ANALYTICAL STUDY OF COLLECTOR SOLAR-GAIN ENHANCEMENT BY MULTIPLE REFLECTORS

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Abstract—We have considered the general case of a collector with four reflectors. An analytical model is developed for study of the effect of an individual reflector on the collector. Reflectors R-1, R-2, R-3, and R-4 face south, north, west, and east, respectively. For a collector with two reflectors R-1 and R-2 on the southern and northern sides, up to 44% enhancement is achieved in the solar gain for December (winter) and up to 14% for May (summer). However, due to shadowing, two additional reflectors R-3 and R-4 on the eastern and western sides cause negligible additional solar gain. Without shadowing, enhancement in solar gain due to reflectors R-3 and R-4 is 14–36% for May and 44–56% for December for the Delhi climate.

INTRODUCTION

A flat-plate collector is the simplest means for converting solar energy into useful heat. A flat-plate collector collects both direct and diffuse components of incident solar radiation. If the collector surface is normal to the incoming solar radiation, the heat gain is a maximum. In practice, it is not always possible to tilt the collector surface towards the sun. In such cases, higher concentrations of solar radiation on the collector are achieved by the addition of one or more reflectors surrounding the collector. A reflector does not only increase the collection area but also redirects the solar radiation closer to normal incidence.

There is published literature on collector–reflector combinations. Tabor¹ studied the effect of a specular reflector on the collector by using a graphical method. McDaniel et al² studied the effect of a planar reflector on the performance of a collector. Seital³ studied the effect of both diffuse and specular reflectors on the collector. Most of the published work deals with one reflector.^{4–7} Garg and Hrishikeshan⁸ and Kaushik et al⁹ considered two reflectors, one facing the southern side and the other the northern side of the collector, and optimized the tilts of these reflectors for maximum solar gain.

We consider a collector with four reflectors, one on each side of the collector. An analytical model is developed to evaluate the optimum reflectors tilt for maximum output from the collector. When the collector faces south and has a 0° tilt with respect to the horizontal, it is found that multiple reflectors can improve collector performance significantly.

ANALYSIS

The configuration of the collector–reflector system is shown in Fig. 1. The reflectors R-1, R-2, R-3, and R-4 are located at the four sides of the collector and are connected edge-to-edge to the collector. The tilts of the collector and reflectors are variable and the whole system has an off-south angle ϕ . Assumptions made in the analysis are: (i) the reflectors are specular with reflectivity $\rho_r = 0.8$. (ii) The collector surface is a blackened flat plate. (iii) The transmittance–absorptance products are functions of the angle of incidence. (iv) Shading by the collector and of reflectors on each other is neglected but shading of reflectors on the collector is considered. (v) Multiple reflection between reflectors is neglected. (vi) Addition of reflectors between the collector and sky does not affect the shape factor. (vii) Each month of the year is represented by an average day. Values of the constants used are given in Table 1. (viii) The analysis takes into account only absorption of solar radiation by the blackened plate.

The total energy absorbed by the collector is

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Fig. 1. A schematic representation of the collector-reflector assembly.

Table 1. Values of A , B , and C for the representative day of each month.

Date	A (W/m^2)	B	C
17 Jan.	1230	0.1420	0.0580
16 Feb.	1217	0.1434	0.0590
16 Mar.	1191	0.1520	0.0690
15 Apr.	1146	0.1750	0.0920
15 May	1110	0.1930	0.1160
11 June	1093	0.2020	0.1300
17 July	1086	0.2066	0.1356
16 Aug.	1102	0.2020	0.1200
15 Sep.	1143	0.1816	0.0980
15 Oct.	1184	0.1630	0.0760
14 Nov.	1214	0.1510	0.0650
10 Dec.	1229	0.1440	0.0590

$$Q = (1 - S_{rc}) \{ (\alpha\tau)_{bc} I_{bc} + (\alpha\tau)_{dc} I_{dc} + \rho_r [(\alpha\tau)_{rc1} I_{br1} f_{rc1} \cos \theta_{rc1} + (\alpha\tau)_{rc2} I_{br2} f_{rc2} \cos \theta_{rc2} + (\alpha\tau)_{rc3} I_{br3} f_{rc3} \cos \theta_{rc3} + (\alpha\tau)_{rc4} I_{br4} f_{rc4} \cos \theta_{rc4}] \}, \quad (1)$$

$$S_{rc} = S_{rc1} + S_{rc2} + S_{rc3} + S_{rc4}, \quad (2)$$

where the first term in the curly bracket of Eq. (1) stands for direct radiation incident on the collector, the second term is the diffuse component, and the third term is energy received by the collector from the reflectors. The first term within the square bracket is the energy from reflector R-1 absorbed by the collector. Similarly, the 2nd, 3rd, and 4th terms give the reflected energy on the collector due to reflectors R-2, R-3, and R-4, respectively. S_{rc1} , S_{rc2} , S_{rc3} , and S_{rc4} are shading factors on the collector due to reflectors R-1, R-2, R-3, and R-4, respectively.

The following relations¹⁰ are used to evaluate the beam and diffuse components of radiation on the collector and the beam radiation on the reflectors:

$$I_{bc} = I_{DN} \cos \theta_c, I_{dc} = CI_{DN} f_{cs}, I_{br} = I_{DN} \cos \theta_r, \quad (3)$$

$$I_{DN} = A \exp(-B/\sin \beta); \quad (4)$$

A , B and C are constants (see Table 1). The shape factor between the collector and the sky is

$$f_{cs} = (1 + \cos s_c)/2. \quad (5)$$

If ϕ is the off-south angle of the system, the unit vectors normal to the collector surface and reflectors are, respectively,

$$\begin{aligned} \bar{n}_c &= \sin s_c \cos \phi \bar{i} + \sin s_c \sin \phi \bar{j} + \cos s_c \bar{k}, \bar{n}_{r1} = \sin s_{r1} \cos \phi \bar{i} + \sin s_{r1} \sin \phi \bar{j} + \cos s_{r1} \bar{k}, \\ \bar{n}_{r2} &= -\sin s_{r2} \cos \phi \bar{i} - \sin s_{r2} \sin \phi \bar{j} + \cos s_{r2} \bar{k}, \bar{n}_{r3} = \sin s_{r3} \sin \phi \bar{i} - \sin s_{r3} \cos \phi \bar{j} + \cos s_{r3} \bar{k}, \\ \bar{n}_{r4} &= -\sin s_{r4} \sin \phi \bar{i} + \sin s_{r4} \cos \phi \bar{j} + \cos s_{r4} \bar{k}. \end{aligned} \quad (6)$$

The unit vector in the direction of the incoming beam radiation is

$$\bar{n}_b = -\cos \beta \cos \gamma \bar{i} - \cos \beta \cos \gamma \bar{j} - \sin \beta \bar{k}, \quad (7)$$

where

$$\sin \beta = \cos L \cos \delta \cos H + \sin \delta \sin L, \sin \gamma = \cos \delta \sin H / \cos \beta; \quad (8)$$

δ is the declination angle and is given by $23.45 \sin[360(284 + N)/365]$.

The angles of incidence on the collector and reflectors are

$$\begin{aligned} \cos \theta_c &= \sin s_c \cos \beta \cos(\gamma - \phi) + \cos s_c \sin \beta, \cos \theta_{r1} = \sin s_{r1} \cos \beta \cos(\gamma - \phi) + \cos s_{r1} \sin \beta, \\ \cos \theta_{r2} &= -\sin s_{r2} \cos \beta \cos(\gamma - \phi) + \cos s_{r2} \sin \beta, \cos \theta_{r3} = -\sin s_{r3} \cos \beta \sin(\gamma - \phi) + \cos s_{r3} \sin \beta \\ \cos \theta_{r4} &= \sin s_{r4} \cos \beta \sin(\gamma - \phi) + \cos s_{r4} \sin \beta. \end{aligned} \quad (9)$$

The unit vectors in the directions of the reflected rays from the reflectors are, respectively, $\bar{n}_{rc1}$, $\bar{n}_{rc2}$, $\bar{n}_{rc3}$, and $\bar{n}_{rc4}$ and may be expressed as

$$\bar{n}_{rc1} = u_1 \bar{i} + v_1 \bar{j} + w_1 \bar{k}, \bar{n}_{rc2} = u_2 \bar{i} + v_2 \bar{j} + w_2 \bar{k}, \bar{n}_{rc3} = u_3 \bar{i} + v_3 \bar{j} + w_3 \bar{k}, \bar{n}_{rc4} = u_4 \bar{i} + v_4 \bar{j} + w_4 \bar{k}, \quad (10)$$

where

$$\begin{aligned} u_1 &= -\cos \beta \cos \gamma + 2 \cos \theta_{r1} \sin s_{r1} \cos \phi, v_1 = -\cos \beta \sin \gamma + 2 \cos \theta_{r1} \sin s_{r1} \sin \phi, \\ w_1 &= -\sin \beta + 2 \cos \theta_{r1} \cos s_{r1}, \end{aligned} \quad (11)$$

$$\begin{aligned} u_2 &= -\cos \beta \cos \gamma - 2 \cos \theta_{r2} \sin s_{r2} \cos \phi, v_2 = -\cos \beta \sin \gamma - 2 \cos \theta_{r2} \sin s_{r2} \sin \phi, \\ w_2 &= -\sin \beta + 2 \cos \theta_{r2} \cos s_{r2} \end{aligned} \quad (12)$$

$$\begin{aligned} u_3 &= -\cos\beta\cos\gamma + 2\cos\theta_{r3}\sin s_{r3}\sin\phi, \quad v_3 = -\cos\beta\sin\gamma - 2\cos\theta_{r3}\sin s_{r3}\cos\phi, \\ w_3 &= -\sin\beta + 2\cos\theta_{r3}\cos s_{r3}, \end{aligned} \quad (13)$$

$$\begin{aligned} u_4 &= -\cos\beta\cos\gamma - 2\cos\theta_{r4}\sin s_{r4}\sin\phi, \quad v_4 = -\cos\beta\sin\gamma + 2\cos\theta_{r4}\sin s_{r4}\cos\phi, \\ w_4 &= -\sin\beta + 2\cos\theta_{r4}\cos s_{r4}. \end{aligned} \quad (14)$$

The expressions for the angles of incidence on the collector for the reflected rays are, respectively,

$$\begin{aligned} \cos\theta_{re1} &= -u_1\sin s_c\cos\phi - v_1\sin s_c\sin\phi - w_1\cos s_c, \\ \cos\theta_{re2} &= -u_2\sin s_c\cos\phi - v_2\sin s_c\sin\phi - w_2\cos s_c, \\ \cos\theta_{re3} &= -u_3\sin s_c\cos\phi - v_3\sin s_c\sin\phi - w_3\cos s_c, \\ \cos\theta_{re4} &= -u_4\sin s_c\cos\phi - v_4\sin s_c\sin\phi - w_4\cos s_c. \end{aligned} \quad (15)$$

EVALUATION OF THE EXCHANGE AND SHADING FACTORS

The fraction of the total collector area illuminated by reflected radiation from a reflector is the exchange factor (f_{re}). It is a function of collector and reflector tilts, the relative dimensions of the collector-reflector assembly, and the position of the sun in the sky. As shown in Fig. 2, if $\bar{e}_1, \bar{e}_2, \bar{e}_3$, and $\bar{e}_4$ are the position vectors of points on the incident solar rays after reflection, then

$$\bar{e}_1 = e_{x1}\bar{i} + e_{y1}\bar{j} + e_{z1}\bar{k}, \quad \bar{e}_2 = e_{x2}\bar{i} + e_{y2}\bar{j} + e_{z2}\bar{k}, \quad \bar{e}_3 = e_{x3}\bar{i} + e_{y3}\bar{j} + e_{z3}\bar{k}, \quad \bar{e}_4 = e_{x4}\bar{i} + e_{y4}\bar{j} + e_{z4}\bar{k}, \quad (16)$$

where

$$\begin{aligned} e_{x1} &= -(a/2)\sin\phi - b_{r1}\cos s_{r1}\cos\phi + C_{e1}u_1, \quad e_{y1} = -(a/2)\cos\phi - b_{r1}\cos s_{r1}\sin\phi + C_{e1}v_1, \\ e_{z1} &= b_{r1}\sin s_{r1} + C_{e1}w_1, \end{aligned} \quad (17)$$

$$\begin{aligned} e_{x2} &= -(a/2)\sin\phi - b_{r2}\cos s_{r2}\cos\phi + C_{e2}u_2, \quad e_{y2} = -(a/2)\cos\phi - b_{r2}\cos s_{r2}\sin\phi + C_{e2}v_2, \\ e_{z2} &= b_{r2}\sin s_{r2} + C_{e2}w_2, \end{aligned} \quad (18)$$

$$\begin{aligned} e_{x3} &= -(b_c/2)\cos\phi - b_{r3}\cos s_{r3}\sin\phi + C_{e3}u_3, \quad e_{y3} = -(b_c/2)\sin\phi - b_{r3}\cos s_{r3}\cos\phi + C_{e3}v_3, \\ e_{z3} &= b_{r3}\sin s_{r3} + C_{e3}w_3, \end{aligned} \quad (19)$$

$$\begin{aligned} e_{x4} &= -(b_c/2)\cos\phi - b_{r4}\cos s_{r4}\sin\phi + C_{e4}u_4, \quad e_{y4} = -(b_c/2)\sin\phi - b_{r4}\cos s_{r4}\cos\phi + C_{e4}v_4, \\ e_{z4} &= b_{r4}\sin s_{r4} + C_{e4}w_4. \end{aligned} \quad (20)$$

Here,

$$\begin{aligned} C_{e1} &= b_{r1}\sin(s_{r1} - s_c)/\cos\theta_{re1}, \\ C_{e2} &= b_{r2}\sin(s_{r2} + s_c)/\cos\theta_{re2}, \\ C_{e3} &= b_{r3}\sin s_{r3}/\cos\theta_{re3}, \\ C_{e4} &= b_{r4}\sin s_{r4}/\cos\theta_{re4}. \end{aligned} \quad (21)$$

Expressions for the exchange factors with respect to different regions of the reflectors are given in Tables 2–5. Shading of reflectors for radiation to the collector is calculated similarly. Here, instead of taking reflected rays from points E_1 to E_4 , we consider the incoming beam of solar radiation as passing from these points and falling on the collector. The position vectors of points for which the incoming beams are incident on the collector are

Fig. 2. Schematic representation of the reflected rays from the reflectors and falling on the collector.

$$\begin{aligned}\bar{e}'_1 &= e'_{x1}\bar{i} + e'_{y1}\bar{j} + e'_{z1}\bar{k}, \bar{e}'_2 = e'_{x2}\bar{i} + e'_{y2}\bar{j} + e'_{z2}\bar{k}, \bar{e}'_3 = e'_{x3}\bar{i} + e'_{y3}\bar{j} + e'_{z3}\bar{k}, \\ \bar{e}'_4 &= e'_{x4}\bar{i} + e'_{y4}\bar{j} + e'_{z4}\bar{k},\end{aligned}\quad (22)$$

where

$$\begin{aligned}e'_{x1} &= -(a/2)\sin\phi - b_{r1}\cos s_{r1}\cos\phi - C'_{e1}\cos\beta\cos\gamma, e'_{y1} = -(a/2)\cos\phi - b_{r1}\cos s_{r1}\sin\phi - C'_{e1}\cos\beta\sin\gamma, \\ e'_{z1} &= b_{r1}\sin s_{r1} - C'_{e1}\sin\beta,\end{aligned}\quad (23)$$

$$\begin{aligned}e'_{x2} &= -(a/2)\sin\phi - b_{r2}\cos s_{r2}\cos\phi - C'_{e2}\cos\beta\cos\gamma, e'_{y2} = -(a/2)\cos\phi - b_{r2}\cos s_{r2}\sin\phi - C'_{e2}\cos\beta\sin\gamma, \\ e'_{z2} &= b_{r2}\sin s_{r2} - C'_{e2}\sin\beta,\end{aligned}\quad (24)$$

$$\begin{aligned}e'_{x3} &= -(b_c/2)\cos\phi - b_{r3}\cos s_{r3}\sin\phi - C'_{e3}\cos\beta\cos\gamma, \\ e'_{y3} &= -(b_c/2)\sin\phi - b_{r3}\cos s_{r3}\cos\phi - C'_{e3}\cos\beta\sin\gamma, \\ e'_{z3} &= b_{r3}\sin s_{r3} - C'_{e3}\sin\beta, \\ e'_{x4} &= -(b_c/2)\cos\phi - b_{r4}\cos s_{r4}\sin\phi - C'_{e4}\cos\beta\cos\gamma,\end{aligned}\quad (25)$$

Table 2. Values of the exchange factors with specified boundary conditions between the reflector R-1 and the collector for six different regions (defined by the boundary conditions).

Region	Boundary conditions	Exchange factor (f_{rc1})
1	$R_1 \geq 0, S_1 < -0.5$ $(R_1 + S_1 W_1) < 0.5 W_1$	$0.5 R_1 / W_1 (0.5 - S_1)$
2	$R_1 > W_1, S_1 \leq 0.5$ $(R_1 + S_1 W_1) \geq 0.5 W_1$	$0.5 [2 - (0.5 - S_1) W_1 / R_1]$
3	$-0.5 \leq S_1 \leq 0.5$ $0 \leq R_1 \leq W_1$	$0.5 (1.5 + S_1) R_1 / W_1$
4	$S_1 > 0.5, R_1 \geq W_1$ $(R_1 - S_1 W_1) > -0.5 W_1$	$0.5 [2 - (S_1 - 0.5) W_1 / R_1]$
5	$0.5 < S_1 \leq 1.5$ $0 \leq R_1 < W_1$	$0.5 R_1 (2.5 - S_1) / W_1$
6	$S_1 > 0.5, R_1 \geq 0$ $(R_1 - S_1 W_1) < -0.5 W_1$	$0.5 R_1 / W_1 (S_1 - 0.5)$

$$S_1 = -(e_{x1}/a) \sin \phi + (e_{y1}/a) \cos \phi, R_1 = (e_{x1}/a) \cos \phi + (e_{y1}/a) \sin \phi, S'_1 = -(e'_{x1}/a) \sin \phi + (e'_{y1}/a) \cos \phi, R'_1 = (e'_{x1}/a) \cos \phi + (e'_{y1}/a) \sin \phi, W_1 = (b_c/a) \cos s_c.$$

Table 3. Values of the exchange factors with specified boundary conditions between the reflector R-2 and the collector for six different regions (defined by the boundary conditions).

Region	Boundary conditions	Exchange factor (f_{rc2})
1	$R_2 \leq 0, S_2 < -0.5$ $(R_2 + S_2 W_2) > 0.5 W_2$	$0.5 R_2 / W_2 (0.5 - S_2)$
2	$R_2 < W_2, S_2 \leq 0.5$ $(R_2 + S_2 W_2) < 0.5 W_2$	$0.5 [2 - (0.5 - S_2) W_2 / R_2]$
3	$-0.5 \leq S_2 \leq 0.5$ $0 \geq R_2 > W_2$	$0.5 (1.5 + S_2) R_2 / W_2$
4	$S_2 > 0.5, R_2 \leq W_2$ $(R_2 - S_2 W_2) < -0.5 W_2$	$0.5 [2 - (S_2 - 0.5) W_2 / R_2]$
5	$0.5 < S_2 < 1.5$ $0 \geq R_2 > W_2$	$0.5 R_2 (2.5 - S_2) / W_2$
6	$S_2 > 0.5, R_2 \leq 0$ $(R_2 - S_2 W_2) > -0.5 W_2$	$0.5 R_2 / W_2 (S_2 - 0.5)$

$$S_2 = -(e_{x2}/a) \sin \phi + (e_{y2}/a) \cos \phi, R_1 = (e_{x2}/a) \cos \phi - (e_{y2}/a) \sin \phi, S'_2 = (e'_{x2}/a) \sin \phi + (e'_{y2}/a) \cos \phi, R'_2 = (e'_{x2}/a) \cos \phi - (e'_{y2}/a) \sin \phi, W_2 = (b_c/a) \cos s_c.$$

$$e'_{y4} = -(b_c/2) \sin \phi - b_{r4} \cos s_{r4} \cos \phi - C_{e4} \cos \beta \sin \gamma, \quad (26)$$

$$e'_{z4} = b_{r4} \sin s_{r4} - C_{e4} \sin \beta.$$

Here,

$$\begin{aligned} C_{e1} &= b_{r1} \sin(s_{r1} - s_c) / \cos \theta_c, \\ C_{e2} &= b_{r2} \sin(s_{r2} + s_c) / \cos \theta_c, \\ C_{e3} &= b_{r3} \sin s_{r3} / \cos \theta_c, \\ C_{e4} &= b_{r4} \sin s_{r4} / \cos \theta_c. \end{aligned} \quad (27)$$

Table 4. Values of the exchange factors with specified boundary conditions between the reflector R-3 and the collector for six different regions (defined by the boundary conditions).

Region	Boundary conditions	Exchange factor (f_{rc3})
1	$R_3 \leq 0, S_3 < -0.5$ $(R_3 + S_3 W_3) > 0.5 W_3$	$0.5 R_3 / W_3 (0.5 - S_3)$
2	$R_3 \leq W_3, S_3 \leq 0.5$ $(R_3 + S_3 W_3) \leq 0.5 W_3$	$0.5 [2 - (0.5 - S_3) W_3 / R_3]$
3	$-0.5 \leq S_3 < 0.5$ $0 \geq R_3 > W_3$	$0.5 (1.5 + S_3) R_3 / W_3$
4	$S_3 > 0.5, R_3 \leq W_3$ $(R_3 - S_3 W_3) < -0.5 W_3$	$0.5 [2 - (S_3 - 0.5) W_3 / R_3]$
5	$0.5 < S_3 \leq 1.5$ $0 > R_3 > W_3$	$0.5 R_3 (2.5 - S_3) / W_3$
6	$S_3 > 0.5, R_3 \leq 0$ $(R_3 - S_3 W_3) > -0.5 W_3$	$0.5 R_3 / W_3 (S_3 - 0.5)$

$$S_3 = -(e_{x3}/a) \cos \phi - (e_{y3}/a) \sin \phi, R_3 = -(e_{x3}/a) \sin \phi - (e_{y3}/a) \cos \phi, S'_3 = (e'_{x3}/a) \cos \phi - (e'_{y3}/a) \sin \phi, R'_3 = -(e'_{x3}/a) \sin \phi - (e'_{y3}/a) \cos \phi, W_3 = -1.$$

Table 5. Values of the exchange factors with specified boundary conditions between the reflector R-4 and the collector for six different regions (defined by the boundary conditions).

Region	Boundary conditions	Exchange factor (f_{rc4})
1	$R_4 \geq 0, S_4 < -0.5$ $(R_4 + S_4 W_4) < 0.5 W_4$	$0.5 R_4 / W_4 (0.5 - S_4)$
2	$R_4 > W_4, S_4 \leq 0.5$ $(R_4 + S_4 W_4) > 0.5 W_4$	$0.5 [2 - (0.5 - S_4) W_4 / R_4]$
3	$-0.5 \leq S_4 \leq 0.5$ $0 \leq R_4 \leq W_4$	$0.5 (1.5 + S_4) R_4 / W_4$
4	$S_4 > 0.5, R_4 \geq W_4$ $(R_4 - S_4 W_4) > -0.5 W_4$	$0.5 [2 - (S_4 - 0.5) W_4 / R_4]$
5	$0.5 < S_4 \leq 1.5$ $0 \leq R_4 < W_4$	$0.5 R_4 (2.5 - S_4) / W_4$
6	$S_4 > 0.5, R_4 \geq 0$ $(R_4 - S_4 W_4) < -0.5 W_4$	$0.5 R_4 / W_4 (S_4 - 0.5)$

$$S_4 = -(e_{x4}/a) \sin \phi + (e_{y4}/a) \cos \phi, R_4 = (e_{x4}/a) \cos \phi + (e_{y4}/a) \sin \phi, S'_4 = -(e'_{x4}/a) \sin \phi + (e'_{y4}/a) \cos \phi, R'_4 = (e'_{x4}/a) \cos \phi + (e'_{y4}/a) \sin \phi, W_4 = -1.$$

The transmittance-absorptance product ($\tau\alpha$) is

$$(\tau\alpha) = 1.03 - 0.11 / \cos \theta_c \text{ for } \cos \theta_c \geq 0.175, (\tau\alpha) = 3.694 - 2.353 \theta_c \text{ for } \cos \theta_c < 0.175. \quad (28)$$

RESULTS AND DISCUSSION

Numerical calculations have been carried out for May and December in a typical year for Delhi. The collector tilt with respect to the horizontal is zero degree ($s_c = 0^\circ$) and the system faces due south ($\phi = 0^\circ$). Numerical values for other parameters are $a = 1.0$ m, $b_c = b_{r1} = b_{r2} = b_{r3} = b_{r4} = 1.0$ m. The values of s_{r1} , s_{r2} , s_{r3} and s_{r4} are selected for maximum solar gain. The analysis has three steps.

Step 1—The tilts of reflectors R-3 and R-4 (s_{r3} and s_{r4}) with respect to the horizontal are 0° . The total

Fig. 3. Variation of the total energy absorbed by the collector with s_{r2} for different values of s_{r1} for December; $s_c = 0^\circ$, $s_{r3} = 0^\circ$, $s_{r4} = 0^\circ$.

energy (Q) absorbed by the collector during sunshine hours is plotted in Fig. 3 as functions of s_{r1} and s_{r2} for December. The optimum tilts of reflectors R-1 and R-2 with respect to the horizontal are 100° and 20° , respectively. Figure 4 is used to evaluate optimum tilts for reflectors R-1 and R-2 for May; the optimum values of s_{r1} and s_{r2} for maximum solar gain on the collector for May are both 60° . *Step II*—The tilts of reflectors R-1 and R-2 (s_{r1} and s_{r2}) remain fixed at their optimum values for December and May as found in Step I. The tilts of reflectors R-3 and R-4 (s_{r3} and s_{r4}) are varied to make the solar gain on the collector a maximum. Figures 5 and 6 show the effects of reflectors R-3

Fig. 4. Variation of the total energy absorbed by the collector with s_{r2} for different values of s_{r1} for May; $s_c = 0^\circ$, $s_{r3} = 0^\circ$, $s_{r4} = 0^\circ$.

Fig. 5. Variation of the total energy absorbed by the collector with s_{r3} for different values of s_{r4} for December; $s_c = 0^\circ$, $s_{r1} = 100^\circ$, $s_{r2} = 20^\circ$.

and R-4 on the solar gain of the collector for December and May, respectively. It is evident that the increase in the solar gain of the collector caused by reflectors R-3 and R-4 is negligible. This result is due to the fact that when the solar gain from R-3 increases, that from R-4 decreases because of shadowing and conversely.

Step III—The values of s_{r1} and s_{r2} are again fixed at their optimum values while s_{r3} and s_{r4} are varied as in Step II. Shadowing of reflectors R-3 and R-4 is avoided if, before noon, $s_{r3} = 0^\circ$ and, for the afternoon, $s_{r4} = 0^\circ$. The increases in the solar gain for these conditions due to reflectors R-3 and R-4

Fig. 6. Variation of the total energy absorbed by the collector with s_{r3} for different values of s_{r4} for May; $s_c = 0^\circ$, $s_{r3} = 60^\circ$, $s_{r4} = 60^\circ$.

for December and May are plotted in Figs. 7 and 8. It is seen that reflectors R-3 and R-4 are more effective in May than in December. The increase in the solar gain reaches 36% in May and 56% in December. Table 6 shows comparisons of the percentages of solar gains for the three steps taken.

Fig. 7. Variation of the total energy absorbed by the collector with s_{r3} for different values of s_{r4} for December; $s_c = 0^\circ$, $s_{r1} = 100^\circ$, $s_{r2} = 20^\circ$; before 12 noon, $s_{r3} = 0^\circ$ and after 12 noon, $s_{r4} = 0^\circ$.

Fig. 8. Variation of the total energy absorbed by the collector with s_{r3} for different values of s_{r4} for May; $s_c = 0^\circ$, $s_{r3} = 60^\circ$, $s_{r4} = 60^\circ$; before 12 noon, $s_{r3} = 0^\circ$ and after 12 noon, $s_{r4} = 0^\circ$.

Table 6. Percentage increments of the solar gain on the collector for selected conditions.

Step	Month	$s_{r1}(^\circ)$	$s_{r2}(^\circ)$	$s_{r3}(^\circ)$	$s_{r4}(^\circ)$	Q_0	Q_m	Solar gain (%)
I	Dec	100	20	0	0	3200	4591	44
	May	60	60	0	0	6836	7761	14
II	Dec	100	20	0	0	3200	4591	44
	May	60	60	30	30	6707	7673	15
III	Dec	100	20	100	100	3200	5007	56
	May	60	60	90	90	6836	9247	36

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NOMENCLATURE

a = Common length of the collector and reflectors R-1 and R-2	H = Hour angle
A, B, C = Solar constants given in Table 1	I = Incident solar intensity (W/m^2)
b_c = Common length of the collector and reflectors R-3 and R-4	$\bar{i}, \bar{j}, \bar{k}$ = Unit vectors along the x, y, and z axes, respectively
$b_{r1}, b_{r2}, b_{r3}, b_{r4}$ = Widths of reflectors R-1, R-2, R-3, and R-4, respectively	L = Latitude of the location
$C_{e1}, C_{e2}, C_{e3}, C_{e4}$ = Scalar magnitudes of the vectors $\bar{E}_1\bar{F}_1, \bar{E}_2\bar{F}_2, \bar{E}_3\bar{F}_3$, and $\bar{E}_4\bar{F}_4$, respectively	N = Day of the year
$\bar{e}_1, \bar{e}_2, \bar{e}_3, \bar{e}_4$ = Unit vectors given in Eq. 16.	$\bar{n}_b$ = Unit vector in the direction of beam radiation
e_{x1}, e_{y1} = Horizontal components of the vector $\bar{e}_1$	$\bar{n}_c$ = Unit vector normal to the collector surface
e_{x2}, e_{y2} = Horizontal components of the vector $\bar{e}_2$	$\bar{n}_{r1}, \bar{n}_{r2}, \bar{n}_{r3}, \bar{n}_{r4}$ = Unit vectors normal to reflectors R-1, R-2, R-3, and R-4, respectively
e_{x3}, e_{y3} = Horizontal components of the vector $\bar{e}_3$	$\bar{n}_{rc1}, \bar{n}_{rc2}, \bar{n}_{rc3}, \bar{n}_{rc4}$ = Unit vectors in the direction of the reflected beams from reflectors R-1, R-2, R-3, and R-4, respectively
e_{x4}, e_{y4} = Horizontal components of the vector $\bar{e}_4$	Q = Total energy absorbed by the collector per unit collector area (W/m^2)
f_{cs} = Collector–sky shape factor	Q_0, Q_m = Total energy absorbed by the collector during sunshine hours without reflectors and with reflectors (W/m^2), respectively
f_{rc} = Exchange factor between the collector and reflector	

$S_{rc1}, S_{rc2}, S_{rc3}, S_{rc4}$ = Shading factors on collectors due to reflectors R-1, R-2, R-3, and R-4, respectively

$s_c, s_{r1}, s_{r2}, s_{r3}, s_{r4}$ = Tilts of the collector and reflectors R-1, R-2, R-3, and R-4 with respect to the horizontal, respectively

u_1, v_1, w_1 = Components of the unit vector $\bar{n}_{rc1}$

u_2, v_2, w_2 = Components of the unit vector $\bar{n}_{rc1}$

u_3, v_3, w_3 = Components of the unit vector $\bar{n}_{rc1}$

u_4, v_4, w_4 = Components of the unit vector $\bar{n}_{rc1}$

Greek letters

β = Solar altitude angle

γ = Solar azimuth angle

δ = Solar declination

ϕ = Wall azimuth angle of the collector and reflectors measured east of south

ρ = Reflectivity of the reflectors

$\theta_c, \theta_{r1}, \theta_{r2}, \theta_{r3}, \theta_{r4}$ = Angles of incidence of beam radiation on the collector and reflectors R-1, R-2, R-3, and R-4, respectively

$\theta_{rc1}, \theta_{rc2}, \theta_{rc3}, \theta_{rc4}$ = Angles of incidence for the reflected beam from reflectors R-1, R-2, R-3 and R-4 and falling on the collector, respectively

$\tau\alpha$ = Transmittance-absorptance product

Subscripts

bc = Beam radiation on the collector

br = Beam radiation on the reflectors

c = Collector

cs = Collector-sky

DN = Normal components of direct radiation

dc = Diffuse radiation on the collector

r1, r2, r3, r4 = Stand for reflectors R-1, R-2, R-3, and R-4, respectively

rc1, rc2, rc3, rc4 = Rays reflected from reflectors R-1, R-2, R-3, and R-4 and falling on the collector, respectively