Influence of hydrodynamic coefficients in the response behavior of triangular TLPs in regular waves

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Abstract

Triangular configuration tension leg platforms (TLPs) are used for deep-water oil/gas exploration. The mechanics of TLP is highly nonlinear due to larger structural displacements and fluid motion-structure interaction. Triangular TLP has major consideration for deep-water application also due to its relative insensitivity with increasing water depth, excellent station keeping characteristics, etc. which makes this as a most cost effective and practical production system for deep waters. This study focuses on the influence of hydrodynamic drag coefficient ($C_d$) and hydrodynamic inertia coefficient ($C_m$) on the nonlinear response behavior of triangular TLP models under regular waves. Two typical triangular TLP models vis-a-vis TLP\textsubscript{1} and TLP\textsubscript{2} are taken for the study at 600 and 1200 m water depths, respectively. Hydrodynamic forces on these TLPs are evaluated using modified Morison equation under regular waves. Diffraction effects are neglected. Various nonlinearities arising due to relative velocity term in drag force, change in tether tension due to TLP movement, and set down effect are being considered in the analysis. The dynamic equation of motion has been solved in time-domain by employing Newmark’s b numerical integration technique. Based on the numerical study conducted, it is seen that the response evaluated using varying hydrodynamic coefficients through the water depth is significantly lesser in comparison to the response with constant coefficients in all activated degrees-of-freedom. However, sway, roll, and yaw degrees-of-freedom are not present due to the unidirectional wave loading considered for the study. The influence of hydrodynamic coefficients in wave period of 15 s is more in comparison with that of 10 s, and is nonlinear. The hydrodynamic coefficients also influence the plan dimension of TLP and its site location (geometry). Therefore, it may
become essential to estimate the range of $C_d - C_m$ values to vary through the water depth, based on Reynolds number ($Re$) or Keulegan-Carpenter number ($K_c$) even before the preliminary design of the TLP geometry.

**Keywords:** Fluid-structure interaction; Varying hydrodynamic coefficients; Offshore TLP

### 1. Introduction

Offshore structures are essentially meant for oil exploration and are usually constructed on the seashore and then towed down to the particular location for anchorage. Triangular tension leg platforms (TLPs) are generally useful for deep-water oil/gas exploration. The tensioned cabling consists of three-tethered leg, each leg being members of multiple parallel cables, terminated at the base of the structure. The composition of the tensioned members, terminated at the base will vary depending on the mooring requirements and the type of the tension member selected. Owing to this tension, the vertical motion such as heave, roll, and pitch are almost restrained while the horizontal motion such as surge, sway, and yaw becomes considerably large. Both this features are attractive because vertical rigidity helps to tie wells for production while horizontal compliance makes the platform insensitive to the primary effects of the waves. However, the second-order slowly varying drifts forces at low frequency caused by crossed modulation between various harmonic components results in low frequency oscillations. The mechanics of TLP is highly nonlinear because of larger displacement of the structure and due to nonlinear interaction between the structure and fluid motion. TLP has a major consideration for deepwater application due to its relative insensitivity with respect to increasing water depth. Therefore, the saving in steel combined with its excellent station keeping characteristics makes TLP as one of the most cost effective and practical production system for deep-water developments.

### 2. Literature review

The empirical force model proposed by Morison et al. (1950) has been most widely used and accepted in determining forces on thin cylindrical members in an offshore structure. However, they pointed out that the computation depends on the knowledge of water particle kinematics and empirically determined hydrodynamic force coefficients. Extensive research effort has been made in determining these force coefficients $d$ and $C_m$. Sarpakaya and Isaacson (1981) showed that these coefficients are functions of Keulegan-Carpenter number ($K_c$), Reynolds number, and roughness parameter of the cylinder. The Keulegan-Carpenter number ($K_c$) is a measure of water particle orbital amplitude with respect to cylinder diameter, and has been defined in terms of amplitude of the water particle velocity. Burrows et al. (1997) discussed the application of Morison wave force coefficients to random seas. They focused on the use of rigid and flexible forms of Morison
equation for estimation of drag and inertia under random wave excitation. They suggested that the hydrodynamic coefficients from large-scale measures may not be so sensitive to their method of estimation and that they are applicable to relative form. Clauss and Birk (1996) presented the hydrodynamic shape optimization of large offshore structures, which is applicable at an early design stage to develop offshore structures with improved sea-keeping qualities. Evolution and comparison of different designs based on significant double amplitude of forces and motion, which is computed for given design spectrum work, is discussed. It is also stated that the design of hull forms with favorable sea-keeping behavior based on engineering skills is necessary to sketch a good design. Chandrasekaran (1999) conducted a parametric study on nonlinear analysis of offshore TLPs subjected to environmental loading. The study was conducted for triangular TLP under regular waves; with water particle kinematics computed using Airy's linear wave theory. He showed that the uncertainties in evaluating proper $C_m$ values will have larger effects on the response behavior of triangular TLP. The hydrodynamic force given by modified Morison equation is nonlinear due to relative velocity squared drag force term and evaluation of force at the instantaneous position of the structure. He also showed that the response behavior of triangular TLP is inertia influenced, but it is not linear due to various other nonlinearities present in the system. Hahn (1995) discussed the effect of sea surface elevation on the response of offshore structures. The wave motion was assumed to be harmonic and structure was modeled by lumping of mass at the position of still water level. He pointed out that the drag force loading does not accommodate the effects of fluid structure interaction, which otherwise effectively increases the damping of the system. Therefore, the response of a floating structure shall be investigated with much attention towards the inertia and drag force intensity resultants. Patel and Witz (1985) stated that in order to ensure sufficient hydrodynamic stability, it is desirable to maximize static tension in the tendons by increasing the excess buoyancy and minimizing distance of C.G. from the keel. However, both these objectives also lower the natural period of the structure and therefore it is preferable to vary the natural period such that it lies outside the wave spectra. However, it can be seen that dynamic and static configuration conflict and there must be compromise between the two. Gudmested (1988) discussed a new approach for estimating wave kinematics for irregular deep water based on measured results of regular water wave kinematics. The development of the approach has been based on the results from the measurements of regular water particle kinematics. He suggested this approach as an improvement in the description of irregular sea-state, if documented carefully. Deep-water structures like TLP are more dynamically sensitive than the other structures placed in moderate water depth. Rainey (1989) derived an equation for potential wave loading on a lattice type offshore structure partially immersed in waves. The new equation is an effective replacement to nonlinear inertia term to describe the effects of vorticity. He discussed that the steady flow effects are comparable in the drag term rather than inertia term in Morison equation. The criterion for this significance is therefore based on the size of wave number and hence it will generally depend on cylinder length aspects. He pointed out that the significant of
end effects depends on the angle of cylinder floating, which varies typically during the wave cycle. Geirmoe and Verley (1980) reported an experimental investigation of forces on cylinder in steady current. They stated that the hydrodynamic damping based on Morison equation may be strongly un-conservative. They also pointed out that the effect of moderate current on the oscillating drag coefficients and the use of Morison equation with ordinary drag coefficients is very un-conservative. Boaghe et al. (1998) investigated the spectral analysis based on dynamic Morison equation. They noted that $C_m$ variation is lesser at high Reynolds number and at $Cd = 0.40$. This value of hydrodynamic drag coefficient $Cd$ is lesser than what is normally obtained at $Re > 10^5$ from the Morison equation where its value is 0.6. They pointed out that the results in terms of non-dimensional coefficients are thus quite encouraging but needs more confirmation.

### 3. Assumptions and structural idealization

- Initial pre-tension in all tethers is equal and remains unaltered over time. It is quite large in comparison to the changes that occurred during the lifetime of TLP. However, total pretension changes with the motion of platform.
- Wave forces are estimated at the instantaneous position of the platform by Morison's equation with Airy's linear wave theory. Wave is considered to act unidirectional in the surge direction only.
- Wave diffraction effects are neglected.
- Change in the pretension is calculated in every time step and writing the equation of equilibrium at that time step modified elements of stiffness matrix of the platform.
- The damping matrix has been assumed to be mass and stiffness proportional, based on their initial values.
- Wave forces on the tethers are assumed to be negligible.
- The low frequency drift oscillation in surge and high frequency tension oscillations of the tethers are not considered in the analysis.

### 4. Mass matrix

The structural mass is assumed to be lumped at each degree-of-freedom. Hence, it is diagonal in nature and constant. The added mass $M_a$ due to the water surrounding the structural members and arising from the modified Morison equation has been considered up to MSL. The presence of off-diagonal term in the mass matrix, indicates contribution in added mass, due to the hydrodynamic loading (i.e. the loading will be attracted only in surge, heave and pitch degree of freedom, due to the unidirectional wave acting in the surge direction on a symmetrical configuration of the platform about the $x$ and $z$ axis). The mass matrix as given by
Chandrasekaran and Jain (2002) is presented below.

\[
\begin{bmatrix}
M_{11} + M_{a11} & 0 & 0 & 0 & 0 \\
M_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & M_{33} + M_{a33} & 0 & 0 \\
0 & 0 & 0 & M_{44} & 0 \\
M_{a51} & 0 & M_{a53} & 0 & M_{55} \\
0 & 0 & 0 & 0 & M_{66}
\end{bmatrix}
\] (1)

5. Stiffness matrix

The coefficient, \(K_{ij}\), of stiffness matrix of triangular TLP has been derived as the reaction in \(i\)th degree-of-freedom by giving unit displacement in the \(j\)th degree-of-freedom, keeping all other degree-of-freedom restrained Chandrasekaran and Jain (2002) support the derivation.

\[
\begin{bmatrix}
K_{uu} & 0 & 0 & 0 & 0 & 0 \\
0 & K_{22} & 0 & 0 & 0 & 0 \\
K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & 0 \\
0 & K_{42} & 0 & K_{44} & 0 & 0 \\
K_{51} & 0 & 0 & 0 & K_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & K_{66}
\end{bmatrix}
\] (2)

6. Damping matrix

Assuming \([C]\) to be proportional to \([K]\) and \([M]\), the element of \([C]\) are determined by equation given below, using the orthogonal properties of \([M]\) and \([K]\).

\[
\phi^T[c]\phi = 2\zeta_i\omega_i m_i
\] (3)

\(\zeta\) is taken as 0.05. The matrix is calculated based on the initial value of \([K]\) and \([M]\) only.

7. Hydrodynamic forces

TLPs are comprised of slender structural members whose diameter is small relative to the incident wavelength. Therefore, the wave train remains relatively unaffected outside the immediate vicinity of the member. Hence, flow separation is important other than the wave diffraction. As given by Isaacson (1983), for the structures in this flow separation regime, the wave forces is generally computed by Morison equation which gives the force per unit length \(F\) on the section of the cylinder.

\[
F' = \rho C_d D u \lambda + \rho D^2 \frac{C_m}{4} u \lambda
\] (4)
where \( q \) is the fluid density, \( D \) is the cylinder diameter, \( u_\theta \) is the incident flow velocity at the location of the section, \( C_d \) and \( C_m \) are the hydrodynamic drag and inertia coefficient, respectively.

Bea et al. (1999) presented wave forces on deck of offshore platforms; they indicated that the total force imposed on a platform deck could be formulated as follows:

\[
F_{bw} = F_b + F_s + F_d + F_i
\]

where \( F_b \) is the Buoyancy force (vertical), \( F_s \) is the slamming force, \( F_d \) is the drag force (velocity dependent), \( F_l \) is the lift force (velocity dependent) and normal to wave direction, and \( F_i \) is the inertia force (acceleration dependent).

As the wave crest encounters the platform deck, there is a transfer of momentum from the water particle to the structure, which is reflected as an initial slamming force. The magnitude of the slamming force relative to the peak wave inundation force will be depending on the characteristics of the deck. However, for non-plated (grated, open) decks like TLPs, this force is much lower.

The horizontal drag force can be computed as:

\[
F_D = 0.5qCdAu_\theta^2
\]

and the vertical lift force can be computed as:

\[
F_l = 0.5qC_lAu_\theta^2
\]

where \( A \) is the vertical deck area subjected to the wave crest.

Due to the geometry of the bay (i.e. grated, openings), the horizontal lift forces are expected to be relatively small compared to the horizontal drag forces. The horizontal inertial force can be computed as:

\[
F_i = pC_mVu_\theta
\]

where \( V \) is the volume of the deck inundated.

At the time of the occurrence of \( F_d \), i.e. at the time of occurrence of the maximum velocity at the wave crest, \( F_i \) will be zero or near zero (acceleration is zero or near zero at the wave crest). The primary issues associated with this formulation are evaluating the elevation of the wave crest (which determines the amount of the deck inundated) and the water velocity in the wave crest. Airy's linear wave theory is used for estimating the water particle kinematics. This facilitates the use of simple equation without involving significant error in deep water condition. The columns of the platform are treated as surface piercing slender members and integration for estimation of forces are carried out from the keel level up to the mean sea level. Modified Morison equation is used to estimate the wave forces acting on the structure. This involves relative velocity and acceleration terms in drag and inertial forces, respectively. The original form of Morison equation is:

\[
F = \int F_d + \int F_i = 0.5qCdDju_\theta A_s p \int C_m pn*D^2/4ii ds
\]
This is modified to account for relative velocity as follows:

\[ F = \int \text{d}t = \int \text{d}F_d + \text{J} \text{d}i? = \partial 0.5qC_dDju_{\text{rel}}u_{\text{rel}} \text{ds} \]

\[ + \rho \pi D^2 / 4 \int (C_m \ddot{u} - (C_m - 1) \dot{x}) \text{ds} \]

where \( u_{\text{rel}} \) is \( u_0 - \ddot{x}_0 \).

Though the numerical integration is very easy to be performed on the above equations, in order to facilitate closed form integrations over depth, linearization of nonlinear drag force term is invoked. This ultimately helps in reducing the computation time drastically. Morison equation for wave loading on a rigid vertical member is given by:

\[ F_x = C_m \rho \pi D^2 / 4 \cdot \frac{1}{2} \rho C_d D L u_u \cdot \delta 12 \Phi \]

where \( L \) is the short length over which the force is calculated (the force sleeves are 0.535 m long in this case). Current, when present, is included by virtue of a non-zero mean value particle acceleration. Eq. (11) is more conveniently written as:

\[ F_X = C_m K f i + C d K d u j j u_0 \]

The loads experienced on members are known to be related to the values of parameters i.e. Reynolds number (Re), Keulegan-Carpenter number (Kc) and roughness. According to Burrows (1992),

\[ K_c = \frac{2 \pi}{D} \sqrt{2} \frac{\sigma^2 u}{\sigma u} \]

\[ K^* = \sqrt{2} \cdot K c \]

where both \( K c \) and \( K^* \) are used as measures of Keulegan-Carpenter number.

8. Hydrodynamic force vector for triangular TLP model taken for the study

The hydrodynamic force vector is calculated in each degree-of-freedom as given below. According to Morison’s equation, the intensity of wave force/unit length on the structure is given as:

\[ / = 0.5 \rho \pi C_d \text{e} \text{e}^2 \left| M + \text{e}^2 + \text{e} \text{e}^2 \right| + \text{e} \text{e}^2 \left( \text{e} - \text{e}^2 \right) \left( \text{e}^2 - \text{e} \right) \]

\[ + 0.5 \rho \pi C_d D^2 \text{e} \text{e}^2 \left[ C_m \text{e} \right] \text{e} \text{e}^2 \]

The last term in the above equation is the added mass term and positive sign is used when \( g \) is below MSL and negative signs used when \( g \) is above MSL. The contribution of added mass up to MSL has already been considered along with the structural mass. The force vector is given by:

\[ \{ F(t) \} = \{ F_1 F_2 F_3 F_4 F_5 F_6 \} \]
Referring to Fig. 1 of the typical triangular TLP model taken for the study, the force vector is derived as below:

F11 Force in the surge degree-of-freedom due to wave.
Force in the $x$ direction in members 1 and 2 at $x = -P_b=3$ plus force in member 3 at $x = 2P_b=3$ plus force in the $x$ direction in pontoon 4 at $x = -P_b=3$ and $z = l$ plus force normal to pontoons 5 and 6 (where $x$ is a variable, $-P_b=3 < x < 2P_b=3$) resolved in the $x$ direction.
F21 Force in the sway degree-of-freedom due to wave.
   Force in the $y$ direction in member 1 at $y = -0.5P_l$ plus force in member 2 at $y = 0.5P_l$ plus force in member 3 at $y = 0$ plus force in the $y$ direction normal to pontoons 5 and 6 (where $y$ is a variable $-0.5P_l < y < 0.5P_l$) resolved in the $y$ direction.

F31 Force in the heave degree-of-freedom due to wave.
   Force in the $z$ direction in pontoon 4 at $x = -P_b/3$ and $z = l$ plus force in members 5 and 6 where $x$ is varying as $-A/3 < x < 2P_b/3$ at $z = l$.

F41 Force in the roll degree-of-freedom due to wave.
   Moments of the above coefficients (i.e. $F_{1l}$, $F_{2l}$ and $F_{3l}$), about the $x$ axis.

F51 Force in the pitch degree-of-freedom due to wave.
   Moments of the above coefficients (i.e. $F_{1l}$, $F_{2l}$ and $F_{3l}$), about the $y$ axis.

F61 Force in the yaw degree-of-freedom due to wave.
   Moments of the above coefficients (i.e. $F_{1l}$, $F_{2l}$ and $F_{3l}$), about the $z$ axis.

9. Equation of motion

The equation of motion of the triangular configuration TLP under regular wave is given below.

$$[M]\ddot{x} + [C]\dot{x} + [K]x = F(t)$$

As the force vector is response dependent and fluctuations in added mass are included at every time instant the force vector is changing at every time step. Owing to the nonlinear nature of the matrices in the dynamic equilibrium equations, the time domain integration of acceleration and velocity method is used as a tool to solve this problem. In this particular case, Newmark's method with average acceleration is used.

10. Numerical studies and discussion

Numerical studies are conducted to highlight the influence of hydrodynamic coefficients $Cd$ and $C_m$ on the nonlinear response of two triangular TLP models namely TLP$_1$ and TLP$_2$. The wave heights and wave periods are selected closer to the natural period of these TLPs so as to study the near-resonating structural response of these models, if any. Burrows et al. (1997) presented the Morison coefficients from analysis of entire random sea states with rigid cylinder settings and also compared these coefficients with random wave-by-wave results presented by Klopman and Kostense (1990) and regular wave results of Bearman et al. (1987). The water particle kinematics is computed based upon linear wave theory for the assumed regular sea state. Based on the variance of the horizontal water particle velocity and acceleration and by using Eq. (13), the Keulegan-Carpenter numbers ($K_c$) were estimated. A range of $Cd$ and $C_m$ are then selected and assumed to vary through the water depth as shown in Fig. 2. Columns 1, 2 and 3 are divided into small segments of 0.1 m height, which are grouped in four major
classifications. The hydrodynamic coefficients $C_d$ and $C_m$ are then selected and continuously varied between their ranges throughout the water depth. The study has been conducted with unidirectional regular waves acting in the surge direction without the presence of steady currents. The stiffness matrix of the platform is evaluated at every time instant to accommodate the contribution from change in tether length and response dependence in the analysis. However, the damping matrix is assumed to be constant throughout the analysis which is based on the initial values of $[K]$ and $[M]$.

11. Coupled surge response

By comparing the maximum positive responses, it is seen that the TLP1 shows 53.62% lesser response with varying $C_d$-$C_m$ values in comparison with that of $d = 0.45$ and $C_m = 1.8$ (case II) for 8 m-10 s; 57.34% lesser for 10 m-10 s and 60.97% lesser for 12 m-10 s, respectively. The response is 43.8% lesser with varying $C_d$-$C_m$ in comparison with $C_d = 0.6$ and $C_m = 1.55$ (case III) for 8 m-10 s; 52.23% lesser for 10 m-10 s and 58.44% lesser for 12 m-10 s, respectively. By keeping the inertia coefficient constant at 1.55 and by varying $C_d$ from 0.45 to 0.7 (case IV), the coupled surge response is 34.4% lesser than the maximum positive response with varying $C_d$-$C_m$ (case I) for 8 m-10 s; 25% lesser for 10 m-10 s and 28.12% lesser for 12 m-10 s, respectively. By keeping the drag coefficient constant at 1.8 and by varying $C_m$ from 0.45 to 0.7 (case V), the coupled surge response is 8.57% more than the maximum positive response with varying $C_d$-$C_m$ (case I) for all the three cases vis-a-vis 8 m-10 s, 10 m-10 s and 12 m-10 s, respectively. By keeping the drag coefficient constant at 0.6 and by varying $C_m$ from 1.8 to 1.5 (case VI), the coupled surge response is 8.6% more than the maximum positive response with varying $C_d$-$C_m$ (case I) for all the three cases vis-a-vis 8 m-10 s, 10 m-10 s and
12 m-10 s, respectively. By keeping the drag coefficient constant at 0.45 and by varying \( C_m \) from 1.8 to 1.5 (case VII), the coupled surge response does not show any change with that of the response with varying \( C_d-C_m \) (case I) for 8 m-10 s and 10 m-10 s and 5.89% more for 12 m-10 s.

It is seen that the TU\(^{\wedge} \) shows 57.33% lesser response with varying \( C_d-C_m \) values in comparison with that of \( d = 0.45 \) and \( C_m = 1.8 \) (case II) for 8 m-15 s; 67.07% lesser for 10 m-15 s and 60.43% lesser for 12 m-15 s, respectively. The response is 50.76% lesser with varying \( C_d-C_m \) in comparison with \( C_d = 0.6 \) and \( C_m = 1.55 \) (case III) for 8 m-15 s; 64% lesser for 10 m-15 s and 56.09% lesser for 12 m-15 s, respectively. By keeping the inertia coefficient constant at 1.55 and by varying \( C_d \) from 0.45 to 0.7 (case IV), the coupled surge response is 25% lesser than the maximum positive response with varying \( C_d-C_m \) (case I) for 8 m-15 s; 22.22% lesser for 10 m-15 s and 13.88% lesser for 12 m-15 s, respectively. By keeping the inertia coefficient constant at 1.80 and by varying \( C_d \) from 0.45 to 0.7 (case V), the coupled surge response is 8.57% more than the maximum positive response with varying \( C_d-C_m \) (case I) for 8 m-15 s; 10% more for both 10 m-15 s and 11.11% more for 12 m-15 s, respectively. By keeping the drag coefficient constant at 0.6 and by varying \( C_m \) from 1.8 to 1.5 (case VI), the coupled surge response is 13.51% more than the maximum positive response with varying \( C_d-C_m \) (case I) for 8 m-15 s; 12.9% more for 10 m-15 s and 12.19% for 12 m-15 s, respectively. By keeping the drag coefficient constant at 0.45 and by varying \( C_m \) from 1.8 to 1.5 (case VII), the coupled surge response is 3.03% more than that of the response with varying \( C_d-C_m \) (case I) for 8 m-15 s; does not show any change for 10 m-15 s and 12 m-15 s, respectively.

By comparing the maximum positive responses, it is seen that the TLP\(_2\) shows 49.38% lesser response with varying \( C_d-C_m \) values in comparison with that of \( C_d = 0.45 \) and \( C_m = 1.8 \) (case II) for 8 m-10 s; 53.93% lesser for 10 m-10 s and 65.21% lesser for 12 m-10 s, respectively. The response is 48.1% lesser with varying \( C_d-C_m \) in comparison with \( C_d = 0.6 \) and \( C_m = 1.55 \) (case III) for 8 m-10 s; 51.76% lesser for 10 m-10 s and 64.04% lesser for 12 m-10 s, respectively. By keeping the inertia coefficient constant at 1.55 and by varying \( C_d \) from 0.45 to 0.7 (case IV), the coupled surge response is 14.63% lesser than the maximum positive response with varying \( C_d-C_m \) (case I) for 8 m-10 s; 17.07% lesser for 10 m-10 s and 18.75% lesser for 12 m-10 s, respectively. By keeping the inertia coefficient constant at 1.80 and by varying \( C_d \) from 0.45 to 0.7 (case V), the coupled surge response is 8.89% more than the maximum positive response with varying \( C_d-C_m \) (case I) for 8 m-10 s and 10 m-10 s; 8.57% more for 12 m-10 s, respectively. By keeping the drag coefficient constant at 0.6 and by varying \( C_m \) from 1.8 to 1.5 (case VI), the coupled surge response is 4.65% more than the maximum positive response with varying \( C_d-C_m \) (case I) for 8 m-10 s and 10 m-10 s; 11.11% more for 12 m-10 s, respectively. By keeping the drag coefficient constant at 0.45 and by varying \( C_m \) from 1.8 to 1.5 (case VII), the coupled surge response is 7.31% lesser than the response with varying \( C_d-C_m \) (case I) for 8 m-10 s and 10 m-10 s; does not show any change for 12 m-10 s.
It is seen that the \textbf{TLP}_2 shows 56.47\% lesser response with varying $C_d-C_m$ values in comparison with that of $Cd = 0.45$ and $C_m = 1.8$ (case II) for 8 m-15 s; 64.13\% lesser for 10 m-15 s and 61.38\% lesser for 12 m-15 s, respectively. The response is 54.32\% lesser with varying $C_d-C_m$ in comparison with $Cd = 0.6$ and $C_m = 1.55$ (case III) for 8 m-15 s; 63.73\% lesser for 10 m-15 s and 58.94\% lesser for 12 m-15 s, respectively. By keeping the inertia coefficient constant at 1.55 and by varying $Cd$ from 0.45 to 0.7 (case IV), the coupled surge response is 21.6\% lesser than the maximum positive response with varying $Cd-C_m$ (case I) for 8 m-15 s; 18.18\% lesser for 10 m-15 s and 20.51\% lesser for 12 m-15 s, respectively. By keeping the inertia coefficient constant at 1.80 and by varying $Cd$ from 0.45 to 0.7 (case V), the coupled surge response is 7.5\% more than the maximum positive response with varying $Cd-C_m$ (case I) for 8 m-15 s; 5.7\% more for 10 m-15 s and 7.14\% more for 12 m-15 s, respectively. By keeping the drag coefficient constant at 0.6 and by varying $C_m$ from 1.8 to 1.5 (case VI), the coupled surge response is 11.9\% more than that of the maximum positive response with varying $C_d-C_m$ (case I) for 8 m-15 s; 15.38\% more for 10 m-15 s and 11.36\% more for 12 m-15 s, respectively. By keeping the drag coefficient constant at 0.45 and by varying $C_m$ from 1.8 to 1.5 (case VII), the coupled surge response does not show any change with that of the response with varying $Cd-C_m$ (case I) for 8 m-15 s and 12 m-15 s; 5.71\% more for 10 m-15 s, respectively.

Figs. 3-5 show the time histories of the coupled surge response of both the TLP models for wave height-wave period of 10 m-10 s. Figs. 6-8 show the time histories of the coupled surge response for wave height-wave period of 10 m-15 s. In all the above figures, it is seen that the TLP\textsubscript{2} exhibits more difference between the responses with varying $C_d-C_m$ values in comparison of the response obtained either with constant $C_d$ and varying $C_m$ or with constant $C_m$ and varying $C_d$. The regular wave of 10 s wave period causes more positive surge offset than 15 s wave period waves. This may be because the lower wave period is closer to the natural periods of the TLP causing a higher response. The variation in the coupled surge response is significant with varying $Cd-C_m$ values when compared with that of constant coefficients. The influence of hydrodynamic coefficients in the wave period of 15 s is more in comparison with that of 10 s and the variation is nonlinear between the different wave heights with the same wave period (in both the time periods).

\section*{12. Coupled heave response}

By comparing the maximum positive responses, it is seen that the \textbf{TLP}_1 shows 66.3\% lesser response with varying $Cd-C_m$ values in comparison with that of $C_d = 0.45$ and $C_m = 1.8$ (case II) for 8 m-10 s; 66.34\% lesser for 10 m-10 s and 66.95\% lesser for 12 m-10 s, respectively. The response is 63.53\% lesser with varying $Cd-C_m$ in comparison with $Cd = 0.6$ and $C_m = 1.55$ (case III) for 8 m-10 s; 63.54\% lesser for 10 m-10 s and 64.48\% lesser for 12 m-10 s, respectively. By keeping the inertia coefficient constant at 1.55 and by varying $Cd$ from 0.45 to 0.7 (case IV), the coupled heave response is 19.35\% lesser than the maximum positive
Fig. 3. Coupled surge response of triangular TLPs for varying $C_d - C_m$ ($H = 10$ m, $T = 10$ s).

Fig. 4. Coupled surge response of triangular TLPs for varying $C_d$ ($C_m = 1.8$) ($H = 10$ m, $T = 10$ s).

Fig. 5. Coupled surge response of triangular TLPs for varying $C_m$ ($Q = 0.45$) ($H = 10$ m, $T = 10$ s).
Fig. 6. Coupled surge response of triangular TLPs for varying $C_d-C_m$ ($H = 10 \text{ m}, T = 15 \text{ s}$).

Fig. 7. Coupled surge response of triangular TLPs for varying $C_d$ ($C_m = 1.8$) ($H = 10 \text{ m}, T = 15 \text{ s}$).

Fig. 8. Coupled surge response of triangular TLPs for varying $C_m$ ($Q = 0.45$) ($H = 10 \text{ m}, T = 15 \text{ s}$).
response with varying \( C_d-C_m \) (case I) for 8 m-10 s; 20% lesser for 10 m-10 s and 18.42% lesser for 12 m-10 s, respectively. By keeping the inertia coefficient constant at 1.80 and by varying \( C_d \) from 0.45 to 0.7 (case V), the coupled heave response does not show any change with that of the maximum positive response with varying \( C_d-C_m \) (case I) for 8 m-10 s; 2.85% lesser for 10 m-10 s and 2.63% lesser for 12 m-10 s, respectively. By keeping the drag coefficient constant at 0.6 and by varying \( C_m \) from 1.8 to 1.5 (case VI), the coupled heave response is 18.42% more than the maximum positive response with varying \( C_d-C_m \) (case I) for 8 m-10 s; 14.63% more for 10 m-10 s and 9.52% for 12 m-10 s, respectively. By keeping the drag coefficient constant at 0.45 and by varying \( C_m \) from 1.8 to 1.5 (case VII), the coupled heave response does not show any change with that of the response with varying \( C_d-C_m \) (case I) for all the three cases namely 8 m-10 s, 10 m-10 s and 12 m-10 s.

It is seen that the TLP1 shows 69.29% lesser response with varying \( C_d-C_m \) values in comparison with that of \( C_d = 0.45 \) and \( C_m = 1.8 \) (case II) for 8 m-15 s; 66.67% lesser for 10 m-15 s and 68.57% lesser for 12 m-15 s, respectively. The response is 67.5% lesser with varying \( C_d-C_m \) in comparison with \( C_d = 0.6 \) and \( C_m = 1.55 \) (case III) for 8 m-15 s; 64.8% lesser for 10 m-15 s and 66.67% lesser for 12 m-15 s, respectively. By keeping the inertia coefficient constant at 1.55 and by varying \( C_d \) from 0.45 to 0.7 (case IV), the coupled heave response is 17.94% lesser than the maximum positive response with varying \( C_d-C_m \) (case I) for 8 m-15 s; 18.18% lesser for both 10 m-15 s and 12 m-15 s, respectively. By keeping the inertia coefficient constant at 1.80 and by varying \( C_d \) from 0.45 to 0.7 (case V), the coupled heave response does not show any change with that of the maximum positive response obtained with varying \( C_d-C_m \) (case I) for 8 m-15 s; 9.09% more for 10 m-15 s and 4.54% more for 12 m-15 s, respectively. By keeping the drag coefficient constant at 0.6 and by varying \( C_m \) from 1.8 to 1.5 (case VI), the coupled heave response is 9.3% more than the maximum positive response with varying \( C_d-C_m \) (case I) for 8 m-15 s; 4.34% more for both 10 m-15 s and 12 m-15 s, respectively. By keeping the drag coefficient constant at 0.45 and by varying \( C_m \) from 1.8 to 1.5 (case VII), the coupled heave response does not show any change with that of the response with varying \( C_d-C_m \) (case I) for all the three cases namely 8 m-15 s, 10 m-15 s and 12 m-15 s.

By comparing the maximum positive heave responses, it is seen that the TLP2 shows 66.94% lesser response with varying \( C_d-C_m \) values in comparison with that of \( C_d = 0.45 \) and \( C_m = 1.8 \) (case II) for 8 m-10 s; 66.93% lesser for 10 m-10 s and 67.4% lesser for 12 m-10 s, respectively. The response is 65.81% lesser with varying \( C_d-C_m \) in comparison with \( C_d = 0.6 \) and \( C_m = 1.55 \) (case III) for 8 m-10 s; 66.11% lesser for 10 m-10 s and 66.15% lesser for 12 m-10 s, respectively. By keeping the inertia coefficient constant at 1.55 and by varying \( C_d \) from 0.45 to 0.7 (case IV), the coupled heave response is 17.5% lesser than the maximum positive response with varying \( C_d-C_m \) (case I) for 8 m-10 s; 17.07% lesser for 10 m-10 s and 18.18% lesser for 12 m-10 s, respectively. By keeping the inertia coefficient constant at 1.80 and by varying \( C_d \) from 0.45 to 0.7 (case V), the coupled heave response is 7.5% lesser than the maximum positive response with varying \( C_d-C_m \) (case I) for 8 m-10 s;
does not show any change for 10 m-10 s; 4.54% lesser for 12 m-10 s, respectively.

By keeping the drag coefficient constant at 0.6 and by varying $C_m$ from 1.8 to 1.5 (case VI), the coupled heave response is 9.09% more than the maximum positive response with varying $C_d-C_m$ (case I) for 8 m-10 s; 8.88% more for 10 m-10 s and 6.38% for 12 m-10 s, respectively. By keeping the drag coefficient constant at 0.45 and by varying $C_m$ from 1.8 to 1.5 (case VII), the coupled heave response does not show any change with that of the response with varying $C_d-C_m$ (case I) for all the three cases vis-à-vis 8 m-10 s, 10 m-10 s and 12 m-10 s.

It is seen that the TLP2 shows 66.41% lesser response with varying $C_d-C_m$ values in comparison with that of $d = 0.45$ and $C_m = 1.8$ (case II) for 8 m-15 s; 67.62% lesser for 10 m-15 s and 68.67% lesser for 12 m-15 s, respectively. The response is 65.35% lesser with varying $C_d-C_m$ in comparison with $Cd = 0.6$ and $C_m = 1.55$ (case III) for 8 m-15 s; 65.64% lesser for 10 m-15 s and 66.42% lesser for 12 m-15 s, respectively. By keeping the inertia coefficient constant at 1.55 and by varying $C_d$ from 0.45 to 0.7 (case IV), the coupled heave response is 18.18% lesser than the maximum positive response with varying $C_d-C_m$ (case I) for 8 m-15 s; 17.78% lesser for 10 m-15 s and 17.02% lesser for 12 m-15 s, respectively. By keeping the inertia coefficient constant at 1.80 and by varying $C_d$ from 0.45 to 0.7 (case V), the coupled heave response is 4.54% lesser than the maximum positive response with varying $C_d-C_m$ (case I) for 8 m-15 s; 6.67% lesser for 10 m-15 s and 6.12% lesser for 12 m-15 s, respectively. By keeping the drag coefficient constant at 0.6 and by varying $C_m$ from 1.8 to 1.5 (case VI), the coupled heave response is 6.38% more than the maximum positive response with varying $C_d-C_m$ (case I) for 8 m-15 s; 4.25% more for 10 m-15 s and 4.08% for 12 m-15 s, respectively. By keeping the drag coefficient constant at 0.45 and by varying $C_m$ from 1.8 to 1.5 (case VII), the coupled heave response does not show any change with that of the response with varying $C_d-C_m$ (case I) for 8 m-15 s, 10 m-15 s and 12 m-15 s.

Figs. 9-11 show the time histories of the coupled heave response of both the TLP models for wave height-wave period of 10 m-10 s. Figs. 12-14 show the time histories of the coupled heave response for wave height-wave period of 10 m-15 s. It is seen that the TLP2 exhibits more difference between the responses with varying $C_d-C_m$ values in comparison of the response obtained either with constant $C_d$ and varying $C_m$ or with constant $C_m$ and varying $C_d$. The variation in the coupled heave response is significant with varying $C_d-C_m$ values when compared with that of constant coefficients and therefore the hydrodynamic coefficients shall be established along with their range of application through the water depth to obtain the true picture of the heave response behavior. The coupled heave responses with constant $C_d-C_m$ combination of (0.45, 1.8) and (0.6, 1.55) show significantly higher values when compared with that of varying $C_d-C_m$ in all the hydrodynamic combinations taken for the study. However, the variation with $C_m = 18$ is more than that of $C_m = 1.55$. This indicates that heave response is inertia dominant.
Fig. 9. Coupled heave response of triangular TLPs for varying $C_d-C_m$ ($H = 10$ m, $T = 10$ s).

Fig. 10. Coupled heave response of triangular TLPs for varying $C_d$ ($C_m = 1.8$) ($H = 10$ m, $T = 10$ s).

Fig. 11. Coupled heave response of triangular TLPs for varying $C_m$ ($Q = 0.45$) ($H = 10$ m, $T = 10$ s).
Fig. 12. Coupled heave response of triangular TLPs for varying $C_d - C_m$ ($H = 10$ m, $T = 15$ s).

Fig. 13. Coupled heave response of triangular TLPs for varying $C_d$ ($C_m = 18$) ($H = 10$ m, $T = 15$ s).

Fig. 14. Coupled heave response of triangular TLPs for varying $C_m$ ($Q = 0.45$) ($H = 10$ m, $T = 15$ s).
13. Coupled pitch response

By comparing the maximum positive responses, it is seen that the TLP1 shows 64.7% lesser response with varying $Cd-C_m$ values in comparison with that of $d = 0.45$ and $C_m = 1.8$ (case II) for 8 m-10 s; 58.97% lesser for 10 m-10 s and 52.27% lesser for 12 m-10 s, respectively. The response is 58.62% lesser with varying $C_d-C_m$ in comparison with $C_d = 0.6$ and $C_m = 1.55$ (case III) for 8 m-10 s; 54.28% lesser for 10 m-10 s and 46.15% lesser for 12 m-10 s, respectively. By keeping the inertia coefficient constant at 1.55 and by varying Cd from 0.45 to 0.7 (case IV), the coupled pitch response is 16.67% lesser than the maximum positive response with varying $C_d-C_m$ (case I) for 8 m-10 s; 18.75% lesser for 10 m-10 s and 19.04% lesser for 12 m-10 s, respectively. By keeping the inertia coefficient constant at 1.80 and by varying C_d from 0.45 to 0.7 (case V), the coupled pitch response is 7.7% more than the maximum positive response with varying $C_d-C_m$ (case I) for 8 m-10 s; 5.89% more for 10 m-10 s and 4.54% more for 12 m-10 s, respectively. By keeping the drag coefficient constant at 0.6 and by varying C_m from 1.8 to 1.5 (case VI), the coupled pitch response does not show any change with that of the maximum positive response with varying $Cd-C_m$ (case I) for 8 m-10 s and 10 m-10 s; and 4.76% lesser for 12 m-10 s. By keeping the drag coefficient constant at 0.45 and by varying C_m from 1.8 to 1.5 (case VII), the coupled pitch response 7.7% more than that of the response with varying $Cd-C_m$ (case I) for 8 m-10 s; 5.89% more for 10 m-10 s and 4.76% lesser for 12 m-10 s.

It is seen that the TLP1 shows 60.97% lesser response with varying $Cd-C_m$ values in comparison with that of $Cd = 0.45$ and $C_m = 1.8$ (case II) for 8 m-15 s; 62.22% lesser for 10 m-15 s and 52.94% lesser for 12 m-15 s, respectively. The response is 57.9% lesser with varying $Cd-C_m$ in comparison with $Cd = 0.6$ and $C_m = 1.55$ (case III) for 8 m-15 s; 58.53% lesser for 10 m-15 s and 46.67% lesser for 12 m-15 s, respectively. By keeping the inertia coefficient constant at 1.55 and by varying Cd from 0.45 to 0.7 (case IV), the coupled pitch response is 12.5% lesser than the maximum positive response with varying $Cd-C_m$ (case I) for 8 m-15 s; 5.88% more for 10 m-15 s and 4.54% more for 12 m-15 s, respectively. By keeping the inertia coefficient constant at 1.80 and by varying Cd from 0.45 to 0.7 (case V), the coupled pitch response 7.7% more than that of the response with varying $Cd-C_m$ (case I) for 8 m-15 s; 5.89% more for 10 m-15 s and 4.76% lesser for 12 m-15 s, respectively. By keeping the drag coefficient constant at 0.6 and by varying C_m from 1.8 to 1.5 (case VI), the coupled pitch response does not show any change with that of the maximum positive response with varying $Cd-C_m$ (case I) for 8 m-15 s and 12 m-15 s cases; 10.52% more for 10 m-15 s, respectively. By keeping the drag coefficient constant at 0.45 and by varying C_m from 1.8 to 1.5 (case VII), the coupled pitch response is 5.88% more than that of the response with varying $Cd-C_m$ (case I) for 8 m-15 s; 10.52% more for 10 m-15 s and 7.7% for 12 m-15 s, respectively.

By comparing the maximum positive responses, it is seen that the TLP2 shows 58.82% lesser response with varying $C_d-C_m$ values in comparison with that of $Cd = 0.45$ and $C_m = 1.8$ (case II) for 8 m-10 s; 61.81% lesser for 10 m-10 s and 55.93%
It is seen that the TLP 2 shows 63.89% lesser response with varying $C_d-C_m$ values in comparison with that of $C_d = 0.45$ and $C_m = 18$ (case II) for 8 m-15 s; 60.97% lesser for 10 m-15 s and 48.38% lesser for 12 m-15 s, respectively. The response is 59.37% lesser with varying $C_d-C_m$ in comparison with $C_d = 0.6$ and $C_m = 1.55$ (case III) for 8 m-15 s; 56.75% lesser for 10 m-15 s and 42.85% lesser for 12 m-15 s, respectively. By keeping the inertia coefficient constant at 1.55 and by varying $C_d$ from 0.45 to 0.7 (case IV), the coupled pitch response is 15.38% lesser than the maximum positive response with varying $C_d-C_m$ (case I) for 8 m-15 s; 6.25% lesser for 10 m-15 s and 12.5% lesser for 12 m-15 s, respectively. By keeping the inertia coefficient constant at 1.80 and by varying $C_d$ from 0.45 to 0.7 (case V), the coupled pitch response is 7.14% more than the maximum positive response with varying $C_d-C_m$ (case I) for 8 m-15 s; 11.11% more for 10 m-15 s and 7.7% more for 12 m-15 s, respectively. By keeping the drag coefficient constant at 0.6 and by varying $C_m$ from 1.8 to 1.5 (case VI), the same trend of difference between the pitch responses as above is seen and for the case of keeping the drag coefficient constant at 0.45 and by varying $C_m$ from 1.8 to 1.5 (case VII) as well.

Figs. 15-17 show the time histories of the coupled pitch response of both the TLP models for wave height-wave period of 10 m-10 s. Figs. 18-20 show the time histories of the coupled pitch response for wave height-wave period of 10 m-15 s. It is seen that the TLP 2 exhibits more difference between the responses with varying $C_d-C_m$ values in comparison of the response obtained either with constant $C_d$ and varying $C_m$ or with constant $C_m$ and varying $C_d$. 
Fig. 15. Coupled pitch response of triangular TLPs for varying $C_d-C_m$ ($H = 10$ m, $T = 10$ s).

Fig. 16. Coupled pitch response of triangular TLPs for varying $C_d$ ($C_m = 18$) ($H = 10$ m, $T = 10$ s).

Fig. 17. Coupled pitch response of triangular TLPs for varying $C_m$ ($Q = 0.45$) ($H = 10$ m, $T = 10$ s).
Fig. 18. Coupled pitch response of triangular TLPs for varying $C_d - C_m$ ($H = 10 \text{ m}, T = 15 \text{ s}$).

Fig. 19. Coupled pitch response of triangular TLPs for varying $C_d$ ($C_m = 1.8$) ($H = 10 \text{ m}, T = 15 \text{ s}$).

Fig. 20. Coupled pitch response of triangular TLPs for varying $C_m$ ($Q = 0.45$) ($H = 10 \text{ m}, T = 15 \text{ s}$).
14. Conclusions

Based on the numerical studies conducted on the response behavior of the two typical triangular TLP models (TLP\textsubscript{1} and TLP\textsubscript{2}) under regular sea state, the following main conclusions are drawn:

• The variation in the coupled surge response is significant with varying Cd-C\textsubscript{m} values when compared with that of constant coefficients. This shows that the surge response of triangular TLP is highly influenced by the values of hydrodynamic coefficients and hence surge response evaluated with constant coefficients will yield a highly conservative value and does not depict the true behavior of the TLP in surge mode.

• The influence of hydrodynamic coefficients in the wave period of 15 s is more in comparison with that of 10 s and the variation is nonlinear between the different wave heights with the same wave period (in both the time periods). This may be due to other nonlinear factors contributing to the response behavior of TLP in regular sea state.

• The influence of Cd-C\textsubscript{m} is more in case of TLP\textsubscript{2} as compared with TLP\textsubscript{1}. This shows that the hydrodynamic coefficients also influence the plan dimensions of TLP and its site location (geometric properties) i.e. selection of a particular geometry suitable for the selected sea state. Therefore, it may become essential to estimate the range of the Morison coefficients based on the Re and (or) Kc even before the preliminary design stage of the TLP geometry.

• In all the hydrodynamic cases taken for the study, it is seen that the coupled responses in all activated degrees-of-freedom are nonlinear i.e. the variation in the response for same wave period (with varying wave height) is not proportional to the variation in the response with same wave height (with different wave period).

• For compliant structures like TLP, application of Morison equation without allowance for correctly estimated Cd and C\textsubscript{m} values, the response behavior would be significantly high with that of the expected real behavior.

• The influence of hydrodynamic coefficients in the response of all activated degrees-of-freedom is significant for the range of their variation selected throughout the water depth.

References


