radius for \( ka = 1.0 \), may act to enhance the actual radar cross sections. Comparisons were made for \( ka = 0.5 \) bet.ween radar cross sections calculated using the computer program and using a low-frequency solution for disk scattering due to Eggimann [SI. Agreement was very good, being within 0.1 DB.

On the basis of the considerations presented here, it is reasonable to conclude that the computer program can be used to compute value of the far-zone backscattered fields for the ideal disk electromagnetic plane wave scattering problem for values of \( h/a \) as large as 5.0 for any values of the aspect angle 0 and may be used for values of \( ka \) as large as 8.0 for restricted values of the aspect angle. The correspondence between the values of the backscattered fields for the ideal disk scattering problem and the thin metallic disk scattering problem is very good for large values of \( Ira \), but may become poor for values of \( ka \) less than or equal to 1.0.

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**REFERENCES**


Nonlinear Reflection and Transmission of Electromagnetic Waves by Moving Plasmas

**Abstract**—Based on the covariance principle of Maxwell's equations and phase-invariance principle (derived from the special theory of relativity), expressions for nonlinear reflection and transmission of electromagnetic waves from weakly ionized plasmas moving perpendicular to the interface have been obtained. It is seen that the effect of nonlinearity is quite appreciable even when the electromagnetic waves are moderately strong.

The problem of the reflection and transmission of electromagnetic waves by a moving dispersive medium has recently been investigated by various authors [1]-[4]. Yeh [1] concluded that, when the plasma moves parallel to the interface, the reflected and transmitted fields for an incident E-plane wave are independent of the movement of the plasma medium. Furthermore, when the plasma medium moves perpendicular to the interface, the reflected and transmitted wave functions are functions of the velocity of the moving plasma medium. However, Yeh assumed the plasma to be cold. We know that when a real absorptive plasma is considered, the incidence of moderately strong electromagnetic waves renders the medium to be nonlinear. The purpose of this communication is to derive expressions for nonlinear reflection and transmission of electromagnetic waves for a weakly ionized plasma moving perpendicular to the interface; the variation of these coefficients with the velocity of the medium and other parameters has been studied.

For a weakly ionized plasma (when electron-neutral particle collisions dominate), the effective collision frequency is governed by c51

\[
\nu_{ef} = \nu \left\{ \frac{T_e}{T} \right\} \frac{T}{T} \quad \text{la}
\]

\[
\nu_{ef} = \nu \left( 1 + \frac{1}{2} \frac{T_e}{T} \right) \quad \text{lb}
\]

when the rise in temperature of electrons due to electromagnetic heating is not very large, i.e., \( T_e = T/\gamma << 1 \). Using the relevant expression for \( T_e = T/\gamma \), we get

\[
\nu_{ef} = \nu \left( 1 + \frac{1}{2} \frac{T_e}{T} \right) \quad \text{lc}
\]

where \( a = e^{211}e^{-z^2/2/\rho T} \) is the nonlinearity parameter. In these expressions, \( y_0 \) is the collision frequency of electrons and, with energetic particles of individual mass \( X \) in the absence of electric field, \( T_e \) is the electronic temperature whereas \( T \) is the gas temperature. \( e \) and \( m \) are the charge and mass of an electron, respectively. \( x \) is Boltzmann's constant, \( \gamma \) is the angular frequency of the incident wave, and \( E_0 \) is the normalizing electric field; * over any quantity indicates its complex conjugate.

Let, the plane \( z = 0 \) be the plasma free space interface, the region \( z > 0 \) corresponding to the free space and the region \( z < 0 \) to a plasma. The permittivity of the plasma medium is given by [5]

\[
\varepsilon = \varepsilon_0 \left( 1 + \frac{1}{2} \frac{T_e}{T} \right)
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\varepsilon = \varepsilon_0 \left( 1 + \frac{1}{2} \frac{T_e}{T} \right)
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\varepsilon = \varepsilon_0 \left( 1 + \frac{1}{2} \frac{T_e}{T} \right)
\]
$z, t'$ are related to the combination of $z$ and $t$ by Lorentz transformation and $a'$ is the frequency in the reference frame $S'$. Using the conditions for the continuity of electric and magnetic vectors at the interface, it can be readily shown that (7), (8)

\[
\begin{align*}
\frac{S_t}{S_0} &= \frac{2ft'}{(ft' + ft^* \gamma)} \quad (7a) \\
\frac{S_0}{S_t} &= \frac{8ai't'So' \gamma}{So' - (ft' + ft^*)^2 (ft' + ft^*) (3ft' - ft^*)} \quad (7b) \\
\frac{S_0}{S_t} &= \frac{8ai't'So' \gamma (2ft' - ft^* + ft^*) (ft^* - ft')(ft^* - 3ft')} {So' - (ft' + ft^*)(ft' + ft^*) (ft^* - ft')(ft^* - 3ft')} \quad (7c)
\end{align*}
\]

In the observer's system $S$, the reflected and transmitted waves take the form

\[
S_r = (S_0 + ax_0) \exp (-i\omega t z) \exp (-iu z) = A_n \exp (-i\gamma t z) \exp (-iu z)
\]

and

\[
\begin{align*}
S_r &= (S_0 + ax_0) \exp \left[ i\left( \frac{a_i}{a_k} - \frac{a_j}{a_k} \right) \right] \\
&= A_n \exp \left[ i\left( \frac{a_i}{a_k} - \frac{a_j}{a_k} \right) \right] (9a)
\end{align*}
\]

When the plasma is moving in the direction of the incident wave, i.e., in $z$ direction, making use of the covariance of Maxwell's equation and the phase invariance of a uniform plane wave, we have the following transformations (Q):

\[
\begin{align*}
\omega' &= \gamma \omega \left( 1 - \frac{v_0}{c^2} \right) \quad (10a) \\
\beta_1' &= \gamma \left( \beta_1 - \frac{\omega}{c^2} \right) \quad (10b) \\
\beta_2' &= \gamma \beta_2 \left( 1 - \frac{v_0}{c} \right) \quad (10c) \\
\omega' &= \gamma (\omega' - eft') \quad (11a) \\
\beta_3' &= -\gamma \left( -\beta_3' + \frac{\omega}{c^2} \right) \quad (11b) \\
\omega' &= \gamma (\omega' - 2y) \quad (He) \\
\beta_4' &= \gamma \left( \frac{\omega'}{\omega^2} - \beta_4' \right) \quad (Hd) \\
\end{align*}
\]

where $y = [1 - \beta_7/c^2]^{1/2}$. Reflection and transmission coefficients are, respectively, defined as

\[
\begin{align*}
R &= \frac{n \cdot S_r}{n \cdot S_i} \quad (12a) \\
T &= \frac{n \cdot S_t}{n \cdot S_i} \quad (12b)
\end{align*}
\]

with velocity of medium after propagating a normalized distance

$w/c = 1$ in reference frame $S$, inside plasma.
So, from (8b) and (gb), the nonlinear reflection and transmission coefficients for moving plasmas are given by [1]

\[ R_{ts} = \frac{A_{ts}^0 - A_{rs}^0}{A_{ts}^0 + A_{rs}^0} \left( \frac{p_{ts}^0}{p_{rs}^0} \right) \cos \theta, \tag{13a} \]

\[ T_{ts} = \frac{G_{ts} - G_{rs}^*}{G_{ts} + G_{rs}} \frac{S_0}{S_0}. \tag{13b} \]

The reflection and transmission of electromagnetic waves for a moving plasma as a function of \( \frac{n}{c} \) for various values of \( \frac{w_0}{n} \) and nonlinear parameter \( a \) are plotted in Figs. 1 and 2, respectively. As pointed out by Yeh [1] and Daly and Gruenberg [10], when the plasma medium moves in the \( z \) direction in the present configuration, the relation \( R_{ts, in} + T_{ts, in} \neq 1 \) in general. As \( \frac{w_0}{n} \) decreases, the plasma becomes more and more transparent, to the incident wave thereby causing a reduction of the reflection coefficient. (Fig. 1). Also it \( k \) seen that, the peak of the reflection coefficient shifts toward the higher velocity side and the nonlinearity tends to increase the reflection coefficient. From Fig. 2 we infer that for overdense plasma, there is not much difference between nonlinear and linear transmission of electromagnetic waves. When plasma medium is moving toward the incident wave (\( c/e \) negative), the transmitted energy is greater than the energy of the incident wave and the transmission coefficient increases monotonically for larger negative values of \( c/e \) [10]. For smaller values of \( \frac{w_0}{n} \), nonlinearity seems to behave notoriously; the nature of variation is demonstrated in Fig. 2 for \( \frac{w_0}{n} = 0.8 \). For positive values of \( e/c \), the nonlinear transmission coefficient shows an oscillatory behavior but for negative values of \( V/c \), the general tendency of nonlinearity is to reduce the transmission coefficient.

The present analysis gives quite satisfactory results for weakly ionized plasma with \( \frac{w_0}{n} \gg v(T) \) [SI]. This method can be easily extended to the case of strongly ionized plasma, following Kaw and Mat [SI] without changing the basic formulation. In conclusion, one observes that, the characteristics of the reflection and transmission of electromagnetic waves for a nonlinear medium, as a function of velocity of the medium are significantly different, from that of a linear dispersive medium.

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A. K. CRE4CR4AVAFITI
Dept. of Physics
Indian Institute of Tech.
Kex Delhi-29, India

### References


Nonlinear Dispersion of Electromagnetic Transient Signals Propagating Through a Partially Ionized Plasma

**Abstract**—The propagation of relatively high-power electromagnetic transient signals through a homogeneous, isotropic, and partially ionized plasma medium is discussed. In particular, the nonlinear dispersion of a transient RF signal caused by the dependence of the electron-neutral collision frequency on the electron temperature is studied. Valuable insights into the effects that plasma nonlinearities have on the distortion of a transient signal result from the numerical evaluation of the solution in integral form.

### I. INTRODUCTION

The distortion that transient electromagnetic signals suffer in propagating through plasma media has been of interest to many engineers and scientists for many years [1]-[31. Haskell and Case [1] have shown that the envelope distortion of transient signals propagating in a lossless and isotropic plasma increases as the carrier frequency approaches the electron-plasma frequency and as the distance between the transmitter and the receiver increases. They have more recently extended their results to the linear propagation of transient signals in lossy plasmas in which the electron-neutral collision frequency is constant [4]. Although they did not achieve an explicit solution, they were able to conclude some of the effects that collisions have on the signal envelope.

Nonlinear effects occur when relatively small and easily obtainable electromagnetic fields are applied to a plasma medium [5]-[7]. Energy transfer from the electromagnetic field to the electrons and neutrals [SI results in the gradual increase of the average electron energy and the change in the rate of electron-neutral collisions when an intense field is applied to the plasma. Ginzburg [6] has shown that this nonlinear effect is significant when the electric field strength is comparable to the plasma field given by

\[ E = \{\alpha - m/Tc(t + \psi) e^{i\omega t} - m/Tc e^{i\omega t}\}. \]

Energy losses by inelastic scattering leads to a loss in the plasma field so that nonlinear effects occur through the proem of elastic collisions between electrons and neutral particles. The amplitude and duration of the signals are assumed to be small enough, however, so that inelastic collisions and ionization are neglected.

### II. THEORY

The transient electric field is assumed to be transverse to its electric field polarized in the +X direction and propagate in the +Z direction. A general wave equation for the electric field intensity \( E \) is given by

\[ \frac{\partial^2 E}{\partial t^2} - \frac{\omega^2} {c^2} E = \frac{ai}{\omega} \frac{\partial E}{\partial r}. \]  

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