Performance analysis and optimum criteria of an irreversible Braysson heat engine

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Abstract

An irreversible cycle model of a Braysson heat engine operating between two heat reservoirs is used to investigate the performance of the cycle affected by the finite-rate heat transfer between the working fluid and the heat reservoirs, heat leak loss between the heat reservoirs and irreversibility inside the cycle. The specific power output is maximized with respect to the cycle temperatures along with the isobaric temperature ratio. The specific power output is found to be a decreasing function of the internal irreversibility parameter and isobaric temperature ratio while there exist the optimal values of the state point temperatures at which the specific power output attains its maximum value for a typical set of operating parameters. Moreover, the maximum specific power output and other cycle parameters are calculated for different sets of operating conditions. The optimally operating regions of the important parameters in the cycle are determined. The results obtained here may provide some useful criteria for the optimal design and performance improvement of a realistic Braysson heat engine.

Keywords: Braysson heat engine; Multi-irreversibilities; Specific power output; Thermal efficiency; Optimally operating region; Optimum criterion

1. Introduction

The Braysson cycle is a hybrid power cycle based on a conventional Brayton cycle for the high temperature heat addition while adopting the Ericsson cycle for the low temperature heat rejection as proposed and investigated by Frost et al. [1] using the first law of thermodynamics. Very recently, some workers have investigated the performance of an endoreversible Braysson cycle [2-4] and a mirror cycle [5] based on the analysis of Brayton [6-23] and Ericsson [24-28] cycles by using the concept of finite time thermodynamics [29-38] for a typical set of operating conditions and obtained some significant results.

In real thermodynamic cycles, there often exist other irreversibilities besides finite-rate heat transfer between the working fluid and the heat reservoirs. For example, the heat leak loss between the heat reservoirs and internal dissipation of the working fluid are also two main sources of irreversibility. In the present paper, we will study the influence of multi-irreversibilities on the performance of a Braysson heat engine cycle.

2. An irreversible Braysson cycle

An irreversible Braysson cycle working between a source and a sink of infinite heat capacities is shown on the $T$-$S$ diagram of Fig. 1. The external irreversibilities are due to the finite temperature difference between the heat engine and the external reservoirs and the direct heat leak loss from the source to the sink while the internal irreversibilities are due to nonsentropic processes in the expander and compressor devices as well as due to other entropy generations within the cycle. The working fluid enters the compressor at state point 4 and is compressed up to state point $1S/1$ in an ideal/real compressor, and then it comes into contact with the heat source and is heated up to state point 2 at constant pressure. After that the working fluid enters the turbine at state point 2 and expands up to state point $3S/3$ in an ideal/real expander/turbine, it rejects the heat to the heat sink at constant temperature and enters the compressor at state point 4, thereby, completing the cycle. Thus we study
the 4-1-2-3-4 closed cycle of an irreversible Braysson heat engine coupled with the heat reservoirs of infinite heat capacities at temperature \(TH\) and \(TL\), respectively.

Assuming the working fluid as an ideal/prefect gas and the heat exchangers as counter flow configurations, the heat transfer to and from the heat engine following Newton's Law of heat transfer will be

\[
Q_H = \frac{U_H A_H (T_2 - T_1)}{\ln((Z_{ff} - Z_i)/(Z_{ff} - r^2))} \quad m c_P (T_2 - T_1) = U_L A_L (T_3 - T_L) \quad \text{(1)}
\]

and

\[
Q_L = U_L A_L (T_3 - T_L) \quad \text{(2)}
\]

where \(U_H\) and \(A_H\) \((J = H, L)\) are the overall heat transfer coefficients and areas on their respective heat exchangers, \(T_i\) \((i = 1, 2, 3)\) are the temperatures of the working fluid at state points 1, 2, and 3, and \(m\) and \(c_P\) are, respectively, the mass flow rate and specific heat of the working fluid.

According to Fig. 1, it is reasonable to consider some heat leak loss directly from the source to the sink, which may be expressed as [26,27,30]

\[
Q_0 = k_0 (T_H - T_L) \quad \text{(3)}
\]

where \(k_0\) is the heat leak coefficient. In addition, it is very significant to further consider the influence of irreversible adiabatic processes and other entropy generations within the cycle. Using the second law of thermodynamics for this cycle model, we have

\[
\eta = \frac{Q_H QL}{\eta_L QH + Q0} = \frac{U_H A_L (T_3 - T_L) / T_3}{m c_P R T_3 \ln(x) + \text{h} (T_H - T_L) u h_{max} A_L (T_3 - T_L) / T_3} \quad \text{(5)}
\]

where \(x = T_2 / T_1\) is the isobaric temperature ratio and \(R\) is the internal irreversibility parameter which is greater than unity for a real cycle and defined as

\[
R = \frac{S_3 - S_4}{S_2 - S_1} > \frac{U_H A_L (T_3 - T_L) / T_3}{m c_P R T_3 \ln(x) + \text{h} (T_H - T_L) u h_{max} A_L (T_3 - T_L) / T_3} \quad \text{(5)}
\]

3. The expressions of several parameters

Using Eqs. (1)-(3) and (5), we can derive the expressions of the specific power output and thermal efficiency, which are, respectively, given by

\[
P = \frac{QH - QL}{A} = U_H a (x - 1) T_1 - RT_d \ln x \quad \text{(6)}
\]

and

\[
\eta = \frac{QH QL}{QH + Q0} = \frac{[(x - 1) T_1 - RT_d \ln x]}{QH + Q0} \quad \text{(7)}
\]
where \( a = (U_h/U_c)Rn_b, b = \ln[(T_m - T)/(T_m - T_1)] \) and \( A = AH + AL \) is the total heat transfer area of the cycle. Using Eqs. (1)-(3) and (5), we can also derive the expressions of the ratios of the hot- and cold-side heat exchanger areas to the total heat exchanger area as

\[
A_H = \frac{aT_1(T_m - T_1)}{aT_1(T_m - T_1) + b}
\]

and

\[
A_L = \frac{aT_1(T_m - T_1)}{aT_1(T_m - T_1) + b}
\]

Using the above equations, one can analyze the optimal performance of a realistic Braysson heat engine.

### 4. Optimal performance characteristics

It is seen from Eq. (6) that the specific power output is a function of two variables (\( T_1, T_x \)) for given values of the isobaric temperature ratio \( x \) and other parameters. Using the extremal condition \( 3P/dT_x = 0 \) and Eq. (6), it can be proven that when the specific power output is in the optimal states, the temperature \( T_x \) should satisfy the following equation

\[
T_x = T_c + \frac{b + \sqrt{b(c-a) + ac}}{a + b}
\]

where \( c = (U_h/U_c)(x - 1)T_1/T_1 \). Using Eqs. (6) and (10), one can generate a three-dimensional graph, as shown in Fig. 2, where \( P^* = P/(ULTL) \) is the dimensionless specific power output and the parameters \( TH = 1200 \text{ K}, TL = 300 \text{ K}, R = 1.1 \) and \( UH/UL = 1 \) are chosen. It is seen from Fig. 2 that the specific power output first increases and then decreases as \( T_1 \) increases. It shows clearly that there is an optimal value of \( T_1 \) at which the specific power output attains its maximum value for a given set of operating parameters, as shown in Fig. 3, where the values of \( T_{in} \) and \( P^*_m \) are listed in Table 1, which also gives the optimal values of other parameters at the maximum specific power output. It is also seen from Figs. 2 and 3 that the specific power output is a monotonically decreasing function of \( x \) and \( R \). This point is easily expounded from the theory of thermodynamics. Because the larger the isobaric temperature ratio \( x \) and internal irreversibility parameter \( R \) are, the larger the total irreversibility in the cycle and consequently the smaller the specific power output.

In order to further reveal the performance characteristics of an irreversible Braysson cycle, Eqs. (6), (7) and (10) are used to plot the \( P^* - \eta \) curves, as shown in Fig. 4, where \( \eta_m \) and \( P^*_m \) are, respectively, the thermal efficiency at the maximum specific power output and the specific power output at the maximum thermal efficiency. The values of \( \eta_m \) are also listed in Table 1. It is seen from Fig. 4 that the specific power output is not a monotonic function of the thermal efficiency. When the specific power output is situated in the regions of the \( P^* - \eta \) curves with a positive slope, it will decrease as the thermal efficiency is decreased. Obviously, these regions are not optimal. It is thus clear that the specific power output should be situated in the region of

![Fig. 2. The variation of the dimensionless specific power output \( P^* \) with respect to temperature \( T_x \) and isobaric temperature ratio \( x \), where the parameters \( T_m = 1200 \text{ K}, T_1 = 300 \text{ K}, R = 1.1 \) and \( U_h/U_c = 1 \) are chosen.](image)

![Fig. 3. The variation of the dimensionless specific power output \( P^* \) with respect to temperature \( T_1 \). The values of the parameters \( TH, TL \) and \( U_H/U_L \) are the same as those used in Fig. 2.](image)

<table>
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<th>( x )</th>
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<th>( T_{in} )</th>
<th>( P^*_m )</th>
<th>( A_s/A_c )</th>
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the $P^* \sim \eta$ curves with a negative slope. When the specific power output is in the region, it will increase as the thermal efficiency is decreased, and vice versa. Thus, the optimal regions of the specific power output and thermal efficiency should be subject to

$$P_{m}^* \leq P^* \leq P_{\text{max}}^*$$  \hspace{1cm} (11)

and

$$\eta_{\text{max}} \geq \eta \geq \eta_m$$  \hspace{1cm} (12)

This shows that $P^*$, $\eta$, $P_{m}^*$ and $\eta_m$ are four important parameters of the Braysson heat engine. $P_{m}^*$ and $\eta_m$ give the upper bounds of the specific power output and corresponding thermal efficiency while $P^*$ and $\eta$ determine the allowable values of the lower bounds of the optimal specific power output and corresponding thermal efficiency.

According to Eqs. (11) and (12), we can further determine the optimal regions of other performance parameters. For example, the optimal regions of the parameters $T_1, T_\sigma, AH/A$ and $AL/A$ may be, respectively, determined by

$$T_{m} \leq T < T_{1 \eta}$$  \hspace{1cm} (13)

$$T_{3 m} > T_{\sigma} > T_{k0}$$  \hspace{1cm} (14)

$$(A/H)_m < (AH/A) < (AH/A)_H$$  \hspace{1cm} (15)

and

$$(A/L)_m > (A/L/H) \geq (A/L/A)_{\eta}$$  \hspace{1cm} (16)

where $T_{m}, T_{k0}, (AH/A)_m$ and $(AL/A)_m$ are, respectively, the values of the $T_1, T_\sigma, AH/A$ and $AL/A$ at the maximum specific power output and have been listed in Table 1, and $T_{1 \eta}, T_{k0}, (AH/A)_H$ and $(AL/A)_H$ are, respectively, the values of the $T_1, T_\sigma, AH/A$ and $AL/A$ at the maximum thermal efficiency and may be calculated numerically from the above equations.

So far we have obtained some optimum criteria for the important performance parameters of the cycle, which may provide some theoretical guidance for the design and improvement of the Braysson cycle.

5. Discussions

As mentioned above, some special cases of the cycle may be directly discussed from the results derived in this paper.

(1) When $R = 1$, the cycle becomes endoreversible. The characteristic curves of the Braysson heat engine are shown by the solid lines in Figs. 3 and 4.

(2) When $k0 = 0$, the heat leak loss is not considered. In such a case, Figs. 2 and 3 are still true, because the specific power output is not affected by the heat leak loss between the heat reservoirs. However, the heat leak loss between the heat reservoirs will directly reduce the thermal efficiency of the cycle. When $k0 = 0$, the $P^* \sim \eta$ curve of the cycle is not a closed loop line through the zero point but an approximate parabola line, as shown in Fig. 5.

(3) When $R = 1$ and $k0 = 0$, we get the cycle model adopted in Refs. [2,3]. It implies the fact that some significant results in Refs. [2,3] may be directly obtained from this paper. For example, the $P^* \sim \eta$ curve plotted in Ref. [3] may be given by the solid line in Fig. 5 as long as the same values of the parameters are chosen.

(4) When $x = 1$, the cycle becomes the Carnot cycle [31-33] and Eqs. (6)-(9) may be, respectively, simplified as

$$P = \frac{UH(1-RT/T)}{I/(T_s - T_f) + (UH/UL)R/(T_f/T_f - T_f)}$$  \hspace{1cm} (17)

$$\eta = \frac{1 - RT/T}{1 + \frac{UH(1-RT/T)}{T_s/T_f + \frac{UHRT_f}{T_s} + \frac{UHRT_f}{T_s + \frac{UHRT_f}{T_s}}}}$$  \hspace{1cm} (18)

$$AH = \frac{1}{2} \frac{RT_f/T_f - T_f}{U_f}$$  \hspace{1cm} (19)

and

$$AL \frac{RT_f/T_f - T_f}{U_f}$$  \hspace{1cm} (20)
It shows clearly that the optimal performance of the Carnot heat engine can be directly obtained from the results derived in this paper. For example, using Eqs. (17) and (18), one can generate the \( P^* \sim \eta \) characteristic curves of the cycle, as shown in Fig. 6.

(5) It is of interest to note that one may use not only the objective function [34] defined in Eq. (6) but also the objective function defined as [34]

\[
P = \frac{Q_H - Q_L}{U_A} = \frac{Q_H - Q_L}{U_A} \frac{(x-1)T_1 - RT_L nx}{a^* T/(T_0 - T_0)} + b
\]

(21)

to optimize the performance of a heat engine, where \( a^* = RHnx \). Comparing Eq. (21) with Eq. (6), one can directly derive the performance characteristics of an irreversible Braysson heat engine optimized by the objective function defined in Eq. (21) through Eqs. (7)-(20), as long as \( U_H \) and \( U_L \) are chosen to be equal to 1 and \( A, AH \) and \( AL \) are replaced by \( UA, UHAH \) and \( U_LA_L \) in Eqs. (7)-(20), respectively.

6. Conclusions

An irreversible cycle model of a Braysson heat engine is established and used to investigate the influence of multi-irreversibilities on the performance of the heat engine. The specific power output is optimized with respect to the state point temperatures for a given set of other performance parameters. The maximum specific power output and some other important parameters are calculated for a typical set of operating conditions. The optimal regions of some important parameters such as the specific power output, thermal efficiency, temperatures of the working fluid, heat-transfer area ratios, and so on, are determined in detail. The influence of the isobaric temperature ratio \( x \) and internal irreversibility parameter \( R \) on the performance of the cycle is analyzed. Some special cases of the cycle are discussed so that the optimal performance of the Carnot heat engine may be directly derived from the results obtained in this paper. In other words, the analysis presented in this paper is general and will be useful to understand the relationships and difference between the Braysson cycle and other cycles and to further improve the optimal design and operation of a real Braysson heat engine for different sets of operating parameters and constraints.

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References


