A new 0–1 integer programming method of feeder reconfiguration for loss minimization in distribution systems

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Abstract

One of the features provided by distribution automation which can result in substantial savings for the utility is feeder reconfiguration for loss minimization. Since the composition of loads for various feeders is different, and their loading patterns vary with time, there is a need for feeder reconfiguration to be carried out whenever there is a change in the loads. In this paper a new 0–1 integer programming method of feeder reconfiguration for loss minimization in distribution systems is proposed. The proposed method is illustrated with an example.

Keywords: Distribution automation; Feeder reconfiguration; Loss minimization

1. Introduction

Distribution feeders supply power to various types of loads, namely, residential, commercial, industrial and agricultural. Each feeder has a different load composition and their daily load variations are dissimilar. Consequently, the peak loads on substation transformers, on individual feeders, or on feeder sections occur at different times. Thus, a particular configuration of the distribution system which is set for minimum loss at a certain instant of time will no longer be a minimum loss configuration at a different instant of time. Hence there is a need for feeder reconfiguration for loss minimization to be carried out whenever there is a change in the loading pattern on the system. This is an important function in distribution automation [1].

Several attempts have been made in the past to obtain an optimal feeder configuration for minimizing losses in distribution systems [2–15]. Civanlar et al. [2] considered one feeder pair at a time for loss reduction and derived a formula to estimate the loss reduction which would result from carrying out a particular switching option. Shirmohammadi and Hong [3] determined a low-loss configuration by applying an optimal load flow analysis to the system with all switches closed. The system is returned to a radial configuration by opening the branches with the lowest current, considering one loop at a time. Baran and Wu [4] used a branch exchange method and suggested a mechanism to reduce the number of switching options. They also developed the approximate power flow method to estimate the loss reduction. Liu et al. [5] developed two loss minimization algorithms in which they consider one feeder pair at a time to get the optimal solution. Huddleston et al. [6] formulated the problem as a quadratic programming problem with constraints on the currents in the system, considering multiple feeders at a time. Glamocanin [7] considered it as a transhipment problem with quadratic costs. Nara and Kitagawa [8] used a simulated annealing method for this problem. In another paper [9] Nara et al. used a genetic algorithm to solve it. Wagner et al. [10] compared various methods and presented a new linear programming method using a stepping-stone algorithm. They also proposed a new heuristic search method. Jasmon and Lee [11] modified the method suggested by Baran and Wu [4] to obtain the criterion for optimal switching. Goswami and Basu [12] presented a power flow based heuristic algorithm for determining the minimum loss configuration of radial distribution networks. Chen and Cho [13] presented a method to derive an optimal switching plan to achieve energy loss minimization for short- and long-term operation of distribution systems. Expert systems and artificial neural network based methods are also presented in Refs. [14] and [15], respectively.
The above methods are based on either heuristics or successive-approximation methods in which one feeder pair is considered at a time to reconfigure the network to reduce losses. An interesting method is proposed by Huddleston et al. [6] in which they consider multiple feeder pairs at a time. In their method the segments at the end of each circuit are reduced to spot loads and are considered for switching to reduce the losses. However, they do not consider the possibility of switching any other segments to further reduce the losses. Thus their method may not give the overall minimum loss configuration for all cases. In this paper a new method based on 0–1 integer programming is proposed for feeder reconfiguration for loss minimization in distribution networks. The proposed method considers multiple switchings at a time and finds the overall minimum loss configuration. It is illustrated using an example system.

2. Proposed method

Consider the example system shown in Fig. 1. The system is composed of three circuits and each circuit is composed of various elements. For the system in Fig. 1, circuit 1 comprises elements 1, 2, 3, 4, 5 and 6, circuit 2 comprises elements 7, 8, 9, 10, 11, 13, 14 and 15, and circuit 3 comprises elements 17, 18, 19, 20, 21 and 22. Elements 12, 16 and 23 (indicated by dotted lines) are the elements which are initially in the open position. It is assumed here that each element in Fig. 1 contains a sectionalizing switch so that any element may be opened for reconfiguration purposes. The system details of Fig. 1 are given in Table 1.

Table 1
Details of the system shown in Fig. 1 (Case 1)

<table>
<thead>
<tr>
<th>Element no.</th>
<th>Start node</th>
<th>End node</th>
<th>Resistance (Ω)</th>
<th>End-node current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>4</td>
<td>0.13</td>
<td>88</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>5</td>
<td>0.13</td>
<td>132</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>6</td>
<td>0.16</td>
<td>44</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>7</td>
<td>0.16</td>
<td>88</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>8</td>
<td>0.07</td>
<td>44</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>9</td>
<td>0.07</td>
<td>66</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>10</td>
<td>0.19</td>
<td>178</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>11</td>
<td>0.19</td>
<td>224</td>
</tr>
<tr>
<td>9</td>
<td>11</td>
<td>12</td>
<td>0.19</td>
<td>44</td>
</tr>
<tr>
<td>10</td>
<td>12</td>
<td>13</td>
<td>0.14</td>
<td>27</td>
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<td>11</td>
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<td>14</td>
<td>0.14</td>
<td>27</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
<td>14</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>11</td>
<td>17</td>
<td>0.14</td>
<td>204</td>
</tr>
<tr>
<td>14</td>
<td>10</td>
<td>15</td>
<td>0.17</td>
<td>44</td>
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<td>15</td>
<td>15</td>
<td>16</td>
<td>0.17</td>
<td>44</td>
</tr>
<tr>
<td>16</td>
<td>16</td>
<td>20</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>3</td>
<td>18</td>
<td>0.19</td>
<td>44</td>
</tr>
<tr>
<td>18</td>
<td>18</td>
<td>19</td>
<td>0.17</td>
<td>66</td>
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<tr>
<td>19</td>
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<td>0.14</td>
<td>44</td>
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<td>20</td>
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<td>0.19</td>
<td>44</td>
</tr>
<tr>
<td>21</td>
<td>21</td>
<td>22</td>
<td>0.14</td>
<td>66</td>
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<td>44</td>
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<tr>
<td>23</td>
<td>9</td>
<td>23</td>
<td>0.07</td>
<td></td>
</tr>
</tbody>
</table>

Elements 12, 16 and 23 can now be associated with variables \( x_6, x_9, x_{14} \) and \( x_{23} \), respectively, which can take values of either 0 or 1 (referred to as 0–1 variables). \( x_6 = 1 \) implies that node 6 is connected to circuit 1 and \( x_6 = 0 \) implies that node 6 is not connected to circuit 1, but connected to circuit 2. Thus the values of \( x_6 \) and \( x_{14} \) will decide whether node 6 is to be switched on to circuit 2 or node 14 is to be switched on to circuit 1. It is important to note here that each of these nodes is connected to any one of the circuits. This implies that the values of \( x_6 \) and \( x_{14} \), in this case, cannot be zero simultaneously. This condition pertaining to these variables can be represented in the form of a constraint as \( x_6 + x_{14} \geq 1 \). For circuit 1, nodes 6 and 9 are referred to as switching nodes and nodes 14 and 23 as neighbouring nodes for this circuit.

Now the current in element 1 can be written as

\[
i_1 = I_{n4} + I_{n5} + I_{n7} + I_{n8} + x_6 I_{n6} + (1 - x_{14})I_{n14} + x_9 I_{n9} + (1 - x_{23})I_{n23}
\]

(1)

In the above expression the coefficient of \( I_{n14} \) is taken as \( (1 - x_{14}) \). This is because node current \( I_{n14} \) will not be contributing to the current in element 1 when node 14 is connected to circuit 2, i.e. when \( x_{14} = 1 \). Similarly, the coefficient of \( I_{n23} \) is taken as \( (1 - x_{23}) \) for the same reason.

The power loss in element 1 can now be written as

\[
L_1 = i_1^2 r_1
\]

(2)
where $r_i$ denotes the resistance of the $i$th element. From (1) and (2), the power loss in element 1 is given by

$$L_1 = r_1 \{ n_4 - n_5 - n_7 + n_8 + n_{14} + n_{23} \}^2 + 2x_n x_i I_{n6} I_{n9}$$

$$- 2x_n x_{14} I_{n6} I_{n14} - 2x_n x_{23} I_{n6} I_{n23} - 2x_n x_{14} I_{n6} I_{n14}$$

$$- 2x_n x_{23} I_{n6} I_{n23} + 2x_n x_{14} I_{n14} I_{n23}$$

$$+ x_n [I_{n6}^2 + 2I_{n6} I_{n4} + I_{n5} + I_{n7} + I_{n8} + I_{n14} + I_{n23}]$$

$$+ x_n [I_{n9}^2 + 2I_{n9} I_{n4} + I_{n5} + I_{n7} + I_{n8} + I_{n14} + I_{n23}]$$

$$+ x_{14} [I_{n14}^2 - 2I_{n14} I_{n4} + I_{n5} + I_{n7} + I_{n8}$$

$$+ I_{n14} + I_{n23} - x_{23} [I_{n23}^2 - 2I_{n23} I_{n4} + I_{n5}$$

$$+ I_{n7} + I_{n8} + I_{n14} + I_{n23}]$\}$$

(3)

It can be seen that in deriving expression (3) $x_i^2$ is equated to $x_i$ since $x_i$ is a 0-1 integer variable. The above expression can be written in a general form as

$$L_1 = K_{01} + K_{11} I_{n6} I_{n9} + K_{21} I_{n6} I_{n14} + K_{31} I_{n6} I_{n23}$$

$$+ K_{41} x_n x_{14} + K_{51} x_n x_{23} + K_{61} I_{n6} x_{14} + K_{71} I_{n6} x_{23}$$

$$+ K_{81} x_n x_{14} + K_{91} x_n x_{23} + K_{101} x_n x_{23}$$

(4)

where $K_{01}, K_{11}, K_{21}, \ldots$ represent the coefficients contributed by element 1 to the terms of the loss function.

The expression for the power loss in the $i$th element in the circuit can thus be written in a general form as

$$L_i = K_{0i} + K_{1i} x_n x_9 + K_{2i} x_n x_{14} + K_{3i} x_n x_{23} + K_{4i} x_n x_{14} + K_{5i} x_n x_{23} + K_{6i} x_n x_{14} + K_{7i} x_n x_{23}$$

$$+ K_{8i} x_n x_{14} + K_{9i} x_n x_{23} + K_{10i} x_n x_{23}$$

(5)

where $K_{0i}, K_{1i}, K_{2i}, \ldots$ represent the coefficients contributed by element $i$ to the terms of the loss function. Expressions similar to (5) can be written for all the elements of circuit 1.

The loss function for circuit 1 can now be obtained by taking the summation over all the elements in the circuit which can be written as

$$L_{F1} = K_0 + K_1 x_n x_9 + K_2 x_n x_{14} + K_3 x_n x_{23} + K_4 x_n x_{14} + K_5 x_n x_{23} + K_6 x_n x_{14} + K_7 x_n x_{23} + K_8 x_n x_{14} + K_9 x_n x_{23}$$

(6)

where $K_0 = \Sigma K_{0i}, K_1 = \Sigma K_{1i}, \ldots$. The above summations are carried over all the elements in the circuit, namely, 1, 2, 3, 4, 5, 6, 12 and 23. In Eq. (6) $K_{0i}, K_{1i}, K_{2i}, \ldots$ represent the coefficients of the loss function.

It may be noted here that in the expressions for the power losses in circuit 1, the open elements 12 and 23 are also included though these were not shown as elements belonging to circuit 1. This is to ensure that the losses in these elements are properly accounted for after the switching operation.

It is possible to generalize the procedure for writing the above expression of losses for each element of a circuit in the system as follows.

3. Procedure for writing the power loss function for each circuit

Step 1. Identify the 0-1 integer variables for the circuit under consideration. For example, for circuit 1 of Fig. 1 these variables are $x_6, x_9, x_{14}$ and $x_{23}$. Of these variables, $x_{14}$ and $x_{23}$ are the variables associated with the neighbouring nodes 14 and 23. Each variable of the circuit is associated with a variable $y_i$ which takes the value 1 if the node is a switching node and -1 if the node is a neighbouring node of the circuit. For circuit 1, $y_i$ takes the values 1, 1, -1 and -1 for the variables $x_6, x_9, x_{14}$ and $x_{23}$, respectively.

Step 2. Identify the load currents that contribute to the loss function in each element of the circuit. The node currents corresponding to the switching nodes should be excluded and the node currents corresponding to the neighbouring nodes are to be included in the above consideration. For example, for element 1 the load currents to be considered are $I_{n4}, I_{n5}, I_{n7}, I_{n8}, I_{n14}$ and $I_{n23}$. $I_{n6}$ and $I_{n9}$, which are the currents at the switching nodes, are not included.

Step 3. Find the contributions of the various elements to the coefficients in the power loss function as explained below.

3.1. Step 3(a). Second-order terms. The number of second-order terms in the loss function of a circuit can be obtained by finding all possible combinations of 0-1 integer variables $x_i$ taken two at a time. For circuit 1 of Fig. 1 since the 0-1 variables $x_i$ are $x_6, x_9, x_{14}$ and $x_{23}$, the second-order terms in the power loss function would contain product terms like $x_6 x_9, x_6 x_{14}, x_6 x_{23}, x_9 x_{14}, x_9 x_{23}$ and $x_{14} x_{23}$. The contribution of an element $i$ to the coefficients of these terms can be obtained as follows.

The contribution of the $i$th element to the coefficient of a second-order term corresponding to $x_p x_q$ is equal to $2r_i y_p I_{n6} y_q I_{n9}$. The values of $y_p$ and $y_q$ are defined as explained earlier in Step 1. Thus, the contribution of an element $i$ to the coefficient of $x_n x_9$ is given by

$$K_{1i} = 2y_p y_q I_{n6} y_9 I_{n9}$$

(7)

Similarly, the contribution of element $i$ to the coefficients of the other second-order terms can be written as follows:

$$K_{2i} = 2y_p y_q I_{n6} y_{14} I_{n14}$$

(8)

$$K_{3i} = 2y_p y_q I_{n6} y_{23} I_{n23}$$

(9)
\[ K_{4i} = 2r_i y_i n_{14} I_{n_{14}} \]
\[ K_{5i} = 2r_i y_i n_{10} I_{n_{23}} \]
\[ K_{6i} = 2r_i I_{14} I_{n_{14} n_{23}} \]

Only those elements in which both the switching currents flow would contribute to the coefficients. Thus, if switching currents at nodes 6 and 9 flow in an element, then it would contribute to the coefficient of \( x_6 x_9 \). But, if the switching current at node 6 does not flow in an element, then this element would not contribute to the coefficients of \( x_6 x_9 \), \( x_6 x_{14} \), and \( x_6 x_{23} \).

For example, the coefficients contributed by element 1 are given by

\[ K_{11} = +2r I_{n_6} I_{n_9} \quad \text{(since } y_6 = 1 \text{ and } y_9 = 1) \]
\[ K_{21} = -2r I_{n_6} I_{n_{14}} \quad \text{(since } y_6 = 1 \text{ and } y_{14} = -1) \]
\[ K_{31} = -2r I_{n_6} I_{n_{23}} \quad \text{(since } y_6 = 1 \text{ and } y_{23} = -1) \]
\[ K_{41} = -2r I_{n_9} I_{n_{14}} \quad \text{(since } y_9 = 1 \text{ and } y_{14} = -1) \]
\[ K_{51} = -2r I_{n_9} I_{n_{23}} \quad \text{(since } y_9 = 1 \text{ and } y_{23} = -1) \]
\[ K_{61} = +2r I_{n_{14}} I_{n_{23}} \quad \text{(since } y_{14} = -1 \text{ and } y_{23} = -1) \]

Since the switching current at node 6 does not flow in element 4, this element does not contribute to the coefficients of \( x_6 x_9 \), \( x_6 x_{14} \), and \( x_6 x_{23} \).

**Step 3(b). First-order terms.** The number of first-order terms would be equal to the number of 0–1 variables in a circuit. For circuit 1 of Fig. 1 the first-order terms would correspond to \( x_6 \), \( x_9 \), \( x_{14} \), and \( x_{23} \). Only those elements in which the switching current flows would contribute to the coefficient of the corresponding terms. For example, if the switching current at node 6 does not flow in an element, then that element does not contribute to the coefficient of \( x_6 \).

The coefficients contributed to the first-order terms by an element \( i \) are obtained as follows. In general, the coefficient of \( x_m \) contributed by an element \( i \) is equal to \( r_i [I_{am}^2 + 2y_i I_{am} I_{L_i}] \), where \( I_{L_i} \) is the sum of the load currents that contribute to the current in element \( i \). For element 1 in Fig. 1, the coefficient of \( x_6 \) is given by

\[ K_{71} = r_1 [I_{n_6}^2 + 2y_6 I_{n_6} (I_{L_1})] \]

Since \( y_6 = 1 \),

\[ K_{71} = r_1 [I_{n_6}^2 + 2I_{n_6} (I_{n_4} + I_{n_5} + I_{n_7} + I_{n_8} + I_{n_{14}} + I_{n_{23}})] \]

Similarly, the coefficient of \( x_{14} \) contributed by element 1 is given by

\[ K_{81} = r_1 [I_{n_{14}}^2 + 2y_{14} I_{n_{14}} (I_{L_1})] \]

Since \( y_{14} = -1 \),

\[ K_{81} = r_1 [I_{n_{14}}^2 - 2I_{n_{14}} (I_{n_4} + I_{n_5} + I_{n_7} + I_{n_8} + I_{n_{14}} + I_{n_{23}})] \]

Since the current at node 6 does not flow through element 4, the contribution of this element to the coefficient of \( x_6 \) is equal to zero.

**Step 3(c). Constant term.** The contribution of an element \( i \) to the constant term \( K_0 \) denoted by \( K_{0i} \) is given by

\[ K_{0i} = r_i (I_{L_i})^2 \]

where \( I_{L_i} \) is the sum of the load currents that contribute to the current in element \( i \). For element 1 in Fig. 1,

\[ K_{01} = r_1 (I_{n_4} + I_{n_5} + I_{n_7} + I_{n_8} + I_{n_{14}} + I_{n_{23}})^2 \]

**Step 4. Determination of the power loss function.** Now the coefficients of the terms in the power loss function of a circuit can be obtained by taking the summation of the contributions of the various elements in the circuit to the respective coefficients:

\[ K_0 = \sum K_{0i}, \quad K_1 = \sum K_{1i}, \quad K_2 = \sum K_{2i}, \ldots \]

where the summation is carried out over all elements in the circuit.

Since the constant term is in terms of only the load currents and does not depend on the 0–1 variables, this does not influence the final result in the minimization of the loss function with respect to the 0–1 variables. Hence, it need not even be computed.

The algorithm of the proposed method can be briefly summarized in the following steps.

**Step (a).** Identify the switchable elements and the corresponding switching and neighbouring nodes for each of the circuits. Assign 0–1 integer variables to each of these nodes in all the circuits.

**Step (b).** Formulate the loss function for each circuit and the total loss function for the system along with the relevant constraints. This constitutes the objective function.

**Step (c).** Minimize the above objective function subject to the constraints formulated in Step (b) using a 0–1 integer programming package. Here GINO (Generalized Interactive Non-linear Optimizer) has been used to solve the optimization problem.

**Step (d).** Based on the solution of the above optimization problem, determine the segments to be switched (opened and closed). If there is no change in the current positions of the open switches, go to step (e). Else, switch the segments as suggested and go to Step (a).

**Step (e).** End.

4. Illustration

For circuit 1 of Fig. 1, the values of the contributions of the various elements to the respective coefficients are given in Table 2.

The power loss function for circuit 1 is given by
Table 2
Contributions of elements in circuit 1 to the coefficients of the power loss function for Case 1 (0-1 variables for circuit 1: x6, x9, x14 and x23)

<table>
<thead>
<tr>
<th>Terms in loss function</th>
<th>Elements in circuit 1</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>x6x9</td>
<td>755.04</td>
<td>0</td>
</tr>
<tr>
<td>x6x14</td>
<td>-308.88</td>
<td>-308.88</td>
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<tr>
<td>x6x23</td>
<td>-503.36</td>
<td>0</td>
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<tr>
<td>x6x14</td>
<td>-463.32</td>
<td>0</td>
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<tr>
<td>x6x23</td>
<td>-755.04</td>
<td>0</td>
</tr>
<tr>
<td>x14x23</td>
<td>308.88</td>
<td>0</td>
</tr>
<tr>
<td>x6</td>
<td>5090.80</td>
<td>2070.64</td>
</tr>
<tr>
<td>x9</td>
<td>7824.96</td>
<td>0</td>
</tr>
<tr>
<td>x14</td>
<td>-2874.69</td>
<td>-1021.41</td>
</tr>
<tr>
<td>x23</td>
<td>-4587.44</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ LF_1 = 755.04x_6x_9 - 997.92x_6x_{14} - 503.36x_6x_{23} - 463.32x_9x_{14} - 2497.44x_9x_{23} + 308.88x_{14}x_{23} + 7851.36x_6 + 14068.56x_9 - 4063.77x_{14} - 7433.36x_{23} \] (19)

Similarly, the power loss function for circuits 2 and 3 of Fig. 1 can be obtained as follows:

\[ LF_2 = 451.44x_{14}x_{16} - 2019.6x_{14}x_6 - 451.4x_{14}x_{20} - 735.68x_{16}x_6 - 2052.16x_{16}x_{20} + 735.68x_6x_{20} + 16540.47x_{14} + 16527.28x_{16} - 24434.96x_6 - 16410.64x_{20} \] (20)

\[ LF_3 = 735.68x_{20}x_{23} - 1936.0x_{20}x_{16} - 1103.52x_{20}x_9 - 735.68x_{23}x_{16} - 3426.72x_{23}x_6 + 1103.52x_{16}x_9 + 8673.28x_{20} + 1635.36x_{23} - 6872.8x_{16} - 13474.56x_9 \] (21)

The total loss function for the entire system in Fig. 1 is given by

\[ LF = LF_1 + LF_2 + LF_3 \] (22)

This power loss function has to be minimized with respect to the 0-1 variables defined for this system subject to the following constraints:

\[ x_6 + x_{14} \geq 1 \] (23a)
\[ x_9 + x_{23} \geq 1 \] (23b)
\[ x_{16} + x_{20} \geq 1 \] (23c)

Solving the above optimization problem, using a 0-1 integer optimization package, the optimal values of the variables are obtained as \( x_6 = 1, x_9 = 1, x_{14} = 0, x_{16} = 0, x_{20} = 1 \) and \( x_{23} = 1 \). Here GINO has been used to solve the optimization problem.

From the above result, it can be seen that the elements which are to be kept open are 11, 15 and 23. The system configuration at the end of the above procedure is shown in Fig. 2.

Now it may be possible to reduce the losses further by repeating the above procedure with Fig. 2 as the initial configuration. Thus, this procedure has to be repeated till no further loss reduction is possible. Table 3 gives the details of all the iterations for this example (Case 1). At the end of the fourth iteration it is observed that there is no change in the open-switch positions suggested by the optimization procedure, thus indicating that no further loss reduction is possible. Fig. 3 is the minimum loss configuration for Case 1.

Now consider the system shown in Fig. 4. This system is the same as that considered by Civanlar et al. [2] and Huddleston et al. [6]. Let this example be referred to as Case 2. Using the procedure explained...
Table 3  
Results of all the iterations for Case 1

<table>
<thead>
<tr>
<th>Iteration no.</th>
<th>0-1 variables</th>
<th>Power losses</th>
<th>Switches in open position</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Initial Final</td>
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</tr>
<tr>
<td>1</td>
<td>$x_5$, $x_9$, $x_{14}$, $x_{16}$, $x_{20}$, $x_{23}$</td>
<td>247.75 233.15</td>
<td>12, 16, 23 11, 15, 23</td>
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<tr>
<td>2</td>
<td>$x_9$, $x_{13}$, $x_{14}$, $x_{15}$, $x_{16}$, $x_{23}$</td>
<td>233.15 225.66</td>
<td>11, 15, 23 10, 14, 23</td>
</tr>
<tr>
<td>3</td>
<td>$x_9$, $x_{10}$, $x_{12}$, $x_{13}$, $x_{15}$, $x_{23}$</td>
<td>225.66 219.19</td>
<td>10, 14, 23 9, 15, 23</td>
</tr>
<tr>
<td>4</td>
<td>$x_9$, $x_{11}$, $x_{12}$, $x_{13}$, $x_{16}$, $x_{23}$</td>
<td>219.19 219.19</td>
<td>9, 15, 23 9, 15, 23</td>
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earlier, the loss functions for the circuits of this system are obtained as follows:

$$LF_1 = 2265.12x_5x_7 - 1853.28x_5x_{11} - 3157.44x_5x_{16} - 463.32x_7x_{11} - 4371.84x_7x_{16} + 645.84x_{11}x_{16} + 15581.28x_{11} - 2121.93x_{11} - 11086.0x_{16} \tag{24}$$

$$LF_2 = 451.44x_{10}x_{11} - 1471.36x_{10}x_{14} - 2207.04x_{10}x_5 - 451.44x_{11}x_{14} - 3706.56x_5x_7 + 2207.04x_{14}x_5 + 14546.4x_{10} + 13990.32x_{11} - 13210.56x_{14} - 5873.04x_5 \tag{25}$$

$$LF_3 = 1538.24x_{14}x_{16} - 1355.0x_{14}x_{10} - 1916.64x_{14}x_7 - 1538.24x_{16}x_{10} - 3157.44x_{16}x_7 - 1103.52x_{10}x_7 + 6217.20x_{14} + 9937.84x_{16} - 3371.28x_{10} - 6145.92x_7 \tag{26}$$

The total loss function for the entire system is given by $LF = LF_1 + LF_2 + LF_3$.

This has to be minimized with respect to the 0-1 variables $x_5$, $x_7$, $x_{10}$, $x_{11}$, $x_{14}$ and $x_{16}$ and subject to the following constraints:

$$x_5 + x_{11} \geq 1 \tag{27a}$$

$$x_7 + x_{16} \geq 1 \tag{27b}$$

$$x_{10} + x_{14} \geq 1 \tag{27c}$$

The optimal values of the variables for this case are obtained as $x_5 = 1$, $x_7 = 1$, $x_{10} = 0$, $x_{11} = 0$, $x_{14} = 1$ and $x_{16} = 1$. From this it can be seen that the elements which are to be kept open are 19, 17 and 26, which are the same as obtained in Ref. [6]. The system configuration at this stage is shown in Fig. 5. If Fig. 5 is taken as the initial configuration and the above procedure is repeated, it is seen that there is no change in the positions of the elements which are open, thus indicating that Fig. 5 is the minimum loss configuration for Case 2.

5. Discussion

Most of the methods available in the literature for the problem of feeder reconfiguration for loss minimization in distribution systems are based on either heuristics or successive-approximation methods in which one feeder pair is considered at a time to reconfigure the system to reduce losses. Huddleston et al. [6] proposed an interesting method by considering multiple feeder pairs at a time. In their method the segments at the end of each circuit are reduced to spot loads and are considered for switching to reduce the losses. However, they do not consider the possibility of switching any other segments to further reduce the losses. Thus, their method may not give the overall minimum loss configuration for all cases.

In the example system shown in Fig. 1 (Case 1), if the method of Ref. [6] were used, it would stop after the first iteration and would not consider the other switching options to reduce the losses further. However, using the proposed method for the same example system, the minimum loss configuration has been obtained after
four iterations with a lower power loss than can be obtained by the method of Ref. [6].

The method proposed in this paper considers multiple switchings at a time and finds the overall minimum loss configuration. Further, in the method of Huddleston et al. [6], the values of the switching currents have to be interpreted to decide on the switchings. For example, for Case 2 (Fig. 4) Huddleston's method gives the optimal value of $I_{s7}$ as 41 A. But the initial value of $I_{s7}$ is 66 A. This indicates that only 25 out of the 66 A at node 7 have to be switched to minimize the losses. But, since it is not a major part of the 66 A, it is not switched. Whereas in the method proposed in this paper, since the problem is formulated as a 0–1 integer programming problem, it is possible to directly decide on the switchings based on the values of the 0–1 integer variables obtained by minimizing the power loss function. For example, in Case 2 the optimal value of $x_7$ is 1, indicating that node 7 need not be switched.

Thus the proposed method, which is based on the 0–1 integer programming problem formulation, would consider multiple switchings at a time and also would give an overall optimal solution for loss minimization in electrical distribution systems. This method always provides the optimal solution for the minimum loss configuration problem.

6. Conclusions

In the present paper a new method based on the 0–1 integer programming problem formulation for feeder reconfiguration for loss minimization in distribution systems is presented. Generalized procedures for writing the loss function are also given. The proposed method is illustrated using an example system and the salient features and advantages of the proposed method have been brought out clearly.

References