On the sensitivity and signal to noise ratio of a step-index fiber optic surface plasmon resonance sensor with bimetallic layers

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Abstract

In this paper, a theoretical analysis of sensitivity and signal-to-noise ratio (SNR) of a step-index fiber optic surface plasmon resonance sensor with bimetallic layers (silver and gold) has been carried out. The numerical treatment is based on Kretschmann's SPR theory and Drude model of metals. The light source considered is a p-polarized one. The potential of different bimetallic ratios (gold as outer one) for the sensing purposes has been analyzed. The effect of different design parameters related to optical fiber as well as sensing layer on sensitivity and signal-to-noise ratio of the sensor has been studied. A comparison between selected ray launching and all guided rays launching in terms of sensitivity and SNR has been performed for different bimetallic ratios. The effect of fiber length and FWHM of a Gaussian input on the sensor's performance has also been studied. All these studies, leads to a significant analysis to achieve the best possible design of a fiber optic SPR sensor with maximum sensitivity and SNR simultaneously for both laboratory and remote sensing applications.

Keywords: Surface plasmon resonance; Optical fiber; Bimetallic layer; Sensitivity; Signal-to-noise ratio; Selected ray launching; All guided rays launching; Remote sensing

1. Introduction

Since last two decades, surface plasmon resonance (SPR) has become a powerful conceptual tool for chemical and biochemical sensing [1-6]. The SPR sensing principle is an optical phenomenon in which a p-polarized light beam satisfies the resonance condition and excites a charge density oscillation (known as surface plasmon wave, SPW) propagating along the metal-dielectric interface. The resonance condition explicitly depends on the incident angle, wavelength of the
light beam, and the dielectric functions of both the metal as well as the dielectric. The resonance appears in the form of a sharp dip of output optical signal either with incident angle (angular interrogation) or wavelength (spectral interrogation). Any change in refractive index near the interface causes a change in the value of resonance parameter (either angle or wavelength). Among several SPR sensing structures the Kretschmann configuration in which the metallic layer is deposited directly on the base of a prism [7-9] is the most popular. Usually, the metallic layer used in SPR measurements consists of either silver or gold. Gold demonstrates a higher shift of resonance parameter to change in refractive index of sensing layer and is chemically stable. Silver, on the other hand, displays a narrower resonance curve causing a higher signal-to-noise ratio (SNR) of SPR chemical sensor, but has a poor chemical stability. The oxidation of silver happens as soon as exposed to air and especially to water, which makes it difficult to get a reliable sensor for practical applications. Treatment of the silver surface by a thin and dense cover is therefore required. Recently, Zynio et al. [10] suggested a new structure of resonant metal film based on bimetallic layers (gold as outer one) on the prism base with angular interrogation sensing. They showed that the new structure displayed high shift of resonance angle as gold films, but also showed narrower resonance curve as silver film, thus providing a higher SNR in addition to protecting silver against oxidation.

In the present work, we have theoretically investigated the above structure with spectral interrogation in place of angular interrogation and have extended the investigation to optical fiber for the selected as well as all guided rays launching configuration. The optical fibers have certain advantages over the bulk prism, e.g., simple and flexible design, facility of miniaturization, multi-channel analysis, remote sensing and so forth [11-15]. The sensitivity and SNR of the optical fiber based bimetallic SPR sensor are evaluated numerically. The effects of fiber parameters like numerical aperture (NA), core diameter and sensing parameters like sensing region length and metallic film thickness have been studied for different ratios of silver and gold layer thickness. The analysis has been extended to remote sensing applications.

2. Theory

2.1. Background

The attenuated total reflection (ATR) method along with Kretschmann configuration is frequently used in SPR measurements. In the case of bimetallic prism-based SPR sensor, the above configuration comprises a high refractive index dielectric (coupling prism), a silver layer of thickness $d_1$, a gold layer of thickness $d_2$, and a low refractive index dielectric (sensing medium) (Fig. 1(a)). A light beam from a broadband source is passed through the coupling prism and, if the resonance condition meets that is when the incident wave-vector exactly matches the surface

![Incident light (TM polarized) Reflected light](image)

![Prism Gold layer of thickness $d_2$ Silver layer of thickness $d_1$ Sensing medium](image)

![Illustration of SPR curve with spectral interrogation](image)

Fig. 1. (a) Kretschmann configuration of a prism-metal sensing layer interface. The fused silica prism, the bimetallic film, and the sensing layer are labeled separately. $h$ is the angle of incidence. (b) Illustration of SPR curve with spectral interrogation.
plasmon wave-vector, the spectrum of output signal demonstrates a dip located at the resonance wavelength (Fig. 1(b)). This resonance wavelength is highly sensitive to refractive index variations of sensing medium. Hence, the refractive index of sensing layer index can be determined by measuring the resonance wavelength.

For the present study we consider the coupling prism and the core of the step-index fiber made of fused silica. The refractive index of fused silica varies with wavelength according to following dispersion relation [16]:

\[ n(X) = C_0 + C_1 k^2 + C_2 k^4 + \frac{C_3}{(\lambda^2 - \lambda_0^2)} + \frac{C_4}{(\lambda^2 - \lambda_1^2)}, \]  

(1)

where the coefficients \( l, C_0, C_1, C_2, C_3, C_4, \) and \( C_5 \) have certain numeric values and \( k \) denotes the wavelength (measured in \( \text{nm} \)). Eq. (1) is valid for the wavelength range lying between 0.5 and 1.6 \( \text{nm} \).

According to Drude formula, the dielectric function \( (\varepsilon_m) \) of any metal can be written as

\[ \varepsilon_m(\lambda) = \varepsilon_{\text{in}} + i \varepsilon_{\text{mil}} = 1 - \frac{k_p^2 k_c}{\lambda^2 (\lambda_c + i \lambda)}, \]  

(2)

where \( k_p \) denotes the plasma wavelength and \( k_c \) denotes the collision wavelength. For both gold and silver we use the full complex dielectric function in our calculations. To obtain the expression for intensity reflection coefficient \( (R) \) for p-polarized incident beam we have used the matrix method for a multilayer system [17].

### 2.2. Sensitivity and signal-to-noise ratio

In the SPR sensor based on wavelength interrogation, resonance wavelength \( (k_{res}) \) is determined corresponding to refractive index of the sensing layer \( (n_s) \). To obtain \( k_{res} \) wavelength of the light beam is varied and the corresponding intensity of the reflected light is measured. At \( k_{res} \), a sharp dip in reflected light intensity is observed as shown in Fig. 1(b). If the refractive index of the sensing layer is altered by \( d n_s \), the resonance wavelength shifts by \( d k_{res} \). There are two important parameters of SPR sensor. These are sensitivity and SNR. Both should be as high as possible for a sensor. The sensitivity \( (S_n) \) of a SPR sensor with spectral interrogation is defined as [3]

\[ S_n = \frac{d n_s}{d k_{res}}. \]  

(3)

To calculate the sensitivity \( k_{res} \) is plotted as a function of \( n_s \). The slope of this curve gives the sensitivity of the sensor.

As mentioned above, in the spectral interrogation based fiber optic SPR sensor, the wavelength of the light is varied and the corresponding transmitted power is measured. At \( k_{res} \) a sharp dip in transmitted power is observed. The value of \( k_{res} \) depends on the refractive index of the sensing medium. The accuracy of the detection of SPR wavelength \( (k_{res}) \) depends on the width of the response curve. Narrower the width higher is the detection accuracy. Therefore, similar to angular interrogation [10], if \( dk_{0.5} \) is the spectral width of the response curve corresponding to 50% reflectivity (Fig. 1(b)), the detection accuracy of the sensor can be assumed to be inversely proportional to \( dk_{0.5} \). The SNR of the SPR sensor utilizing spectral interrogation can, therefore, be written as

\[ \text{SNR} = \frac{d n_s}{d k_{res}}. \]  

(4)

### 2.3. Fiber optic SPR sensor with bimetallic layer

In the fiber optic SPR sensor, prism is replaced by the core of a multimode step-index optical fiber. The cladding around the central region of the fiber is removed and is coated with a silver layer. This silver layer is further covered with a thin gold layer, which is finally covered with the sensing layer (Fig. 2). The light from a polychromatic source is launched into one of the ends of the fiber and is detected at the other end. Depending on the launching condition, the light source can excite both meridional rays and skew rays in the fiber. Further, there is no fixed analytical expression for power distribution if the skew rays are also excited in the fiber. Therefore, to simplify the analysis we consider the meridional rays only. For the short fiber length, there can be two cases: (a)
selected ray launching, and (b) all guided rays launching. We consider them below to find the sensitivity and SNR of the sensor.

2.3.1. Selected ray launching

In this case, selected guided rays at a desired angle are launched into the fiber from a broadband source. For the experiment, selected ray launching may be achieved by using a tunable optical filtering device or an annular beam mask at the launch end of the fiber. The number of ray reflections, \( N_{\text{ref}} \), in the fiber sensing area is a function of the angle of the ray with the normal to the core-cladding interface \( (\theta) \), core diameter \( (D) \), and the length of the sensing region \( (L) \). This relationship is given by

\[
N_{\text{ref}} = \frac{L}{D \tan \theta}
\]

In order to determine the effective normalized transmitted power, \( P_{\text{trans}} \), the reflectance \( (R) \) for a single reflection is raised to the power of the number of reflections \( (N_{\text{ref}}) \) that specific launched angle undergoes with the sensor interface. Therefore, in the case of selected ray launching the normalized power transmitted is given by

\[
\text{trans} = (R)^{N_{\text{ref}}}
\]

It may be noted that, in the case of fiber, the minimum angle of the guided ray is equal to the critical angle of the fiber.

2.3.2. All guided rays launching

In this case, rays at all possible angles are launched simultaneously from a white light source (such as tungsten halogen lamp) after collimating with proper optics into a multimode optical fiber using a lens (microscope objective). The light beam is focused on the axis and at the input face of the fiber so that only meridional rays can be considered for the analysis. The NA of the lens is chosen larger than that of the fiber so that all the guided rays can be excited in the fiber. The power, \( dP \), arriving at the fiber-end face between the angles \( h_0 \) and \( h_0 + dh_0 \) is proportional to \( \left( \tan \frac{h_0 \cos^2 \theta}{h_0} \right) \), where \( h_0 \) is the angle of the ray from the axis for the ray outside the fiber. Using Snell’s law and \( 9 = 90° \), we can write [18]

\[
dP/n = \frac{n_2 \sin h \cos h}{(1-n_j \cos^2 \theta)^2} \sin \theta d\theta.
\]

Using Eqs. (6) and (7), the generalized expression for the normalized transmitted power, \( P_{\text{trans}} \), in the case of all guided rays can be written as

\[
P_{\text{trans}} = \frac{\int_{h_0}^{\pi/2} \int_{n_1}^{n_2} \sin \theta \cos \theta d\theta}{(1-n_j \cos^2 \theta)^2}.
\]

where

\[
\theta = \sin^{-1} \left( \frac{n_{\text{cl}}}{n_1} \right)
\]

is the critical angle of the fiber whereas \( n_l \) and \( n_1 \) are the refractive indices of the cladding and core, respectively. Here we have neglected the polarization effect of different launched rays because in the SPR sensors the sensitive area is generally far from the input end of the fiber [14].

2.4. Remote sensing

The cases discussed above are for non-remote sensing applications. In the case of remote sensing a large length of the fiber is used. In the case of a highly multimoded fiber, the attenuation coefficient varies from mode to mode causing a faster loss of higher order modes along the fiber length. Further, due to the increase in fiber length and mode number an increase in propagation delay is observed due to perpetual mode mixing. However, if the modes are assumed to form a continuum, the
dependence of power distribution on time, fiber length, and the continuous mode parameter can be described by a simple differential equation [19]. This assumption is very important in the context of remote sensing where the very long fibers are used and the power distribution of modes varies with length. The differential equation for length dependent power distribution with above assumptions comes out to be

$$\frac{\partial P}{\partial z} = -A \theta^2 P + \frac{D_0}{e} \frac{\partial}{\partial \theta} \left( \theta \frac{\partial P}{\partial \theta} \right).$$

where $P$ is the power that depends on both angle ($h$) of the mode as well as length ($z$) of the fiber. The term $Ah^2$ is the angle dependent attenuation loss caused at the core-cladding interface and parameter $D_0$ is related to the coefficient of coupling between the neighboring modes. The $z$-dependent solution can be obtained from the substitution

$$P = Q e^{-cz}$$

and the Eq. (10) takes the form

$$D_0 \frac{\partial}{\partial \theta} \left( \theta \frac{\partial Q}{\partial \theta} \right) = \left( Ah^2 - c \right) Q,$$

where $c$ is power attenuation constant related to the steady state solution $Q$. This equation is satisfied by the Laguerre-Gaussian polynomials with attenuation ($c$) associated with each solution increasing with the order of the polynomial (and, hence, of the mode). The solutions of the least loss have the form exp $f(z)$ with $T_e(z) = (\gamma)^{1/2}$ as steady state Gaussian width. The power loss associated with this distribution is

$$c_e = 2 \pi A D_0 T_e z^2.$$

Since the solutions of the least loss are Gaussian in nature, it is justifiable to assume the initial input launched at one of the ends of a multimode fiber as Gaussian, given by

$$P_0 = P_{0e} \exp \left( -\frac{\theta^2}{T_{0e}^2} \right).$$

For the same reason, it is logical to try the solution

$$P(0,z) = f(z) e^{\left( -\frac{\theta^2}{T_0^2} \right)}.$$
cates that with an increase in NA, the SPR curve gets broader and the resonance wavelengths become farther to one another. In addition, the sensitivity is more if the difference (resolution) between refractive indices is large. This is evident at large value of NA. The trend persists even for other ratios of silver to gold film thickness. These results suggest that the NA of an optical fiber must be selected carefully depending on which parameter (SNR or sensitivity) is more important for the desired application. If both parameters are important then that value of NA should be chosen which gives the maximum value of the product of these parameters. We have also studied the above effect for rays launched at angles greater than the critical angle. It was found that the SNR increases with a very small decrease in the sensitivity. This observation is useful when one uses a fiber with higher NA to get satisfactory values of both SNR and sensitivity simultaneously. To have the satisfactory values, angle of the ray should not be close to 90°. As the angle approaches 90° the depth of the SPR curve decreases.

Fig. 4 depicts the effect of fiber core diameter (D) on the SNR of the fiber optic SPR sensor for the NA of the fiber equal to 0.18. The values of other parameters used for calculation are the same. It may be seen from the figure that the SNR increases with the increase in the core diameter. If the core diameter is kept constant and the sensing length (L) is varied then a decrease in the SNR with the increase in sensing length is observed as shown in Fig. 5. Results in Figs. 4 and 5 have been plotted for selected ray launching at around critical angle of the fiber. These results can be understood in terms of broadening of SPR curve. According to Eq. (5), for the fixed angle of the ray, the number of ray reflections depends on the core diameter and the exposed sensing length. More the number of reflections, broader the SPR curve. Increase in core diameter decreases the number of reflections and hence sharpen the SPR curve thereby increases the SNR. In contrary, increase in exposed sensing length increases the number of reflections and hence increases the SNR. Therefore to have high SNR the number of ray reflections must be less which can be achieved with smaller sensing region and thicker core.
fiber. We have also studied the effect of core diameter and the length of the sensing region on the sensitivity of the SPR sensor. It has been found that both these parameters do not affect the sensitivity of the sensor utilizing selected rays launching.

Fig. 6(a) and (b) show the effect of total thickness of the bimetallic layer \(d\) on the SNR and the sensitivity of the sensor respectively. Both SNR as well as sensitivity increase with an increase in the total bimetallic thickness. But the variation in sensitivity is not as prominent as in SNR. The reason is the variation in interaction between surface plasmon mode (SPM) and fiber mode with a change in bimetallic thickness. Thicker the bimetallic layer, smaller the interaction between SPM and fiber mode. Small interaction causes less absorption of light power around resonance wavelength, and the SPR curve shifts upwards. The up-shift results in narrowing of SPR curve and hence increase in SNR. Since this up-shift does not change the resonance wavelength appreciably, the sensitivity does not change by a significant amount. We have also studied the effect of silver to gold thickness ratio on the SNR and sensitivity. The respective results are shown in Fig. 7(a) and (b). It may be noted from these figures that when there is no silver layer, the sensitivity is maximum and the SNR is minimum. As we increase the thickness of the silver layer (keeping the total bimetallic thickness constant) SNR rises whereas the sensitivity decreases. This is because the silver layer decreases the width of the SPR curve while the gold layer increases the separation between the resonance wavelengths for a given refractive index change of the sensing medium. This once again suggests that the bimetallic combination of silver and gold must be chosen carefully keeping the desired application in mind.

All the studies mentioned above have also been carried out for all guided rays launching in the fiber. For a given refractive index of the sensing layer, the transmitted intensity is calculated using Eq. (8) for different wavelengths of the light beam. The integral in Eq. (8) was evaluated numerically. From plot of transmitted light intensity versus wavelength curve the SPR wavelengths were determined for different values of the refractive indices.
of the sensing layer. The effects and trends of all concerned parameters were more or less similar to selected ray launching case (Figs. 3-7). In order to differentiate, we have compared the SNR and sensitivity of the selected and all guided rays cases in Fig. 8(a) and (b) keeping the values of all other parameters same. The results have been obtained for the refractive index separation of 0.015. The variation of SNR with percentage of silver is almost same for both kinds of launching but the values of SNR for all guided rays launching are almost 1.5 times of those corresponding to selected rays launching. Similarly in case of sensitivity, the variation with silver percentage is almost same but in terms of values, the selected ray launching keeps an edge over all guided rays launching case. The very same trend persists even for other values of parameters with different refractive index separations.

The sensor’s performance has also been evaluated taking very large length of the fiber (remote sensing case). Fig. 9(a) and (b), respectively, show the variation of SNR and sensitivity with the percentage of silver for the different values of steady state parameter $T_s$. The variation of both SNR as well as sensitivity is as same as that observed above. That is, the SNR is better for an excess of silver whereas sensitivity is large for excess of gold. But the most significant result is that how the SNR and sensitivity vary for fibers of different lengths. Fig. 10(a) and (b) are the corresponding figures for different values of initial FWHM (full width at half maximum) of the Gaussian input source. After a certain fiber length, both the SNR and sensitivity approach toward their corresponding steady state values. This is because the length-dependent power distribution profile saturates to
a steady state profile after a certain length of the fiber (from Eqs. (15)-(17)). However, the variations of SNR and sensitivity critically depend on FWHM (and hence on the ratio \( T_0/T_f \)). If the value of \( T_s/T_f \) is greater than 1, a drop in SNR and an increase in sensitivity is observed and the reverse in opposite case. This happens because a Gaussian input remains Gaussian, and its width monotonically transits to the steady state. This transition function is hyperbolic tangent if the input width \( (T_s) \) is smaller than the steady state width \( (T_f) \) and the hyperbolic cotangent in the opposite case [19].

Table 1 gives the values of SNR and sensitivity for four different cases with total bimetallic thickness of 50 nm and \( d_n = 0.015 \). The other parameters used are \( NA = 0.18, D = 600 \text{ lm}, L = 10 \text{ mm}, d = 50 \text{ nm} \), \( T_s = 0.50, T_f/T_s = 0.5 \), and fiber length \( (c_z) = 2 \). In all the four cases, the largest SNR and lowest sensitivity occur when only silver layer (1:0) is used while the smallest SNR and maximum sensitivity are obtained when prism base or fiber core are coated with gold only (0:1). The other bimetallic thickness combinations have SNR and sensitivity lying between these two cases. It may be noted that the prism-based SPR sensor demonstrates higher values of both SNR and sensitivity than fiber-based SPR sensor. But on the basis of the results presented above about the effect of various parameters on SNR and sensitivity, the better values of SNR as well as sensitivity can be obtained in the case of fiber based sensor by choosing appropriate values of the parameters. However, the advantages of optical fiber based SPR sensor over prism based SPR sensor should also be taken into account.

Fig. 9. Numerical simulations for the variation of (a) signal-to-noise ratio, and (b) sensitivity with percentage of silver in bimetallic layer for different values of steady state parameter \( T_s \). The other parameters used are \( D = 600 \text{ lm}, L = 10 \text{ mm}, d = 50 \text{ nm}, NA = 0.18, d_n = 0.015, \) fiber length = 2, and \( T_0/T_s = 0.5 \).

Fig. 10. Numerical simulations for the variation of (a) signal-to-noise ratio, and (b) sensitivity with fiber length for different values of FWHM of initial Gaussian input. The other parameters used are \( D = 600 \text{ lm}, L = 10 \text{ mm}, d = 50 \text{ nm}, NA = 0.18, d_n = 0.015, T_s = 0.75 \) and thickness ratio = 3:1.
Table 1
Comparison of SNR and sensitivity (Sn in lm per RIU) for four different cases with total bimetallic thickness (d) = 50 nm and dn = 0.015 RIU

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<th>Ratio Ag/Au</th>
<th>Prism case</th>
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<th>Fiber (all guided)</th>
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when the two sensors are being compared. One of the major advantages of fiber optic SPR sensor over prism based SPR sensor is remote sensing.

4. Conclusions

To conclude, the effects of various design parameters on the performance of a fiber optic SPR sensor with bimetallic layer, taking both selected ray as well as all guided rays launching have been analyzed numerically. The numerical results show that with NA, SNR decreases whereas sensitivity increases. This can be improved in the case of selected ray launching by taking launching angle greater than the critical angle. Further, the increase in the fiber core diameter increases the signal to noise ratio whereas the decrease in the exposed sensing length increases the SNR in both kinds of launching. However, the effect of core diameter and sensing length on the sensitivity of the sensor is negligible. In addition, the SNR and the sensitivity of the sensor are high for large bimetallic thickness (around 50 nm). For a fixed thickness of the bimetallic layer, greater extent of silver causes high SNR and low sensitivity. Further, the comparison between the selected ray and all guided rays launchings has been carried out. All guided ray launching is better when larger SNR is required whereas selected ray launching provides higher sensitivity. In the case of remote sensing applications (i.e. for very large length of the fiber), the sensitivity and SNR are not affected by the change in the fiber length irrespective of what the FWHM of the Gaussian input is.

In all, significant theoretical results have been presented for a bimetallic SPR sensor with different light distributions in multimode step index optical fibers, which is the most critical item in context of the unpredictable random variations in the environment. In this viewpoint, we believe that the proposed theory with meridional rays can be considered as a sufficient starting point for a more complete understanding of the sensor's performance along with its experimental realization.

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References