitch response when case F is compared to case D. It is observed that for a higher structural damping ratio there is no effect in surge and pitch response when only horizontal excursion of the mooring line is considered, whereas a reduction in structural damping affects heave responses significantly when cases A, B and C are compared. Whereas it affects surge response significantly when cases D, E and F are compared.

It is also seen that there is decreases of 16.64, 6.32 and 2.16% in surge response, increases of 59.88, 50.53 and 44.28% in heave response, and increases of 0.26, 0.26 and 0.26% in pitch response, when case D is compared to case A, case E is compared to case B, and case F is compared to case C, respectively. Consideration of vertical excursion gives rise to significant changes in surge and heave response, when cases A and D, cases B and E and cases C and F are compared.

Figs. 12 and 13 gives the comparison between the steady state response time history of sway and heave of the Spar platform for different structural damping ratio for cases A, B, C and D.

From Fig. 12 it is seen that considering horizontal excursion of the mooring line sway response is almost zero for cases A, B and C, whereas when both horizontal and vertical excursion of the mooring line is considered then there is a slight sway response for cases D, E and F. The maximum values of positive response for cases D, E and F are 0.866, 0.277 and 0.135 m, whereas the maximum value of negative response for cases E and F are -0.091 and -0.107 m.

From Fig. 13 it is seen that considering only horizontal excursion of the mooring line the maximum values of heave response is -1.710, -1.779 and -1.845 m for cases A, B and C, whereas when both horizontal and vertical excursions of the mooring
Fig. 12. Steady state time history of sway response.

Fig. 13. Steady state time history of heave response.
line is considered then the maximum values of heave responses for cases D, E and F is -2.734, -2.678 and -2.662 m. There is not much difference in heave response for cases D, E and F. It is observed that consideration of vertical excursion in mooring line increases heave responses and there is also a set down in the heave direction for the Spar platform.

11.4. Effect of inertia coefficient $C_m$

Two cases are taken when $C_m$ is equal to 2 (case A) and 1.8 (case B) considering the horizontal excursion of the mooring line and two cases are taken for when $C_m$ is equal to 2 (case C) and 1.8 (case D) considering both the horizontal and vertical excursion of the mooring line. Surge force, heave force and pitch moment reduces when $C_m$ reduces from 2 to 1.8 while calculating the force using Morison's equation. Table 7 gives the response for $C_m$ equal to 2 and 1.8 for different cases.

Table 7 shows that there is a decrease of 15.79% in surge, a decrease of 7.25% in heave and a decrease of 10.38% in pitch response when $C_m$ is equal to 1.8 (case B) is considered in comparison to $C_m$ equal to 2 (case A). There is a decrease of 15.93% in surge, increase of 0.41% in heave and decrease of 10.36% in pitch response when $C_m$ is equal to 1.8 (case D) is considered in comparison to $C_m$ equal to 2 (case C). Reduction in inertia coefficient affects the surge, heave and pitch response. It is seen that there is a decrease of 6.32% in surge, an increase of 50.51% in heave and an increase of 0.26% in pitch response, when case C is compared to case A. Similarly there is a decrease of 6.47% in surge, an increase of 62.96% in heave and an increase of 0.29% in pitch response, when case D is compared to case B. Consideration of vertical excursion gives rise to significant changes in surge and heave response, when cases A and C and cases B and D are compared.

Fig. 14 gives the comparison between the steady state response time history of heave of the Spar platform for different inertia coefficient values, $C_m$ for cases A, B, C and D.

From Fig. 14 it is seen that considering only the horizontal excursion of the mooring line the maximum values of the heave response are -1.779 and -1.650 m for cases A and B, respectively, whereas when both horizontal and vertical excursion of the mooring line is considered then the maximum values of heave responses for

<table>
<thead>
<tr>
<th>Inertia coefficient</th>
<th>Case</th>
<th>Maximum displacement (m)</th>
<th>Maximum rotation (radian)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Surge</td>
<td>Heave</td>
</tr>
<tr>
<td>$C_m = 2.0$</td>
<td>A</td>
<td>-15.769</td>
<td>-1.779</td>
</tr>
<tr>
<td>$C_m = 1.8$</td>
<td>B</td>
<td>-13.279</td>
<td>-1.650</td>
</tr>
<tr>
<td>$C_m = 2.0$</td>
<td>C</td>
<td>-14.773</td>
<td>-2.678</td>
</tr>
<tr>
<td>$C_m = 1.8$</td>
<td>D</td>
<td>-12.419</td>
<td>-2.689</td>
</tr>
</tbody>
</table>
cases C and D are -2.678 and -2.689 m, respectively. There is not much difference in heave response for cases C and D. It is observed that consideration of vertical excursion in the mooring line increases heave responses and also there is a set down in heave direction for the Spar platform.

11.5. Effect of drag coefficient $C_d$

Two cases are taken for $C_d$ equal to 1 (case A) and 0 (case B) considering horizontal excursion of the mooring line and two cases are taken for $C_d$ equal to 1 (case C) and 0 (case D) considering both horizontal and vertical excursion of the mooring line. Surge and heave force reduces as the total force decreases when drag coefficient is zero while, calculating force using Morison’s equation. Table 8 gives the response for $C_d$ equal to 1 and 0 for different cases.

<table>
<thead>
<tr>
<th>Drag coefficient $C_d$</th>
<th>Case</th>
<th>Maximum displacement (m)</th>
<th>Maximum rotation (radian)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Surge</td>
<td>Heave</td>
</tr>
<tr>
<td>$C_d = 1$</td>
<td>A</td>
<td>-15.769</td>
<td>-1.779</td>
</tr>
<tr>
<td>$C_d = 0$</td>
<td>B</td>
<td>-11.910</td>
<td>-1.667</td>
</tr>
<tr>
<td>$C_d = 1$</td>
<td>C</td>
<td>-14.773</td>
<td>-2.678</td>
</tr>
<tr>
<td>$C_d = 0$</td>
<td>D</td>
<td>-15.485</td>
<td>-2.676</td>
</tr>
</tbody>
</table>
Table 8 shows that there is a decrease of 24.47% in surge, a decrease of 6.30% in heave and an increase of 0.52% in pitch response when $C_d$ equal to 0 (case B) is considered in comparison to $C_d$ equal to 1 (case A). There is a increase of 4.82% in surge, a decrease of 0.07% in heave and an increase of 1.29% in pitch response when $C_d$ equal to 0 (case D) is considered in comparison to $C_d$ equal to 1 (case C). Reduction in drag coefficient affects surge and heave response more when only horizontal excursion is considered. It is also seen that there is a decrease of 6.32% in surge, an increase of 50.51% in heave and an increase of 0.26% in pitch response, when case C is compared to case A. There is an increase of 30.01% in surge, an increase of 60.52% in heave and an increase of 1.03% in pitch response, when case D is compared to case B. Consideration of vertical excursion gives rise to significant changes in surge and heave response, when cases A and C and cases B and D are compared.

Although the Spar exhibits an inertia dominated force regime, the influence of the drag coefficient is appreciable in surge response but not in the heave response.

Fig. 15 gives the comparison between the steady state response time history of heave of the Spar platform for different drag coefficient values, $C_d$, for cases A, B, C and D.

From Fig. 15 it is seen that when considering only horizontal excursion of the mooring line the maximum values of heave response is $-1.779$ and $-1.667$ m for cases A and B, respectively, whereas when both horizontal and vertical excursions of the mooring line is considered then the maximum values of heave responses for cases C and D is $-2.678$ and $-2.676$ m, respectively. There is not much difference
in heave response for cases C and D. It is observed that consideration of vertical excursion in the mooring line increases heave responses and there is also a set down in heave direction for the Spar platform.

12. Conclusions

Based on the numerical study conducted on the Spar platform, the following conclusions can be drawn:

1. Modeling the nonlinear force-excursion (horizontal and vertical) relationship of the mooring lines with different slopes (stiffness) gives the reasonably accurate behavior of Spar responses, whereas modeling the force-excursion (horizontal and vertical) relationship of the mooring line with multilinear segments can result in unrealistic Spar responses.

2. Inclusion of vertical spring plays an important role on the dynamic behavior of Spar response. Also it is seen from the study that neither drag coefficient, inertia coefficient and structural damping ratio variations affects the heave response, whereas these parameters do influence the response, when only horizontal excursion is considered.

3. Considering the effect of vertical excursion of the mooring lines, the heave natural period of the spar is decreased, which makes spar's natural period in heave degrees-of-freedom nearer to the frequently occurring wave periods.

4. The horizontal force-excursion relation of a single mooring line depends mainly on the initial horizontal force. Reducing the initial horizontal force in the mooring line, lowers the Spar response. It is necessary to understand the correct nonlinear behavior of the mooring lines as it directly effects the Spar response.

5. The coupling of the stiffness matrix of the Spar platform play an important role in the dynamic behavior of offshore Spar platforms as the response is significantly affected by considering the coupled stiffness matrix.

6. The influence of structural damping is very minimal on the overall response of the Spar platform for higher structural damping ratios when only horizontal excursion of the mooring line is considered, whereas it is considerable for lower structural damping ratios when horizontal excursion of the mooring line and when both horizontal and vertical excursions of the mooring line is considered.

7. It is necessary to evaluate the proper value of $C_m$ so that wave force can be properly estimated as it has significant effect on the response of the Spar platform.

8. The effect of $C_d$ is important as drag force affects the total force although the Spar is a large diameter structure and is relatively inertia dominated.

References


Triantafyllou, M.S., Kardomateas, G., Bliek, A., 1982. The statics and dynamics of the mooring lines of