SECOND-ORDER DRIFT FORCE RESPONSE OF OFFSHORE GUYED TOWERS

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Abstract—An iterative frequency domain method of analysis is presented for determining the response behaviour of Guyed Offshore Towers to low-frequency, second-order wave drift forces generated in a random sea environment. For the response analysis, the tower is idealized as a shear beam with a rotational spring at the bottom support. The guylines are replaced by a non-linear spring. The second-order drift force is considered to be proportional to the square of the wave elevation and is simulated using a drift force coefficient and the time history of a slowly varying wave envelope in random sea. The responses due to drift forces are obtained in frequency domain by incorporating the non-linearities produced due to non-linear guy lines. An example problem is solved under different random sea states to compare the response behaviour of the tower obtained by the second-order wave force, the first-order wave force and a combination of the two.

INTRODUCTION

Offshore Guyed Tower platforms belong to the group of compliant offshore platforms which is most suited for the deep-water search for oil and gas. The basic feature of compliant offshore platforms is that they are designed to move with the waves, in at least some of their degrees of freedom. As far as excitation at wave frequencies is concerned, the system opposes wave forces by inertial effects. This is in contrast to the fixed platform, which opposes wave forces by stiffness effect in the structure foundation. Hence the fundamental natural frequency of a compliant system is designed to be well below the frequency range of waves having significant energy in the most extreme storm or swell likely to be encountered. By their very nature, therefore, compliant systems are susceptible to excitation at much lower frequencies than those of waves. Analysis of these structures for first-order wave forces would then invariably predict very low responses. The effect of second-order wave forces, which have generally significant energy content at low frequencies, on the response of such platforms must be included in their overall analysis.

Investigation of the second-order drift forces, exciting low frequency responses, has been extensively conducted for floating structures like buoys, semi-submersibles, etc. Potential theory was used to derive the second-order drift forces by Pinkster (1979), Pinkster and Huijsmans (1982), Standing et al. (1981), Chakrabarti and Cotter (1984), Eatock Taylor (1986) and Karppinen (1979), where the wave drift forces were defined as the second-order term in the integrated fluid pressure on the submerged body and was found to be proportional to the square of the wave amplitude. Salvesan et al. (1982), Wahab (1974), Pijfers and Brink (1977) and Ferretti and Berta (1980) studied
the slowly varying motion caused by the second-order viscous drift force due to interaction between current, wind and wave force. The drift force was defined as the mean value of the total force over a wave period and was derived using Morison's equation. The time histories of wave drift force for random waves were generated with the help of the time history of the square of the slowly varying wave envelope (hence, second order) and the drift force operator (i.e. drift force coefficient plotted against frequency of encounter).

Peak mooring forces were analysed by Hsu and Blenkarn (1970), which were caused by the vessel's slow drift oscillation in a random sea. They calculated the wave drift force based on conservation of momentum principles and the concept of "radiation stress" introduced by Longuet-Higgins and Steward (1964). Remery and Hermans (1971) also investigated the slow drift oscillations of a moored object in random seas. The drift force was calculated as a function of the square of the reflected and scattered wave elevation. The reflection coefficient was determined from experimental tests on a rectangular barge in head waves.

Kim and Yue (1989) have presented a new method for predicting slowly varying wave drift excitations in multidirectional seas. The method retains the assumption of narrow bandedness in frequency but treats the wave directional spreading exactly. Arvid Naess (1989) has emphasised the problem of estimating the extreme values of the combined first order and slow drift response of an offshore structure, subjected to a stationary irregular sea wave. The study indicates a strong coupling between the two orders. Kato (1990) has presented an approximate solution for calculating the psdf of total second-order responses, including first-order as well as second-order motions. A statistical interference between first- and second-order responses was also shown. Grue (1988) has examined wave drift damping and low-frequency oscillations of a moored elliptic cylinder. For a long incoming wave he has obtained positive damping; but for short incoming waves damping forces were found to be negative. Langley (1987) has derived the probability density function for low-frequency, second-order forces and motions using a method which represents the random sea state as a sum of regular waves. Grue and Palm (1986) have analysed the influence of a uniform current on slowly varying forces and displacements and the result shows that current may have a great impact on the slowly varying motion of a moored body.

Jain and Datta (1991) have carried out investigations on Articulated Towers for the response due to viscous drift forces. They have calculated the slowly varying drift force by assuming that the drift force is proportional to the product of the square of the wave envelope and the drift force coefficient in regular waves.

In the present investigation, the low-frequency oscillation of a Guyed Tower due to second-order viscous drift forces in a random sea environment is determined and analysed to study its importance on the overall response of the tower (i.e. due to first- and second-order wave force). For the analysis, an iterative frequency domain method is employed using Fast Fourier Transform to obtain the non-linear response of the Guyed Tower. The non-linearities included in the first-order analysis are those produced due to relative velocity squared drag force, non-linear restoring force provided by the guy lines and the effect of the instantaneous position of the tower. The second-order wave force is generated by a simulation procedure, assuming it to be proportional to
the square of the wave envelope. The response analysis due to the second-order wave force considers the effect of the non-linear restoring force provided by the guylines.

ANALYSIS

Guyed Tower model

A prototype Guyed Offshore Tower has been considered to carry out the present investigations. The tower is shown in Fig. 1. The bottom of the tower may either be hinged or provided with a restricted rotational restraint. For the present study the tower has been idealised as a uniform shear beam with a rotational spring at the base of the tower, as shown in Fig. 2. Varying degrees of rotational movement of the tower may be achieved by varying the rotational spring stiffness. The guylines have been idealised by a non-linear spring. The contribution of buoyancy to the restoring force in these structures is generally small when compared with that from the guylines stiffness, and is, therefore, neglected in this study (Dutta, 1984). Further, it is assumed that the dynamics of the guylines does not significantly influence the global tower motion. Thus, the stiffness of the non-linear spring is obtained from a separate static analysis of the guylines under its own weight and current-induced forces. The typical force excursion relationship of the guylines for the example problem is shown in Fig. 3.

The offshore tower is discretised into a number of plane two-dimensional beam elements. At each node, the dynamic degrees of freedom consist of sway translation and in-plane rotation. The equation of motion in structural coordinates takes the form:

$$[M] \{\ddot{x}\} + [C] \{\dot{x}\} + [K] \{x\} = F(t).$$

(1)

In the mass matrix $[M]$, added mass of the element is considered up to still-water level. The effect of variable added-mass due to the instantaneous position of the tower is assumed to be negligible and is ignored in the formulation of the problem. The
damping matrix \([C]\) of the system is assumed to be a linear combination of (constant) mass and stiffness matrices; \(\{\ddot{x}\}, \{\dot{x}\}\) and \(\{x\}\) are, respectively, the vectors of structural acceleration, velocity and displacement; the stiffness matrix \([K]\) is made time-invariant by considering it in only the initial tangent stiffness of the non-linear spring representing the guy system. The correction to the resistive force at any instant of time, necessary for the non-linearity of the spring system, is incorporated in the right-hand side load vector. Thus, the load vector depends not only on the structural velocity [see Equation (2)] but also on the structural displacement. The vector of the nodal loads is computed.
by assuming the intensity of hydrodynamic loading to vary linearly along the element length.

**Simulation of the ocean environment**

The Wave Superposition technique given by Jain (1984) and Goda (1970) has been adopted for simulating the water particle kinematics, which in turn are used in the Morison equation to simulate the representative time histories of the hydrodynamic loading at various nodes along the depth of the structure. The widely used modified Morison equation for calculating the hydrodynamic load intensity per unit length normal to the beam element is given as:

\[
f_i(t) = 0.5 \beta_w C_d D_d |\hat{u}_i - \hat{x}_i| (\hat{u}_i - \hat{x}_i) + 0.25 C_m \beta_w \pi D_m^2 \ddot{u}_i - 0.25 (C_m - 1) \beta_w \pi D_m^2 \dddot{x}_i
\]

where

- \(D_d\) = effective drag diameter of the tower
- \(D_m\) = effective inertia diameter of the tower
- \(C_d\) = drag coefficient
- \(C_m\) = inertia coefficient
- \(\beta_w\) = mass density of sea water
- \(\dddot{x}_i\) = structural velocity normal to the instantaneous position of tower axis
- \(\dddot{u}_i\) = water particle velocity normal to instantaneous position of tower axis
- \(\dddot{u}_i\) = water particle acceleration normal to instantaneous position of tower axis.

Hence the hydrodynamic loading is dependent both on displacement and velocity of the structure.

**Simulation of drift forces**

For the present investigation, the time histories of the drift forces are obtained using a method proposed by Pijfers and Brink (1977). As per this method, a mean hydrodynamic force at various nodes is calculated, which is the time average of all forces acting on the structure, obtained through Morison's concept. The wave drift force is then defined as the mean force as a consequence of the waves and can be calculated by subtracting the true current force from the hydrodynamic force. The method is based on the assumption that the wave spectrum is sufficiently narrow. A random, long-crested sea may then be written as an amplitude modulated signal:

\[
\delta(t) = \text{Re} \Omega(t) = \text{Re} [R(t) \{\exp i \Gamma(t)\}]
\]

where

- \(\delta(t)\) = wave amplitude
- \(R(t)\) = slowly varying wave envelope
- \(\Omega(t)\) = a complex variable whose real part provides the wave amplitude
- \(\Gamma(t)\) = the phase angle, equal to \(\omega_c t + \epsilon(t)\), where \(\omega_c\) is the central frequency of the narrow band spectrum and \(\epsilon(t)\) is a slowly varying phase angle.
The above formulation leads to the expression for the so-called momentary frequency of the spectrum, given by:

\[ \hat{\Gamma}(t) = \omega_c + \epsilon(t). \] (4)

The slowly varying drift force is then calculated assuming that the drift force is proportional to the product of the square of the wave envelope \( R(t) \) and the drift force coefficient in regular waves at the momentary frequency \( \hat{\Gamma}(t) \). Hence, mathematically the drift force \( D_i(t) \) at any node \( i \) of the tower is then given by:

\[ D_i(t) = 0.5 \beta \omega_g C_i(\hat{\Gamma}) R^2(t). \] (5)

To evaluate the value of \( D_i(t) \) as given above, we require:

(a) time histories of \( R^2(t) \)

(b) time histories of \( \hat{\Gamma}(t) \)

(c) drift force coefficient \( C_i \) at each momentary frequency \( \hat{\Gamma}(t) \).

**Determination of time histories of \( R^2(t) \) and \( \hat{\Gamma}(t) \)**

The wave envelope \( R \) is the modulus of the complex variable, that is:

\[ R^2 = (Re \omega)^2 + (Im \omega)^2. \] (6)

Also

\[ \hat{\Gamma} = \tan^{-1} \left( \frac{Im \omega}{Re \omega} \right) \] (7)

and

\[ \hat{\Gamma} = \left[ (Re \omega)^* (Im \omega) - (Im \omega)^* (Re \omega) \right] / \left[ (Re \omega)^2 + (Im \omega)^2 \right] \]

\[ = \frac{1}{R^2} \left[ (Re \omega)^* (Im \omega) - (Im \omega)^* (Re \omega) \right]. \] (8)

The irregular wave amplitude \( \delta(t) \) is given by the sum of a large number of regular waves, that is, using the concept of random wave generation:

\[ \delta(t) = Re \Omega(t) = \sum_n A_n \cos(\omega_n t + \Theta_n) \] (9)

in which \( A_n \) is calculated as a function of the wave spectral density and is obtained according to the well-known formula

\[ A_n = [2 S_\delta(\omega_n) \Delta \omega_n]^{1/2}. \] (10)

in which \( S_\delta(\omega_n) \) is the spectral density ordinate of the wave spectrum at frequency \( \omega_n \); \( \Theta_n \) is the random phase angle between 0 and \( 2\pi \) radians; \( \omega_n \) is the randomly varying frequency and \( \Delta \omega_n \) is the frequency interval. Since \( \delta(t) \) is considered as \( Re \Omega(t) \), the imaginary part of \( \Omega(t) \) is found to be

\[ Im \Omega(t) = \sum_n A_n \sin(\omega_n t + \Theta_n). \] (11)

The time domain derivatives of \( Re \Omega \) and \( Im \Omega \) are, therefore:

\[ Re \Omega = - \sum_n A_n \omega_n \sin(\omega_n t + \Theta_n) \] (12)
Second-order drift force response

\[ Im \Omega = \sum_n A_n \omega_n \cos(\omega_n t + \Theta_n). \] (13)

Using Equations (9)-(13), the time histories of \( R^2(t) \) and \( \hat{\Gamma}(t) \) can be determined from Equations (6) and (8).

**Determination of drift force coefficient**

The drift force coefficient \( C_i \) at any given momentary frequency \( \pi \) is obtained from the plot of \( C_i \) vs \( \sigma_e \), where \( \sigma_e \) is the frequency of encounter. For a given regular wave of amplitude \( \delta_a \) and frequency \( \sigma \), the drift force coefficient \( C_i \) is given by

\[ C_i = \overline{D_i}/(0.5 \beta_w g \delta_a^2), \] (14)

where \( \overline{D_i} \) is the mean drift force and is found by subtracting the true current force from the mean hydrodynamic force, that is:

\[ \overline{D_i} = \overline{F_i} - F_{ci} \] (15)

where

- \( F_{ci} \) = current force
- \( \overline{F_i} \) = mean hydrodynamic force
- \( i \) = node number.

The mean hydrodynamic force \( \overline{F_i} \) at node \( i \) is obtained by determining the mean hydrodynamic forces over one period of encounter \( T \), that is:

\[ \overline{F_i} = \frac{1}{T} \int_0^T f_i(t) \, dt \] (16)

where \( f_i(t) \) = hydrodynamic force at node \( i \).

The hydrodynamic force \( f_i(t) \) is calculated from Morison's equation, namely:

\[ f_i(t) = 0.5 \beta_w C_d D_d |\dot{U}_i| (\ddot{U}_i) + 0.25 C_m \beta_w \pi D_m^2 \dot{X}_i - 0.25 \pi \beta_w (C_m - 1) D_m^2 \dot{X}_i. \] (17)

The relative velocity, \( \dot{\bar{U}} \), comprises the mass transport velocity, \( V_{mi} \), the water particle velocity, \( \dot{U}_i \), the current velocity, \( \dot{U}_{ci} \), and the structural velocity, \( \dot{X}_i \), and is given by:

\[ \dot{\bar{U}}_i = V_{mi} + \dot{U}_i + \dot{U}_{ci} - \dot{X}_i \] (18)

in which \( V_{mi} \), in deep water, according to Stokes theory, is given by:

\[ V_{mi} = \delta_a^2 K \sigma \exp(-2Ky), \] (19)

where \( K \) is the wave number, and \( y \) is the function of water depth. Since the computation of \( f_i(t) \) as given by Equation (17) requires \( \dot{X}_i \) and \( \ddot{X}_i \), the response of the tower is to be obtained for the regular wave of amplitude \( \delta_a \) and frequency \( \sigma \). The frequency of encounter, to include the current velocity, is given by Pizfers and Brink (1977):

\[ \sigma_e = \sigma (1 + \sigma \dot{U}_i/g). \] (20)
Iterative frequency-domain solution

In the present work the non-linear equation of motion—Equation (1)—is solved by the iterative frequency domain approach similar to that proposed by Jain and Datta (1987). The difference between the solution of Jain and Datta (1987) and the present solution is in treating the non-linear restoring force due to guylines in the right-hand side load vector. As a consequence, the solution technique will be presented in brief, highlighting only the changes that are necessary to accommodate the non-linear restoring guyline forces.

The equation of motion (1) can be transformed into the frequency domain as

\[
\psi_p = \begin{pmatrix} \psi_{pc} \\ \psi_{ps} \end{pmatrix} = 0 \quad p = 1, \ldots, n. \quad (21)
\]

Considering the response in the \( p \)th harmonic and separating the cosine and the sine terms, the equations for evaluating \( X_{pc} \) and \( X_{ps} \) are found to be:

\[
\psi_{pc} = [K - (p \sigma)^2 M] X_{pc} - (p \sigma) C X_{ps} - F_{pc} \quad (22)
\]

\[
\psi_{ps} = (p \sigma) C X_{pc} + [K - (p \sigma)^2 M] X_{ps} - F_{ps} \quad (23)
\]

where \( \psi_p \) is a vector of order \( 4N \), each of the components \( \psi_{pc} \) and \( \psi_{ps} \) is of the order of \( 2N \), \( N \) being the total number of the nodes and \( 2N \) being the total number of degrees-of-freedom; \( X_{pc}, X_{ps}, \) etc., are components of Fourier transform \( x(t), f(t), \) etc.

The determination of the Fourier components \( F_{pc} \) and \( F_{ps} \) of load vector \( f(t) \) is not straightforward since \( f(t) \) depends on the structural velocity and displacement as explained below. In general, the load vector \( f(t) \) may be expressed as:

\[
f(t) = K^d S + K^m \ddot{U} + K^R x \quad (24)
\]

in which \( K^d \) and \( K^m \) are, respectively, the drag and inertia coefficient matrices of order \( (2N \times N) \) and \( K^R \) is a \( (2N \times 2N) \) diagonal matrix having non-zero value only at the \( g \)th diagonal element, where the \( g \)th row corresponds to the translational degree of freedom at the node where the guyline is attached; \( S \) and \( \ddot{U} \) are vectors of order \( N \), where

\[
S_j = |V_j(t)| \dot{V}_j(t) \quad (25)
\]

The relative velocity is defined by

\[
V_j(t) = \dot{u}_j - \dot{x}_j \quad (26)
\]

where \( \dot{u}_j \) is the horizontal fluid velocity and \( \dot{x}_j \) is the horizontal structural velocity, both at the node \( j \).

Since the dynamic loading is dependent on the structural velocity and displacement, solution of Equation (21) requires iteration. Determination of \( X_p \) by the exact Newton Raphson iteration scheme can be very expensive even if the number of iterations required for convergence is relatively small. In order to make the iteration scheme less expensive, the Jacobian in the Newton–Raphson scheme is evaluated approximately by ignoring the derivatives of the \( p \)th harmonic of the nodal loads with respect to the \( q \)th harmonic of any of the nodal displacements \( (q \neq p) \) as a first approximation.
With this approximation, the equation for the $r$th iteration becomes

$$J_{pp} X_p^r = J_{pp} X_p^{r-1} - \psi_p^r,$$

(27)
in which $\psi_p$ is the function defined by Equation (21); the superscript $r$ describes the iteration number; $J_{pp}$ is the $(4N \times 4N)$ Jacobian matrix, and is found as follows:

$$J_{pp} = \frac{\partial \psi_p}{\partial X_p} = \begin{bmatrix} K - (p\sigma)^2 M & -(p\sigma) C \\ (p\sigma) C & K - (p\sigma)^2 M \end{bmatrix} - \begin{bmatrix} \frac{\partial F_{pc}}{\partial X_{pc}} & \frac{\partial F_{pc}}{\partial X_{ps}} \\ \frac{\partial F_{ps}}{\partial F_{pc}} & \frac{\partial F_{ps}}{\partial X_{ps}} \end{bmatrix} = \overline{K}_p - \overline{F}_p.$$ 

(28)

An element of $\overline{F}_p$ can be evaluated, for example, in the following manner:

$$\frac{\partial F_{pc}}{\partial X_{pc}} = \frac{2}{T} \int_0^T \frac{\partial f(t)}{\partial X_{pc}} \cos(p\sigma t) \, dt$$

$$= \frac{2}{T} K^d \int_0^T \frac{\partial S}{\partial X_{pc}} \cos(p\sigma t) \, dt + \frac{2}{T} K^R \int_0^T \frac{\partial x}{\partial X_{pc}} \cos(p\sigma t) \, dt$$

$$= 2 \frac{(p\sigma)}{T} K^d \int_0^T [V(t) \cdot 0] \sin(2p) \sigma t \, dt$$

$$+ \frac{1}{T} K^R \int_0^T [I] \cos(2p) \sigma t \, dt + \frac{1}{T} K^R \int_0^T [I] \, dt$$

(29)

where $V(t)$ is a diagonal matrix of order $N$, its $j$th element being $|V_j(t)|$ and 0 is a square null matrix, also of order $N$; $I$ is an identity matrix of order $(2N \times 2N)$.

Similar expressions can be obtained for other derivatives which constitute the elements of $\overline{F}_p$. With the help of the expressions for the derivatives as deduced in Equation (29), the $\overline{F}_p$ matrix can be finally put in the following form:

$$\overline{F}_p^r = 2 (p\sigma) \begin{bmatrix} -K^d \{D_{ee} \cdot 0\} + K^R/2 (p\sigma) & K^d \{D_{es} \cdot 0\} \\ -K^d \{D_{es} \cdot 0\} & K^d \{D_{ss} \cdot 0\} + K^R/2 (p\sigma) \end{bmatrix}$$

(30)
in which $D_{ee}, D_{es}, D_{se}$ and $D_{ss}$ are all diagonal matrices of order $N$; their $j$th elements are, respectively, $R_{sj}$, $(R_{oj} + R_{cj})$, $(R_{oj} - R_{cj})$, $R_{sj}$; and 0 is a square null matrix also of order $N$, in which $R_{oj}$, $R_{cj}$ and $R_{sj}$ are given by:

$$R_{sj} = \frac{1}{T} \int_0^T |V_j(t)| \, dt$$

(31)

$$R_{cj} = \frac{1}{T} \int_0^T |V_j(t)| \cos(2p) \sigma t \, dt$$

(32)

$$R_{sj} = -\frac{1}{T} \int_0^T |V_j(t)| \sin(2p) \sigma t \, dt.$$

(33)
Substituting Equations (22), (23) and (28) in (27), the iteration equation becomes:

\[ \overline{K_p} \overline{X^r_p} - \overline{F_p} \overline{X^r_p} = F^r_p - \overline{F_p} X^r_{p-1}, \]

that is,

\[
\begin{bmatrix}
K - (p\sigma)^2 M & - (p\sigma) C \\
(p\sigma) C & K - (p\sigma)^2 M
\end{bmatrix}
\begin{bmatrix}
X^r_{pc} \\
X^r_{ps}
\end{bmatrix}
+ 2(p\sigma)
\begin{bmatrix}
K^d (D^r_{cc} : 0) - K^d (D^r_{cs} : 0) \\
K^d (D^r_{sc} : 0) - K^d (D^r_{ss} : 0)
\end{bmatrix}
\begin{bmatrix}
X^r_{pc} \\
X^r_{ps}
\end{bmatrix}
= \begin{bmatrix}
F^r_{pc} \\
F^r_{ps}
\end{bmatrix}
+ 2(p\sigma)
\begin{bmatrix}
K^d (D^r_{cc} : 0) - K^d (D^r_{cs} : 0) \\
K^d (D^r_{sc} : 0) - K^d (D^r_{ss} : 0)
\end{bmatrix}
\begin{bmatrix}
X^{r-1}_{pc} \\
X^{r-1}_{ps}
\end{bmatrix}.
\]

In the iteration process, \( D^r_{cc}, \) etc., are the harmonic components of the loading; \( F^r_{pc} \) and \( F^r_{ps} \) are found from the displacements obtained in the previous iteration, namely \( X^r_{pc} \) and \( X^r_{ps} \). Examination of Equation (34) reveals that the second term on either side of the equation acts as a damping force. The implication and role of this damping term are discussed in detail in Jain and Datta (1987). The efficiency of the solution of Equation (34) depends upon how the damping term \( \overline{F_p} \) is evaluated.

Assuming \( R_{cj} \) and \( R_{sj} \) as zero, the \( \overline{F_p} \) takes the form:

\[ \overline{F_p} = 2(p\sigma)
\begin{bmatrix}
K^R/2 (p\sigma) & K^d (D^r_{cc} : 0) \\
-K^d (D^r_{cs} : 0) & K^R/2 (p\sigma)
\end{bmatrix}. \]

The computation of \( \overline{F_p} \) now involves only the computation of the time average of \( |V_k(t)| \) at each node. In order to facilitate the computation further, normal mode theory is applied, as in Jain and Datta (1987), leading to the following modal equations:

\[
\begin{align*}
HZ^r_{pc} - PZ^r_{ps} - GZ^r_{ps} &= \phi^T F^r_{pc} - GZ^r_{ps-1} \\
PZ^r_{pc} + HZ^r_{ps} + GZ^r_{pc} &= \phi^T F^r_{ps} + GZ^r_{pc-1}
\end{align*}
\]

in which \( H \) and \( P \) are diagonal matrices of size \((Q \times Q)\) whose \( i \)th diagonal elements are \( \{\omega_i^2 - (p\sigma)^2\} \) and \( \{2\xi_i \omega_i (p\sigma)\} \), respectively; \( \omega_i \) is the \( i \)th undamped natural frequency of the structural system; \( \xi_i \) is the \( i \)th (modal) damping ratio; and

\[ G = \phi^T \overline{F_p} \phi. \]

In order to make the computational scheme more efficient, the off-diagonal terms of matrix \( G \) are ignored, resulting in decoupling of Equations (37) and (38).

The diagonal terms of \( G \) may be looked upon as modal hydrodynamic damping, the \( i \)th diagonal element being represented by

\[ G^r_{ii} = \phi^T \overline{F_p} \phi_i = 2\omega_i p\sigma \xi^r_{hi}. \]

\( \xi^r_{hi} \) is the hydrodynamic damping ratio in the \( i \)th mode and for the \( r \)th iteration and is a function of the time average of the absolute relative at each node, as explained in Equation (36).

Once \( Z^r_{pc} \) and \( Z^r_{ps} \) are obtained in a particular iteration cycle, the time histories of response can easily be determined by suitable substitutions. These time histories are used to evaluate the loading functions for the next iteration until convergence is achieved. The solution is taken to have converged if the difference in r.m.s. of \( \{z(t)\} \)
in two successive cycles becomes less than or equal to 0.5%. Application of the above procedure for the case of random waves consists of simulating the time histories of hydrodynamic loading over a certain record length of time and then obtaining their Fourier transforms to use in the iterative method.

**NUMERICAL RESULTS AND DISCUSSION**

For the present investigation two irregular sea states have been considered, namely $H_{1/3} = 6.0 \text{ mt}$, $T_z = 5.5 \text{ sec}$ (6.0/5.5) and $H_{1/3} = 15.0 \text{ mt}$, $T_z = 9.0 \text{ sec}$ (15.0/9.0). Each sea state has been characterised by one-sided Pierson–Moskowitz (P–M) spectrum and each has been simulated for a duration of about 17.56 min. Typical time histories of wave envelope square $R^2(t)$ and the momentary frequency $\Gamma(t)$ for sea state 6.0/5.5 are shown in Figs 4 and 5.
Figures 6 and 7 show the variation of the drift force coefficient (at node 17) with the encounter frequency. The non-linearly varying drift force coefficients do not differ significantly with the wave amplitudes for current $1.5 \text{ m/sec}$, as shown in Fig. 7, in the low frequency range ($f \leq 1.0 \text{ rad/sec}$).

However, for small current, namely $0.5 \text{ m/sec}$, the drift force coefficient varies with wave amplitude even for low-frequency range (Fig. 6). Since the oscillation of the Guyed Offshore Tower is predominantly guided by the low-frequency response, the frequency range of interest, that is, the range of instantaneous frequencies falls within $0 \leq f \leq 1.0 \text{ rad/sec}$. Within this frequency range, the values of $C_i(\dot{f})$ for any value of $\dot{f}(t)$, obtained from any of the curves corresponding to wave amplitude $\sigma_a = 1, 2$ and $3 \text{ m}$, are nearly the same for current $1.5 \text{ m/sec}$ (refer to Fig. 7). For the present
study, the values of $C_l(\dot{\Gamma})$ are obtained from the curve corresponding to $\delta_u = 1$ and current velocity 1.5 m/sec is uniform throughout the depth.

Figure 8 shows the power spectral density function (psdf) of drift force at node 17 corresponding to sea state 6.0/5.5. The psdf is broad-banded over the frequency range of interest. The typical time histories of the response due to first- and second-order wave force are shown in Figs 9 and 10, respectively. The power spectral densities of these responses are shown in Figs 13 and 14.

Low-frequency excitation of the structure at the first natural frequency due to second-order drift force is evident from the plot of psdf shown in Fig. 14. Similar to this is the case when the responses obtained from the first- and second-order wave forces are

![Figure 8. psdf of drift force.](image1)

![Figure 9. Time history of tip displacement due to first-order wave force.](image2)
numerically added together, as shown in the time history plot in Fig. 11 and its psdf plot in Fig. 15.

In contrast, however, when the Guyed Tower is analysed for the combined wave force of first and second order, the peak of the psdf curve (Fig. 16) occurs near the second natural frequency for the sea state considered. However, other smaller peaks are at the first natural frequency of the system, and the peak energy frequency of the sea state.

Table 2 compares the responses due to the first-order wave force, the second-order wave force, numerically added first- and second-order responses and the response due to the combined first- and second-order wave forces. It is seen that the contribution
of the low-frequency excitation due to second-order drift force alone is quite significant compared with that due to the primary wave force (refer to Figs 13 and 14). However, the resonating effect of the second-order wave force in the low-frequency range is drastically reduced for the response due to the combined effect of first- and second-order wave forces. This is mainly caused by the effect of hydrodynamic damping of the system, which does not appear in the response calculations for the second-order wave force alone.

In Figs 17–20 the same responses are shown for sea state 15.0/9.0. The nature of the responses to second-order wave forces is nearly the same as those observed for the previous sea state considered. The response due to first-order wave force does not
show any peak at the second natural frequency of the structure. Thus, first-order wave force response is purely governed by predominant wave frequencies. The responses due to the combined effect of first- and second-order wave forces indicate some low-frequency contribution arising due to second-order effect. However, they are not as significant compared with the previous sea state, and on the contrary the peak occurring at the predominant wave frequency also decreases. Thus, the significance of the second-order effect in the overall response of the structure depends upon the sea state being considered.

Table 2 compares the responses due to the wave forces and their combinations. For the 15.0/9.0 sea state, the response due to second-order wave force alone is also comparable to that due to primary wave force (refer to Figs 17–20). However, for
Fig. 16. psdf of tip displacement due to combined first- and second-order wave forces.

Table 1. Details of the idealised guyed tower

<table>
<thead>
<tr>
<th>Feature</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deck height</td>
<td>480 mt</td>
</tr>
<tr>
<td>Mean sea level</td>
<td>457 mt</td>
</tr>
<tr>
<td>Height at guy node</td>
<td>442 mt</td>
</tr>
<tr>
<td>Guy diameter</td>
<td>9 cm</td>
</tr>
<tr>
<td>Number of guy</td>
<td>20</td>
</tr>
<tr>
<td>Guy system initial tension</td>
<td>1680 KN</td>
</tr>
<tr>
<td>$EI$</td>
<td>50,000 GNm</td>
</tr>
<tr>
<td>Deck weight</td>
<td>69,000 KN</td>
</tr>
<tr>
<td>Structural weight</td>
<td>3790 KN/m</td>
</tr>
<tr>
<td>Base restraint</td>
<td>20 Gnm/rad</td>
</tr>
<tr>
<td>Effective drag diameter</td>
<td>35.05 mt</td>
</tr>
<tr>
<td>Effective inertia diameter</td>
<td>5.79 mt</td>
</tr>
<tr>
<td>$Cd$</td>
<td>0.90</td>
</tr>
<tr>
<td>$Cm$</td>
<td>2.00</td>
</tr>
<tr>
<td>Structural critical damping ratio</td>
<td>2%</td>
</tr>
<tr>
<td>Height at node 17</td>
<td>457 mt</td>
</tr>
<tr>
<td>Structure's fundamental frequency</td>
<td>0.2689 rad/sec</td>
</tr>
<tr>
<td>Structure's second frequency</td>
<td>1.3260 rad/sec</td>
</tr>
</tbody>
</table>

Table 2. Comparison of mean square values of tip displacement response for different sea states

<table>
<thead>
<tr>
<th>Sea states</th>
<th>First-order ($m^2$)</th>
<th>Second/order ($m^2$)</th>
<th>First and second numerically added ($m^2$)</th>
<th>First and second combined wave force ($m^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_s = 6.0$ m</td>
<td>0.0802</td>
<td>0.25430</td>
<td>0.3012</td>
<td>0.10468</td>
</tr>
<tr>
<td>$T_s = 5.5$ sec</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_s = 15.0$ m</td>
<td>2.6001</td>
<td>2.3865</td>
<td>4.0865</td>
<td>2.07540</td>
</tr>
<tr>
<td>$T_s = 9.0$ sec</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
this sea state, hydrodynamic damping of the system also reduces the resonating effect of the second-order wave force in the low frequency when response is obtained for the combined effect of both forces.

CONCLUSIONS

The responses of a prototype Offshore Guyed Tower to first-order wave force and second-order wave drift force in a random sea are determined by an iterative frequency domain method which duly considers the non-linearities produced by variable guy line tension force and wave loading. The results of the numerical study lead to the following conclusions:
The second-order drift force is a broad-band process and has significant energy at the low-frequency range.

(b) The response due to second-order wave drift force alone shows resonating effect at the structure's fundamental frequency and could be significant and comparable to the order of first-order wave force.

(c) When second-order wave drift force is combined with the first-order wave force, low-frequency response caused due to second-order effect attenuates because of hydrodynamic damping.

(d) The significance of low-frequency excitation by the second-order wave force effect in the overall response of the system depends upon the sea state being considered.
REFERENCES


Hsu, F.H. and Blenkarn, K.A. 1970. Analysis of peak mooring forces caused by slow vessel drift oscillation in random seas. OTC 1159, pp. 1135–1145.


