Model based online diagnosis of unbalance and transverse fatigue crack in rotor systems

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Abstract

A model based technique for online identification of malfunctions in rotor systems is discussed. Presence of fault changes the dynamic behavior of the system. This change is taken into account by equivalent loads acting on the undamaged system model. Equivalent loads are fictitious forces and moments acting on the undamaged system model, which generate a dynamic behavior identical to that of the real damaged system. The mathematical representation of equivalent loads is referred to as Fault Model. The work focuses on developing a fault model for a transverse fatigue crack in shaft and testing it through simulated studies. The basic principle of the technique is validated for unbalance identification, through numerical simulations as well as by experiments on a real rotor system.

1. Introduction

Online monitoring and diagnosis of rotating machinery is enticing the researchers more and more as it provides early indication of the impending failure. Online detection of the incipient faults becomes more vital when the malfunction is of catastrophic nature like that of a fatigue crack.

Occurrence of fault changes the dynamic behavior of the rotor system significantly. The conventional signal based monitoring methods make use of the vibration signatures to detect the faults. But it gives only qualitative, and yet limited information about the type, location and extent of fault and needs experts to diagnose it. On the other hand, model based techniques allow use of a priori information of the system in identification process and give more precise and reliable information about the growing faults (Bach and Markert, 1997).

Presence of non-linear fault like a crack brings non-linearity in the originally linear system equations. This entails use of time consuming integration schemes. The present approach, which was developed within the BRITE EURAM project MODIAROT (Markert et al., 1999), avoids non-linearity in the system equations by representing the fault-induced change in the dynamic behavior of the system in terms of
equivalent loads. To identify the fault parameters, theoretical equivalent loads are compared with those from measured data via curve fitting algorithm (Markert et al., 2001).

2. Background theory of identification method

2.1. Theory

The model based technique for diagnosis of rotor systems, which was developed in Bach and Markert (1997), is summarized here. The vibrations \( r_0 \partial t P \) at \( N \) degrees of freedom of the undamaged rotor system due the load \( F_0 \partial t P \) (e.g. gas forces, residual unbalances) during normal operation are described by linear equations of motion

\[
M_0 \partial^2 r_0 + (B_0 + QG_0) r_0 + K_0 \omega = F_0 (0)
\]

where \( M_0, B_0, G_0 \) and \( K_0 \) are the mass, damping, gyroscopic and stiffness matrices of the system respectively and \( \omega \) is the angular velocity of the shaft.

Occurrence of fault changes the dynamic behavior of the system; the extent of the change depends on vector \( b \), which describes the fault parameters like type, magnitude, location etc of the fault. For example, crack depth \( a \) and its location \( n \) describe a transverse crack. The fault-induced change in the vibrational behavior is represented by additional loads on the undamaged system

\[
M_0 \partial^2 (0 + (B_0 + QG_0) f) + (B_0 + QG_0) (0 + K_0 \omega) = F_0 (0 + AF(J, \omega, \omega, t))
\]

The fault-induced residual vibrations represent the difference of the previously measured normal vibrations \( r_0 \partial t P \) of the undamaged system from vibrations \( r \partial t P \) of the damaged system. The equations of motion for the residual vibrations are given by subtracting Eq. (2) from (1)

\[
M_0 \partial^2 (0 + (B_0 + QG_0) f) + (B_0 + QG_0) (0 + K_0 \omega) = AF(J, \omega, \omega, t)
\]

The system matrices remain unchanged, the rotor model stays linear. Only the equivalent loads induce the change in the dynamic behavior of the undamaged linear rotor model. To identify the fault parameters, the difference of the theoretical fault model and the equivalent loads from measured data is minimized by a least squares fitting algorithm.

2.2. Fault model for unbalance

The mathematical representation of the equivalent loads acting on the damaged system is termed as Fault Model. Fault models are being developed for prominent known faults like unbalance, transverse fatigue crack, rotor-stator rub, rotor bow, misalignment etc, (Markert et al., 1999; Platz and Markert, 2001).

The fault model for unbalance, developed in Platz and Markert (2001), describes the effect of static and kinetic unbalances at the position \( n \) of the rotor. The static unbalance due to a shift of the mass center of a mounted disk is described by a single unbalance \( u \) at a phase angle \( d_u \), resulting in an equivalent force \( DF_u \). A kinetic unbalance due to the angular misalignment of the disk's principal axis produces a gyroscopic moment \( DM_n \) and depends on the axial and polar moment of inertia \( H_A \) and \( H_P \) and the phase angle \( d_n \). In Fig. 1, the angular misalignment \( a \) of the disk is shown. With \( u \partial t P \) as angle of shaft rotation, the unbalance forces and moments are summarized in the vector of equivalent loads.
combining the components $z$ and $y$ to complex variables $z + iy$. Due to unbalance at node $n$ equivalent loads act only at node $n$. At all other nodes no equivalent loads exist. The fault parameters $b_u$ are given by

\[ P_u = [n, u_n, g_n, a_n]^T \] (5)

2.3. Modal expansion

For calculating the equivalent loads from the Finite Element Model using Eq. (3), measured vibrations must be available for damaged system $i_m(t)$, as well as for undamaged system $r_{0m}$, $\delta P$. Since measurement is done only at a few DOFs in practice, the vibrations at non-measured DOFs are estimated using modal expansion. Modal expansion is a simple and rapid technique, which reconstructs the non-measured vibrations in time domain as linear combination of $j$ mode shapes, $j$ being number of measurement DOFs (see Markert et al., 2001 for details). The residual vibrations at all DOFs $\Delta \tilde{r}(t)$ are then estimated as

\[ \Delta \tilde{r}(t) = Q \Delta \tilde{r}_m(t) \] (6)

where $Q$ is a constant matrix and $\Delta \tilde{r}_m(t) = i_{0m}(t) - r_m \delta P$.

2.4. Least squares fitting

For identifying the fault parameters, the theoretical equivalent loads $DF\delta b_i$, $t \delta P$ from fault models are fitted into the equivalent loads $DF\delta \Phi$ from the measured data by varying the fault parameters. Least
squares algorithm is used in time domain to achieve the best curve fit (Markert et al., 2001). The objective function to be minimized is given as

$$\int \left[ E \Delta F(\beta, t) \right]^2 dt = \min$$

The algorithm iterates for the values of fault parameters $b_i$ for all suspected faults taken into account. Eventually, the fault type, its position and extent are identified.

### 2.5. Probability measures

The quality of fit achieved in the least squares fitting is exploited for estimating the probability of occurrence of the identified faults, (Markert et al., 1999, 2001). Coherence $p_1$, which is normalized cross-correlation of the equivalent forces from measured data and fault models, gives probability of presence of a particular fault. Second measure, Intensity $p_2$, is normalized correlation of equivalent loads of a particular identified fault and those of total identified faults. This in turn gives the contribution of a particular fault when multiple faults are present.

### 3. Fault model for a transverse fatigue crack

A fault model for a transverse fatigue crack in shaft is developed and tested for the first time. Though a simple breathing crack is considered here, fault model for non-linear crack can also be developed in a similar way.

Equations of motion for a shaft containing a transverse fatigue crack are given as, (Gasch, 1976),

$$M \ddot{r}_c + (B_o + QG_0)ic(t) + K(r_c, t)r_c(t) = F_0(0 + F_{static}) \tag{8}$$

where $K\delta r_c, tP$ is non-linear and time variant, and

$$r_c(t) = r(0 + r_{static}) \tag{9}$$

where $r_{static} = K_j^i F_{static}$ is the static deflection of the uncracked shaft with $F_{static}$ as the static load vector due to gravity. Thus, the equations of motion are non-linear and parametric. The stiffness matrix $K\delta r_c, tP$ can be split,

$$K(w) = Ko + AK(w) \tag{10}$$

where $DK\delta r_c, tP$ describes the change in the stiffness of the system because of the crack and is a function of displacement and rotation angle of the shaft.

If weight dominance is assumed for the elastic deflection (i.e. $r \approx C r_{static}$) then the opening and closing of the crack is determined by the static deflection $r_{static}$ rather than vibrational displacement $r(t)$ so that stiffness becomes independent of $r(t)$. If stability is guaranteed, (Gasch, 1993),

$$Mof(0 + (B_o + QG_0)f(0 + KoKO) = -AK(0r_{static} - AK(f)r(0 + F_0(0) \tag{11}$$

Deducting Eq. (1) from Eq. (11), equations of motion for residual vibrations are given by

$$Mo\ddot{r}(0 + (B_o + QG_0)Af(0 + K_oAr(t) = -AK(0(r_{static} + r(0) \tag{12}$$

The RHS of Eq. (12) can be represented by equivalent load acting on the system as

$$AF_n(\beta, r, t) = -AK(0(r_{static} + r(0) \tag{13}$$
Derivation of the stiffness matrix $K$ for cracked beam element from the Stress Intensity Factors is well known, (Audebert, 1997; Popadopoulos and Dimarogonas, 1987; Tada et al., 1985). Several investigations to describe the open-close behavior of a transverse crack under weight dominance have been reviewed in Audebert (1997), Wauer (1990) and Dimarogonas (1996). According to Sekhar and Prabhu (1992), the coefficients of stiffness matrix vary periodically; the variation may be expressed by a cosine series with

$$K_{element} = K(\varphi(t)) = \sum_{q=0}^{4} K_q \cos q\varphi(t)$$

where $K_q$ are fitting coefficient matrices that are determined from the known stiffness behavior at certain angular positions (Sekhar and Prabhu, 1992). This assumption is valid as long as the rotor does not rotate very close to critical speed. The stiffness matrix $K_{element}$ can be decomposed into a stiffness matrix for the closed crack and a time varying stiffness matrix $D K_{element}(\varphi(t))$. Thus, equivalent load in Eq. (13) for crack in element $n$ becomes

$$\Delta F_n(\beta_{crack}, \dot{r}, r, t) = -\Delta K_n(\varphi(t))(r_{static} + r(t))$$

where $\beta_{crack}$ are the crack parameters namely crack depth $a$ and its location $n$.

$$\beta_{crack} = [n, a]^T$$

4. Experimental setup

A test rig for experimental validation of the model based identification technique has been built at Darmstadt University of Technology, Germany. The test rig, shown in Figs. 2 and 3, consists of a 20 mm diameter and 780 mm long shaft supported on two double row self aligning ball bearings and is driven by a motor. Two disks of masses 6.27 and 4.45 kg are mounted symmetrically on the shaft and the whole system

Fig. 2. Photograph of the test rig (max speed 6000 rpm).
The test rig was investigated extensively for non-linearity and validity of the FE model through modal testing. Since a very accurate mathematical model is the key of model based techniques, the inevitable errors in the FE model are minimized by finite element model updating technique. Among the various updating methods, (Friswell and Mottershead, 1996; Ewins, 2000; Modak et al., 2000), output error method was found to be the most suitable for this application. Ultimately, updated FE model, which represents the dynamic behavior of the system very closely (see Fig. 4) is generated.

5. Numerical simulations

The effects of different numerical approximations and errors in measurements on the accuracy of identification results are studied by performing virtual experiments on the mathematical (FE) model of the
system. Similar studies were performed in Platz et al. (2000) for unbalance and rubbing using a simple model with only 6 DOFs.

5.1. Unbalance identification

For only static unbalance, the fault model is modified as

\[
\Delta F_u(\beta, \hat{\nu}, \nu, t) = \begin{bmatrix} 0 & 3 & 2 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix} = \mu_e \phi_{\nu}^{\nu_0} \Omega^2 \begin{bmatrix} \{e^{i\nu(t)} \} \\ \{e^{i\nu_1(t)} \} \\ \{e^{i\nu_2(t)} \} \\ \{e^{i\nu_3(t)} \} \end{bmatrix}
\]

with fault parameters as location, amount and angle of unbalance. (i.e. \(bu = [n, u_n, \theta_u]^T\)).

Residual unbalances of 4e)4 kg m each are assumed to be present at 45 and 90° on first disk (node 4) and second disk (node 9) respectively. Additional unbalances of 2e)4 kg m (u4) at 180° (d4) and 8e)4 kgm (u9) at 45° (d9) simulate unbalanced disks respectively. The different cases for which identification on generated data is performed are described below.

**Case 1:** This is the reference case described above.

**Case 2:** The time signals for displacements, velocities and accelerations are recorded at 48 (all) DOFs. No other effects are considered.

**Case 3:** Measurements are done at only 8 DOFs. Non-measured vibrations are estimated using modal expansion technique.

**Case 4:** Measurements are done at only 4 DOFs. Non-measured vibrations are estimated using modal expansion technique.

**Case 5:** Influence of noise in measured data is studied by adding a band-limited random noise of variance one and zero mean.

**Case 6:** Since only accelerations are measured in practice, the velocities and displacements have to be calculated using numerical integration. Hence, only accelerations at 48 (all) DOFs are measured and trapezoidal rule is used for integration.

**Case 7:** To simulate the actual measurements more closely, the influences in cases 3, 5 and 6 are combined, i.e., only accelerations are measured at 8 DOFs and noise is added.

The results are shown in Table 1. It is evident from Fig. 5 that equivalent loads are present only on those DOFs where the unbalances are present. Also, high values of probabilities at these DOFs show that these are the only faults present in the system. It is seen that the unbalances are identified with good accuracy even when data is noisy. The error in identified unbalances is of considerable magnitude in Case 4, where measurements are done at only 4 DOFs. Error gets reduced greatly with increase in measured DOFs. This implies that the error involved is mainly because of the estimation of non-measured data. The effect of modal expansion is distribution of the equivalent forces on other DOFs (see Fig. 6).

5.2. Crack identification

A transverse fatigue crack of depth 4 mm (20% of shaft diameter) is assumed to be present at the center of the shaft (between nodes 6 and 7). The numerical data is generated from the FE model of the cracked rotor. Following cases are considered.
Table 1

Unbalance identification: results of numerical simulations

<table>
<thead>
<tr>
<th>No</th>
<th>Test case</th>
<th>Identified unbalances</th>
<th>Error in $u$ (in %)</th>
<th>Probability measures (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$</td>
<td>\eta_1</td>
<td>10^{-4}$ (kg)</td>
</tr>
<tr>
<td>1.</td>
<td>Reference case</td>
<td>2</td>
<td>180</td>
<td>8</td>
</tr>
<tr>
<td>2.</td>
<td>Sensors at 48 (all) DOFs</td>
<td>2</td>
<td>180</td>
<td>8</td>
</tr>
<tr>
<td>3.</td>
<td>Sensors at 8 DOFs</td>
<td>2.09</td>
<td>189</td>
<td>7.22</td>
</tr>
<tr>
<td>4.</td>
<td>Sensors at 4 DOFs</td>
<td>2.64</td>
<td>96</td>
<td>3988</td>
</tr>
<tr>
<td>5.</td>
<td>Noisy data</td>
<td>2.007</td>
<td>178</td>
<td>7.99</td>
</tr>
<tr>
<td>6.</td>
<td>Numerical Integration</td>
<td>2</td>
<td>180</td>
<td>8</td>
</tr>
<tr>
<td>7.</td>
<td>Combination of 3, 5 and 6</td>
<td>2.089</td>
<td>189</td>
<td>7.22</td>
</tr>
</tbody>
</table>

**Case 1**: This is the reference case described above.

**Case 2**: The time signals for displacements, velocities and accelerations are recorded at 48 (all) DOFs. No other effects are considered.
Case 3: The second order smallness term \( -DK\dot{t}\Phi_{t\dot{t}}P \) in the Eq. (12) was neglected in Platz and Markert (2001). The effect neglecting this term is studied here.

From Fig. 7, it can be noticed that equivalent loads are present only on nodes 6 and 7, between which crack is present. The technique identifies the crack successfully for Case 1, where measured vibrations are assumed to be available at 48 (all) DOFs. The exact nature of the equivalent loads acting on one node of a cracked beam element is shown in Fig. 8. The equivalent loads on the other node of the cracked beam element are exactly opposite of the loads shown in Fig. 8. The results are presented in Table 2, which show that neglecting the term \( -DK\dot{t}\Phi_{t\dot{t}}P \) in the Eq. (12) can lead to significant error in identification process and should, therefore, be included in the fault model.

![Fig. 7. Equivalent loads: Case 2.](image)

![Fig. 8. Equivalent loads on a node of a cracked beam element.](image)

<table>
<thead>
<tr>
<th>No</th>
<th>Test case</th>
<th>Crack depth (mm)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Reference case</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Sensors at 48 (all) DOFs</td>
<td>4</td>
<td>0.00</td>
</tr>
<tr>
<td>3.</td>
<td>Neglecting term (-AK(*)r(f))</td>
<td>4.474</td>
<td>11.85</td>
</tr>
</tbody>
</table>
6. Experimental validation

6.1. Signal processing

Significant error can get introduced in the calculation of residual vibrations because of differences in rotor speeds, phase and sampling times, and inexact repeatability of the measurements. Different rotor speeds are compensated by adjusting the time scale of the normal vibrations to the time scale of the current measured vibrations. Differences in sampling times are eliminated by interpolating the normal vibrations to the sampling times of the current measured vibrations. Phase shifts are avoided by recording a trigger signal during the measurements. To study the order of magnitude of this error, residual vibrations were calculated for different sets of normal vibrations at neighbouring speeds (difference of less than 10 rpm) on undamaged test rig. Error of maximum of 10% was observed, which, after applying above-mentioned corrections, reduced to less than 5%.

6.2. Unbalance identification on test rig

Normal vibrations were recorded for 1500 rpm at 4 DOFs at the bearing locations using acceleration sensors. An unbalance $u$ of $4.002 \times 10^{-4}$ kgm was then applied on the first disk at an angle $d$ of zero degrees. The vibrations of damaged system were recorded with an effort to keep the speed as close to 1500 rpm as possible. The identification procedure was applied to find out the location, amount and angle of applied unbalance. The identification results, along with the results of simulations for identical conditions, are compiled in Table 3.

In Fig. 9, equivalent loads identified from measured data using fault model in Eq. (17) are shown. The equivalent load present on node 9 is an erroneous result. Equivalent loads in Fig. 10, which are from

<table>
<thead>
<tr>
<th>No</th>
<th>Test case</th>
<th>Identified unbalance</th>
<th>Error (%)</th>
<th>Probability measures (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$</td>
<td>u</td>
<td>$ 10^4 (kgm)</td>
</tr>
<tr>
<td>1</td>
<td>Applied experimentally</td>
<td>4.002</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>Experiment with sensors at 4 DOFs</td>
<td>2.676</td>
<td>1</td>
<td>33.13</td>
</tr>
<tr>
<td>3</td>
<td>Simulation with sensors at 4 DOFs</td>
<td>2.820</td>
<td>1</td>
<td>29.53</td>
</tr>
<tr>
<td>4</td>
<td>Simulation with sensors at 8 DOFs</td>
<td>3.785</td>
<td>0</td>
<td>0.05</td>
</tr>
<tr>
<td>5</td>
<td>Simulation with sensors at 48 (all) DOFs</td>
<td>4.002</td>
<td>0</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 3
Unbalance identification: Comparison of experiments and simulations

Fig. 9. Equivalent loads: experimental (Case 2).
simulations with same measurement locations, also show a similar component on node 9. However, this component vanishes in Fig. 11 where simulations are done with increased (eight) number of measurement DOFs. The error gets minimized to exceptional level when measured DOFs are increased from four to eight. (see Table 3). Hence, this error can be attributed to the estimation of non-measured data.

7. Conclusions

The proposed model based technique is able to determine the position, extent and orientation of unbalance analytically as well as on real rotor system with a reasonable accuracy. A fault model for transverse fatigue crack is developed and tested numerically. As seen in Sections 5.1 and 6.2, the error involved in identification is due largely to the estimation of non-measured data, which itself is subject of research. The potential of the technique can be exploited for online condition monitoring and diagnosis of rotor systems since faults can be identified without ambiguity even when multiple faults are present.

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