General performance characteristics of an irreversible vapour absorption refrigeration system using finite time thermodynamic approach

PramodKumarBhardwaj\textsuperscript{a}, S.C. Kaushik\textsuperscript{a,*}, Sanjeev Jain\textsuperscript{b}

\textsuperscript{a} Centre for Energy Studies, Indian Institute of Technology, Delhi, HauzKhas, New Delhi 110016, India
\textsuperscript{b} Department of Mechanical Engineering, Indian Institute of Technology, Delhi, HauzKhas, New Delhi 110016, India

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Abstract

This communication presents finite time thermodynamic analysis of vapour absorption refrigeration system considering both external as well as internal irreversibility. System is considered as consisting of an irreversible heat engine between the generator and absorber and an irreversible refrigerator between the evaporator and condenser. Optimization is made w.r.t. source/sink side heat exchanger areas, source/sink side heat transfer time and heat engine cycle and refrigeration cycle time. An optimal relation between overall coefficient of performance and maximum cooling load is obtained. Two internal irreversibility parameters (say, $R_{\text{is}}^g$ for generator-absorber assembly and $R_{\text{is}}^e$ for evaporator-condenser assembly) have been introduced in the analysis, having fractional value for irreversible (real) system while $R_{\text{AS}} = R_{\text{cs}} = 1.0$ corresponds to endoreversible system. A detailed parametric study shows that internal irreversibility parameters have more drastic effect on performance reduction than any other external irreversibility parameter. It is also found that internal irreversibility parameters are being dominant factor for performance reduction but out of these two parameters, internal irreversibility parameter of generator-absorber assembly is more sensitive for performance reduction than the internal irreversibility parameter of evaporator-condenser assembly.

Keywords: Finite time thermodynamics; Vapour absorption refrigeration system; External and internal irreversibility; Optimal area distribution; Optimal cycle time distribution

1. Introduction

According to classical thermodynamics, Carnot coefficient of performance is the highest performance parameter of any system having reversible processes while practical systems are irreversible and they cannot achieve Carnot COP. Thus, Carnot COP does not have practical utility. Therefore, it is necessary to establish general performance characteristics with irreversible (real) processes [1-5].

Vapour absorption refrigeration cycles have many advantages over conventional vapour compression refrigeration cycles, like these help to save the primary energy sources and can be driven by low grade energy, more importantly, help a lot to keep clean environment. But major problem with these systems is low performance than conventional one, hence there is a great need to optimize these systems [6-12,22,24].

Wijeysundera [19] has used an ideal three-heat-reservoir cycle with constant internal irreversibilities and external heat transfer irreversibilities to model the absorption refrigeration machine of a solar operated absorption cooling system. He obtained analytical expressions for the variation of the entropy transfer with storage tank temperature and the variation of the coefficient of performance (COP) with the cooling capacity of the plant. These expressions give the operating points for the maximum cooling capacity and the maximum COP. The effect of internal and external irreversibilities on the second-law efficiency of the plant is examined. Bautista et al. [21] have analyzed a generalized endoreversible refrigeration cycle characterized by three heat
sources with finite heat capacities. They have predicted optimal coefficient of performance and the relationship of that with a given rate of refrigeration. The physical influence of the involved finite heat capacities, together with the thermal resistances, on the performance of the refrigeration cycle has been studied. Chen [23] analyzed a two-stage combined refrigeration system taking into account the influence of multi-irreversibilities, such as finite rate heat transfer, heat leak between the heat reservoirs and internal dissipation of the working fluid, on the performance of the refrigeration system. The maximum coefficient of performance with non-zero specific cooling rate is calculated, and other corresponding performance parameters, such as the temperatures of the working fluid in the isothermal processes, the optimal distribution of the heat transfer areas and the power input of the refrigeration system, are determined. Chen [7], has analyzed the vapour-absorption refrigeration system by considering that system is endoreversible (i.e., externally irreversible and internally reversible) and found out a relation between optimal overall coefficient of performance and cooling load. But, he did not consider the effect of internal irreversibility and made optimization only w.r.t. working fluid temperatures and area of heat exchangers.

In this paper, a more general optimization of vapor-absorption refrigeration system is established and the optimal analytical bounds on source/sink side heat transfer time, heat engine cycle and refrigeration cycle time and source/sink side heat exchanger areas are determined. System is considered externally as well as internally irreversible. For studying the effect of internal irreversibility, two internal irreversibility parameters ($R_{AS}$ for generator-absorber assembly and $R'_{AS}$ for evaporator-condenser assembly) are introduced in the analysis, defined by second law of thermodynamics following Wu and Kiang [13], Goktun and Ozkaynak [14], Goktun [11], Bhardwaj [25]. The absorption-refrigeration system is considered as combined cycle, consisting of a heat engine and a refrigeration cycle. Actually temperature of working fluid on generator and condenser side will vary in sub-cooling and superheating regions but in order to simplify the analysis mean effective constant temperature of working fluid are introduced on generator and condenser side following Chen [7], Yan and Chen [9], Bhardwaj [20,25]. This assumption is made to simplify the analysis for studying the external irreversibility of finite heat transfer with surroundings. Applying the concept of finite time thermodynamics, a relation is developed between the overall coefficient of performance and cooling load and hence an optimal bound on overall coefficient of performance corresponding to maximum cooling load is found out. The optimal working regions of the coefficient of performance and cooling load are determined. A detailed parametric study has been carried out and effects of various input parameters have been studied. It is found that the effect of internal irreversibility parameters is more prominent on overall coefficient of performance reduction than the other external irreversibility parameters.

2. System description and analysis

Vapour absorption refrigeration system as shown in Fig. 1, has basically four components-generator, absorber, evaporator and condenser. Sub-system consisting of gener-
Heat Source \( T_G \) \[ Q_G \]

Generator \( T_a \)

Condenser \( T_c \)

Absorber \( T_a \)

Evaporator \( T_e \)

Space to be cooled \( T_E \)

Fig. 1. Schematic diagram of vapour absorption refrigeration system.

2.1. Heat engine cycle analysis

The input heat from heat source to the generator is given by

\[ Q_G = U_g A_g (T_G - T_g) t_g \]  
(1)

Similarly, the heat rejection from absorber to the heat sink

\[ Q_A = U_a A_a (T_a - T_A) t_a \]  
(2)

Work output of the generator-absorber assembly is given by

\[ W = U_g A_g (T_G - T_a) t_g - U_a A_a (T_a - T_A) t_a \]  
(3)

Second law requires,

\[ \Delta S = \frac{U_g A_g (T_G - T_g) U_a A_a (T_A - T_a)}{T} \]  
(4)

Where \( RAS \) is internal irreversibility parameter for heat engine cycle. \( RAS = 1 \) corresponds to endoreversible system and \( RAS < 1 \) is for irreversible (real) system. The total heat transfer area of heat engine cycle is \( Ah = A_g + A_a \).

In Section 2.1, under the given constraint of given heat input to the generator, an optimization is made w.r.t. source/sink side heat exchanger areas and source/sink side heat transfer time of generator-absorber assembly and an optimal thermal efficiency of heat engine cycle is found out while in Section 2.2, for a given refrigeration effect in the evaporator, an optimization is made w.r.t. source/sink side heat exchanger areas and source/sink side heat transfer time of the evaporator-condenser assembly and an optimal COP expression of refrigeration cycle is found out. Finally, in Section 2.3, the overall coefficient of performance, which is multiplication of the optimal thermal efficiency of generator-absorber assembly and COP of the evaporator-condenser assembly, is further maximized w.r.t. heat engine and refrigeration cycle time and a general optimal relation between overall coefficient of performance and cooling load is obtained.

2.1.1. Optimization w.r.t. source/sink side heat transfer area

\[ \frac{\partial \eta}{\partial A_g} = 0 \quad \text{gives} \quad \frac{A_g}{A_g} = 1 + \sqrt{\frac{U_g t_g}{U_g(RAS t_g)}} \]  
(8)

and

\[ \frac{\partial \eta}{\partial A_a} = 0 \quad \text{gives} \quad \frac{A_a}{A_a} = 1 + U_a U_g(RAS t_a) \]  
(9)

Using Eqs. (8) and (9), we have

\[ \frac{A_g}{A_g} = \sqrt{\frac{U_g RAS t_g}{U_g t_g}} \]  
(10)

Substituting the optimal values of \( A_g \) and \( A_a \) into Eq. (7), we have

\[ \eta = 1 - \frac{TA/RAS}{(T_G - \frac{Q_G}{A_g} \left( \frac{1}{U_g t_g} + \frac{1}{U_a RAS t_a} \right))^2} \]  
(11)

2.1.2. Optimization w.r.t. source/sink side heat transfer time

\[ \frac{\partial \eta}{\partial t_g} = 0 \quad \text{gives} \quad \frac{t_g}{t_g} = 1 + \left( \frac{U_g RAS}{U_g} \right)^{1/3} \]  
(12)

and

\[ \frac{\partial \eta}{\partial t_a} = 0 \quad \text{gives} \quad \frac{t_a}{t_a} = 1 + \left( \frac{U_a RAS}{U_g} \right)^{1/3} \]  
(13)
Using Eqs. (12), (13), we get
\[ \left( \frac{U_e R_{\Delta S}^3}{U} \right)^{1/3} = 14 \] (14)

Now, after substituting these optimal values of the cycle time into Eq. (11), the optimal efficiency of the heat engine is given by
\[ \eta = \frac{\frac{T_A}{R_{AS} T_G} \cdot \frac{T_A}{R_{AS} T_G}}{T_G - \frac{U_e A_e}{R_{AS} \eta}} \] (15)
where
\[ A_h = A_e + A_g, \quad h = h_e + h_g \]
and
\[ K_1 = \frac{U_e A_e R_{\Delta S}}{(U_g 1/3 + (U_g R_{\Delta S})^{1/3})} \]

2.2. Refrigeration cycle analysis

The heat extracted from the space to be cooled to the evaporator is given by
\[ Q_E = U_e A_e (T_e - T_c) \] (16)

Similarly, heat rejected from condenser to the heat sink
\[ Q_C = U_c A_c (T_c - T_c) \] (17)

Work input to the refrigeration cycle is given by
\[ W = U_c A_c (T_c - T_c) - U_e A_e (T_e - T_e) \] (18)

Second law requires,
\[ A_c (T_c - T_c) - A_e (T_e - T_e) = 0 \] (19)

Where \( R_{ss} \) is internal irreversibility parameter of refrigeration cycle. \( R_{ss} < 1 \) equal to sign holds for endoreversible system and less than sign holds for irreversible (real) system. Similar to Section 2.1, total heat transfer area and cycle time for refrigeration cycle are \( A_c + A_e \) and \( t = t_e + t_e \), respectively.

Under the constraint of given cooling effect, the working fluid temperatures may be obtained using Eqs. (16) and (19) and given by:
\[ T_e = T_E - \frac{Q_E}{U_c A_e} \] (20)

and
\[ T_c = T_C - \frac{Q_c}{U_c A_c} \] (21)

after substituting the value of \( T_e \) into Eq. (17), we have
\[ \text{COP} = \left[ \frac{T_c}{Q_c} \right]^{-1} - \left[ \frac{T_e}{Q_e} \right]^{-1} \] (22)

(i) Optimization w.r.t. source/sink side heat transfer area
\[ 9 \text{ COP} \left( \frac{dA_e}{3A_e} \right) = 0 \quad \text{gives} \quad A_e = A_c \] (23)

and
\[ 9 \text{ COP} \left( \frac{dA_c}{3A_c} \right) = 0 \quad \text{gives} \quad A_c = 1 + \frac{U_{c_1} R_{AS}}{U_{c_2}} \] (24)

using Eqs. (23) and (24), we have
\[ A_c = \frac{\frac{T_A}{R_{AS} T_G} \cdot \frac{T_A}{R_{AS} T_G}}{T_G - \frac{U_e A_e}{R_{AS} \eta}} \] (25)

On substituting the optimal values of \( A_c \) and \( A_e \) into Eq. (22), we have
\[ \text{COP} = \left[ \frac{T_c}{R_{AS}^3} \right]^{-1} \] (26)

(ii) Optimization w.r.t. source/sink side heat transfer time:
\[ 9 \text{ COP} \left( \frac{dt_e}{3t_e} \right) = 0 \quad \text{gives} \quad t_e = 1 + \frac{U_{c_1}}{U_e R_{AS}} \] (27)

and
\[ 9 \text{ COP} \left( \frac{dt_c}{3t_c} \right) = 0 \quad \text{gives} \quad t_c = 1 + \frac{U_c R_{AS}}{U_e} \] (28)

Using Eqs. (27), (28), we have
\[ t_e = \left( \frac{T_c}{U_{c_1} R_{AS}} \right)^{1/3} \] (29)

Substituting the optimal values of cycle time from Eqs. (27), (28) into Eq. (26), the optimal COP of an irreversible refrigeration system operating between reservoirs at temperatures \( T_e \) and \( T_c \) is given by
\[ \text{COP} = \left[ \frac{T_c}{R_{AS}^3} \right]^{-1} \] (30)

where
\[ A_c = A_c + A_g, \quad t_c = t_e + t_e \]

and
\[ K_2 = \frac{U_c U_e R_{AS}}{(U_c R_{AS}^3 + (U_c R_{AS})^{1/3})} \]

2.3. Optimization of overall coefficient of performance of the system

Overall coefficient of performance of the vapor-absorption refrigeration system is given by
\[ \varepsilon = \left[ 1 - \frac{\frac{T_A}{R_{AS} T_G} \cdot \frac{T_A}{R_{AS} T_G}}{T_G - \frac{U_e A_e}{R_{AS} \eta}} \right]^{-1} \] (31)

Our problem is how to choose \( t_1 \) and \( t_2 \) to maximize the overall coefficient of performance \( \varepsilon \) of the system. We take overall coefficient of performance \( \varepsilon \) as objective function and find its optimum subject to the condition of cooling effect \( Q_E \) and total cycle time \( \tau = t_1 + t_2 \) where, \( t_1 \) and \( t_2 \) are the heat engine and refrigeration cycle time, respectively) remains unvaried. For such type of constraint optimization
problems, Langrangian undetermined multiplier method is a suitable method, so we introduce the Lagrangian operator

\[ L = \varepsilon + \lambda (T_1 - T_2) \]  

(32)

Euler-Lagrangian equations

\[ \frac{dL}{dt_1} = 0 \quad \text{and} \quad \frac{dL}{dt_2} = 0 \]

give

\[ t_1 = \frac{1}{R_{AS}} \left[ \frac{T_A}{\sqrt{K_2 A_T}} \right] \]

\[ + R \left( \frac{1}{R_{AS}} \sqrt{K_A h} - \frac{1}{\sqrt{K_A h}} \right) \]

\[ \times \left[ \frac{1}{R_{AS}} \sqrt{K_A h} \varepsilon T_G + \frac{T_A}{R_{AS}} \right] \]

(33)

\[ t_2 = \frac{1}{R_{AS}} \left[ \frac{T_C}{\sqrt{K_2 A_T}} \right] \]

\[ - R \left( \frac{1}{R_{AS}} \sqrt{K_A h} - \frac{1}{\sqrt{K_A h}} \right) \]

\[ \times \left[ \frac{1}{R_{AS}} \sqrt{K_A h} \varepsilon T_G + \frac{T_A}{R_{AS}} \right] \]

(34)

where cooling load \( R = \frac{Q_E}{\varepsilon} \).

On substituting these optimal values of \( h \) and \( t_2 \) into Eq. (31), we have an optimal relation between overall coefficient of performance and cooling load

\[ \varepsilon = \left[ 1 - \frac{T_A}{R_{AS} T_G} - \frac{R}{\varepsilon T_G} \left( \frac{1}{K_2 A_T} \right) \right] + \frac{T_C}{R_{AS} T_E} - 1 \]

\[ + R \left( \frac{T_A}{\varepsilon T_G} \right) \left( \frac{1}{K_2 A_T} \right) \]

\[ \times \left( \frac{1}{R_{AS} \sqrt{K_2 A_T}} \right) \]

(35)

Rearranging and solving above Eq. (35) for the cooling load \( R \), we have

\[ R = \frac{K_A \varepsilon (e - e)}{T_G} \left( \frac{T_C}{R_{AS} T_E} - T_E \right) \]

\[ \times \left[ \frac{T_A}{R_{AS} T_E} \left( \frac{1}{K_2 A_T} \right) - \frac{T_C}{R_{AS} T_E} \left( \frac{1}{K_2 A_T} \right) \right] \]

(36)

where

\[ \varepsilon_R = \frac{T_E}{T_0} - \frac{T_C}{T_0} \]

\[ \frac{dR}{d\varepsilon} = 0 \]

\[ \varepsilon = \frac{T_E}{T_0} - \frac{T_C}{T_0} \]

\[ \left( -T_G + T_E \right) \left( T_0 / \left( 1 + e \right) \right) \]

\[ = K_A \frac{K_A h}{K_2 A_T} + \frac{T_A}{R_{AS} T_E} \left( \frac{1}{K_2 A_T} \right) \]

\[ \times \left[ \frac{T_G}{T_0} \left( 1 + e \right) \right] \]

(37)

Maximum cooling load at optimal overall coefficient of performance of the system is

\[ R_m = \frac{K_A \varepsilon (e - e)}{T_G} \left( \frac{T_C}{R_{AS} T_E} - T_E \right) \]

\[ \times \left[ \frac{T_A}{R_{AS} T_E} \left( \frac{1}{K_2 A_T} \right) - \frac{T_C}{R_{AS} T_E} \left( \frac{1}{K_2 A_T} \right) \right] \]

(38)

\( \varepsilon_m \) is optimal coefficient of performance corresponding to maximum cooling load \( (R_m) \). This is also the lowest bound of optimal coefficient of performance for the system as shown in Fig. 2.

Special cases

(i) Our analysis is more general and Rubin efficiency may be achieved as a special case of our analysis when \( U_g A_g = U_A \) and \( R_{ps} = 1.0 \)

then Eq. (7) of Section 2.1, becomes

\[ \eta = 1 - \frac{T_A}{T_G} \]

(39)

where

\[ \frac{\varepsilon_R}{\eta} = \frac{\varepsilon}{\eta} \]

which is well known Rubin’s efficiency of heat engine with given input heat condition. However, with given input heat condition, efficiency cannot be achieved in the form of Curzon-Ahlborn efficiency.
(ii) Similarly for refrigeration cycle analysis, well-known COP expression may be achieved with special case of our analysis when
\[ U/A, = U/A, = K (\text{say}) \quad \text{and} \quad R_{as} = 1.0 \]
then Eq. (22) of Section 2.2, becomes

\[
\text{COP} = \left[ \frac{\text{TC}}{T_E - \frac{\text{Q}_{\text{c}}}{K_r}} - 1 \right]^{-1} \tag{40}
\]
where
\[ \tau = \frac{\text{tetc}}{(\text{te} + \text{tc})} \]
which is well-known COP expression with a constraint of given cooling effect for refrigeration cycle and available in the literature.

However in our analysis, efficiency and COP are further optimized w.r.t. source/sink side heat exchanger area and source/sink side heat transfer time and then finally overall coefficient of performance (ε) is optimized w.r.t. the heat engine cycle time (t1) and refrigeration cycle time (t2). So, naturally our final expressions would be more general and different from both the Rubin as well as Curzon-Ahlborn expressions.

(iii) When cooling load (R) = 0 and \( R_{as} = R'_{as} = 1 \), then Eq. (35) becomes,
\[
\epsilon = \frac{1 - \frac{\text{TA}}{\text{TG}}}{\left( \frac{\text{TC}}{\text{TG}} - 1 \right)} \left( \frac{\text{TC} - \text{TE}}{\text{TC}} \right)
\]
which is nothing but Carnot COP of a combined cycle operated among the four thermal reservoirs and may be considered as consisting of heat engine cycle working between \( \text{TG} \) and \( \text{TA} \), and refrigeration cycle working between \( \text{TC} \) and \( \text{TE} \).

3. Results and discussion

It can be seen from Fig. 2 and Eq. (36) that for each value of cooling load there are two coefficient of performance parameters say \( \epsilon_I \) and \( \epsilon_n \) (graphically shown in Fig. 2). Out of these two, one (say \( \epsilon_n \)) is lower than the \( \epsilon_{\text{m}} \) and another is (say \( \epsilon_I \)) higher than \( \epsilon_{\text{m}} \). Obviously, higher one \( \epsilon_I \) is optimal value of overall coefficient of performance. The relation between overall coefficient of performance and cooling load for different values of internal irreversibility parameters \( R_{as} \) and \( R'_{as} \) is shown in Figs. 3 and 4. During the variation of any one input parameter, all other parameters are keeping constant as \( TE = 288 \text{ K}, \ TG = 393 \text{ K}, \ T_{1} = 313 \text{ K}, \ T_{2} = 318 \text{ K}, \ R_{as} = R'_{as} = 1.0, \ A/A = 0.5, \ U_{1} = U_{2} = U_{r} = 10.0 \text{ kW-K-1-m}^{-2} \).

For these typical input parameters, Eq. (41) gives Carnot COP of the system \( \epsilon = 2.20 \), Eq. (37) gives optimal overall coefficient of performance for endoreversible system (say, \( RAS = R'_{as} = 1.0 \)) \( \epsilon_{\text{m}} = 0.48 \) whereas using the analytical expression given by Chen [7], the optimal coefficient of performance of an endoreversible system for the same input parameters has been predicted as \( \epsilon_{\text{m}} = 0.49 \). However, for irreversible system (say, \( RAS = R'_{as} = 0.95 \)) it may be predicted using Eq. (37) and comes to be \( \epsilon_{\text{m}} = 0.32 \). Figs. 5-11 show the rate of change in \( \epsilon_{I} \) and \( \epsilon_{n} \) with input parameters (viz. \( \text{TE}, \ TG, \ \text{TA}, \ TC, \ R_{as}, \ R'_{as}, \ Abh/A \) and \( A/A \)). Figs. 5-8 show that both \( \epsilon_{I} \) and \( \epsilon_{n} \) increase rapidly with increasing \( \text{TE} \) than \( TG \) while these performance parameters decrease more rapidly with increasing \( \text{TA} \) than \( TC \). Figs. 9 and 10 show that both performance parameters (\( \epsilon_{I} \) and \( \epsilon_{n} \)) decrease in prominent way with increasing internal irreversibility (decreasing value of internal irreversibility parameters \( R_{as} \) and \( R'_{as} \)), than external irreversibility parameters. With increasing internal irreversibility (\( R_{as} \) and \( R'_{as} \) is going down from 1.0 to fraction value say 0.91) optimal overall coefficient of performance \( \epsilon_{I} \) decrease about 61% and 57% while decreasing rate of \( \epsilon_{n} \) is about 23% and 38%, with \( R_{as} \) and \( R'_{as} \), respectively. Obviously, internal irreversibility is dominant factor for overall optimal parameters.

Fig. 3. Cooling load (R) vs. overall coefficient of performance (\( \epsilon \)), for varying \( R_{as} \).

Fig. 4. Cooling load (R) vs. overall coefficient of performance (\( \epsilon \)), for varying \( R'_{as} \).
coefficient of performance reduction, but out of these two parameters ($R_{AS}$ and $R'_{AS}$), $R_{AS}$ is more sensitive for performance reduction than $R'_{AS}$.

Fig. 11 shows how distribution of heat exchanger areas affect the performance? Study shows that to keep high overall coefficient of performance, area distribution on the side of evaporator-condenser assembly should be more than that of distributed on generator-absorber side.

Fig. 12 shows the effect of cooling load on overall coefficient of performance. If cooling load is zero, expression of overall coefficient of performance given by Eq. (35) reduces to Carnot COP and with increasing cooling load, performance decreases and has lower value than Carnot COP.

4. Conclusions

Vapour absorption refrigeration system is considered as consists of an irreversible heat engine and an irreversible refrigerator. Optimization is made w.r.t. source/sink side heat exchanger areas, source/sink side heat transfer time and heat engine cycle and refrigeration cycle time. An analytical optimal relation between overall coefficient of performance and cooling load is found out. A detailed parametric study has been made and the effect of various input parameters is shown in the form of graphical Figs. 2-12. Parametric study shows that internal irreversibility has more prominent effect on overall coefficient of performance reduction than external irreversibility. It is also found that internal irreversibility parameter of generator-absorber assembly is more sensitive for overall coefficient of performance reduction than the internal...
irreversibility parameter of evaporator-condenser assembly. 
Eq. (37) provides an optimal overall coefficient of performance corresponding to maximum cooling load as well as lower limit of optimal bound on overall coefficient of performance.

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