we tentatively accept the hypothesis that the order is \( m \). However, \( l_0(m + 1) \leq 5 \) is not a sufficient condition since \( E[r(m + 1)] = 0 \) may not necessarily imply that the process is of \( m \)-th order moving average. The process may be mixed type, for which \( E[r(m + 1)] \) happens to be zero. To avoid such a mistake, further tests are necessary. This can be done by forming the joint distribution of \( r(m + 1), r(m + a), \ldots, y(m + k) \). Let

\[
\begin{align*}
|f(r(m + 1), r(m + a), \ldots, y(m + k) | & = 1 \quad \sqrt{2\pi | \Psi \gamma \Psi^T |} \exp \left( - \frac{1}{2} \Psi \gamma \Psi^T \right) \\
& \text{where the elements of} \\
\Psi^T \gamma \Psi^T & = \gamma^T (m + 1) \gamma (m + 2) \ldots (m + k).
\end{align*}
\]

The necessary and sufficient condition that the order is \( m \) is that, for all \( k \), \( E[y(k)] \) must be equal to zero. The probability that \( y \) lies inside the hyperellipsoid \( \gamma^T (m + 1) \gamma (m + 2) \ldots (m + k) \leq 0 \).

\[
st_{n-1} \text{ is the spherical symmetric volume element in a } k \text{-dimensional space. Find } Z \text{ so that the probability is } 0.95 \text{; then we accept, the null hypothesis if the sample of } y \text{ satisfies } \gamma^T (m + 1) \gamma (m + 2) \ldots (m + k) \leq 0.
\]

For example, a first order moving-average process is

\[
y(t) = Bo(l) + Pdt - 1.
\]

Sow, to test whether \( E[y(2)] \) and \( E[y(3)] \) are both equal to zero, we form the \( W \) matrix

\[
\psi_1 = r_{22} = \frac{1}{N} \left[ r_2(0) \right] \quad \psi_2 = r_{22} = \frac{1}{N} \left[ r_2(1) \right]
\]

(15) \( \psi_2 = r_{22} = \frac{1}{N} \left[ 12_{ij}(0,1) \right] \).

(16) The probability that \( y(2) \) and \( y(3) \) both lie inside the two-dimensional hyperellipsoid \( \gamma^T y \gamma = p \) can be calculated by

\[
\int_0 \exp \left( - \frac{1}{2} \gamma \Psi \gamma \Psi^T \right) \text{dr} = 1 - \exp \left( - \frac{1}{2} \gamma \Psi \gamma \Psi^T \right)
\]

This probability will be 95 percent if \( V_\gamma = 6.00 \). Therefore, we accept the hypothesis that the order is 1 when \( y^T \gamma \gamma \gamma = 0 \) is 6.00. Note that \( W \) is of the order \( 1/N \). Hence, increasing \( N \) will decrease the dimension of the hyperellipsoid.

111. CONCLUSION

Once the order \( m \) is determined, the initial estimation of the unknowns \( \gamma \) can be determined from (10). Further refinement of the estimators can be made if we use the constraints that

\[
E[r(m + 1)] = 0, \quad \text{for } i \geq 1.
\]

(18)


The importance of determining the order prior to the estimation of the parameters \( B \) lies in minimizing the risk of estimating the parameters of a model that is not fitted to the data. The efficient and consistent estimation of the parameters can only be obtained if the model we use is consistent with the process. The method of testing the hypotheses discussed in this note can be used to test whether the moving-average model we proposed for identification is consistent with the unknown system.

Optimal Fixed Lag Smoothing for Time Delayed System with Colored Noise

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Abstract—The aim of this note is to report an algorithm for the fixed lag smoothing problem of a time delayed system whose observations contain colored noise. This is derived by looking at the appropriate components of a filtering algorithm for a nontime delayed higher dimensional system.

Priemer and Vacroux [1] have recently reported an algorithm for the fixed lag smoothing estimate of a time delayed system containing white plant and observation noises. The relevant equations have been obtained from their earlier results [2] for the filtering version of the same problem using the method of orthogonal projections. It has been pointed out by Farooq and Mahalanabis [3] that a more convenient approach for deriving the filtering equations in [2] is to convert the time delayed problem into a higher dimensional nontime delayed one through the technique of state augmentation. The aim of the present note is to suggest an extension of the augmentation approach in [a] to derive relations for the fixed lag smoothing estimate of a time delayed system containing white plant and observation noises. The relevant equations have been obtained from their earlier results [2] for the filtering version of the same problem using the method of orthogonal projections.

Consider a time delayed system containing \( L \) delays

\[
x(k + 1) = \sum_{i=0}^{L} A_i x(k + i) + w(k)
\]

(2) whose observations are modeled by

\[
y(k) = H(k)x(k) + r(k)
\]

where \( x \) is the \( n \)-vector state and \( y \) is the \( m \)-vector output. The \( m \)-vector observation noise \( r(k) \) is assumed to be a colored sequence. The follo-ng propagation model contains

\[
r(k + 1) = B(k + 1)x(k) + v(k + 1)
\]

(3) The noises \( w(k) \) and \( v(k) \) are assumed to be zero mean, white, Gaussian sequences with covariances specified as follows:

\[
E[w(k)w'(k)] = \delta(k)\delta_l
\]

\[
E[v(k)v'(k)] = \delta(k)\delta_l
\]

\[
E[w(k)v'(k)] = 0, \quad \text{for all } l, k \geq 0.
\]

REFERENCES


ACCHOTMENT

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TECHNICAL NOTES AND CORRESPONDENCE
The problem considered here is to obtain unbiased and minimum variance estimates \( S(k) \) of the state vector \( X(k) \) in the presence of noise. The corresponding error covariances, where \( S \) represents the fixed lag between the points of estimation and observation [5]. A new state vector \( X(k) \) is introduced which has the partitioned form

\[
S(k) = [z'(k)jz'(k - 1); \ldots; z'(k - L - 1)]
\]

(4)

From (1), the \( n(L + X - 1) \)-dimensional vector \( S(k) \) can be seen to follow the propagation model

\[
S(k + 1) = C(k + 1)S(k) + D(k + l)w(k)
\]

(5)

where the matrices \( C(k) \) and \( D(k) \) are defined as follows:

\[
C(k) = \begin{bmatrix}
I & \pi/\pi(L+N) & \pi/\pi \end{bmatrix} \text{O} \pi(L+N) & \pi/\pi(L+N)
\end{bmatrix}
\]

\[
D(k) = \begin{bmatrix}
\pi/\pi \n(L+N)
\end{bmatrix}
\]

where

\[
F(k) = [A(j,k)A(k-j,k) \ldots x(-j,0)x \ldots x(0,0)]
\]

I and 0 are, respectively, identity and null matrices having indicated dimensions. The observation equation (2) is also modified to

\[
y(k) = G(k)S(k) + r(k)
\]

(6)

where

\[
G(k) = [H(k) \; \pi/\pi(L+N)]
\]

The system represented by (5) and (6), with the observation, noise \( r(k) \) given by (3), is in a form for which filtering solutions have been reported in the literature [5]-[8]. From a computational point of view, the algorithm in [8] appears attractive and is utilized in the sequel to obtain the filtered estimate of the vector \( S(k) \). Components of the estimate and covariance equations for the augmented system correspond to the desired smoothed estimate and covariance equations of the original system. These are obtained as

\[
i(k - j/k) = 2(k - j/k - 1) + K(j,k)F(k)H(k) - H(k)2(k/k - 1)
\]

- \( B(k)Y(k) - H(k - 1)f(k - 1,k - 1) \)

\[
\lambda(p = \lambda)
\]

\[
2(k/k - 1) = Y_{ij}(k)f(k - p - 1,k - 1)
\]

(7)

\[
K(j,k) = [P(k - j,k/k - 1)H(k) - P(k - j,k - 1,k - 1)] - \cdot H(k - 1)B(k)K(j,k)
\]

(8)

where

\[
V(k) = [P(k) + H(k)P(k,k - 1)H(k) - C(k) - CT(k)
\]

- \( B(k)H(k) - lP(k - 1,k - 1) - 1B(k)k(k - 1)]

(9)

and

\[
iT(k) = H(k)P(k,k - 1,k - 1)H(k) - lB(k)
\]

(10)

Further

\[
P(k - j,k - j/k) = P(k - j,k - j,k - 1) - K(i,k)H(k)
\]

- \( B(k)H(k - 1)\pi/\pi(k - j,k - 1,k - 1) \)

(12)

\[
P(k,k/k) - 1) = \sum_{i=0}^{L} P(k - l,k - j/k - 1) - \pi/\pi(0,k - l)A(k)
\]

(13)

Equations (7)-(14) form a recursive chain starting with the initial conditions \( f(0,j/l) = P(0,j/l) = 0 \). For \( i = 0,1, \ldots, L \)

Since \( y(0,i) \) is nonexistent, \( f(0,j/l) \), and \( f(0,0) \) are modified at \( i = 0 \) to read:

\[
f(0,j,l) = f(0,j,l) + KG,O)\pi/\pi(O,k - 0,1) - 1)
\]

(7a)

\[
K(j,O) = P(0,j,l)H(O)\pi/\pi(O,0,1) + R(O)I
\]

(9a)

\[
P(0,j,l) - P(0,j,l - 1) - K(0,j,O)\pi/\pi(0,j/l) I
\]

(12a)

for \( i = 0,1, \ldots, L \).

With these computed data, (7)-(14) have to be processed for \( k > 0 \) and for \( j = 0,1, \ldots, L ; i = 1,2, \ldots, L ; j = 1,2, \ldots, L \).

Continuous time versions of the above equations can be obtained through a formal limiting procedure [5]. It is also possible to obtain the fixed point and fixed interval smoothing algorithms for the problem considered in this note by suitably redefining the vector \( X@ \).

REFERENCES


Derivation of an Optimal Filter for Distributed Parameter Systems via Maximum Principle

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Abstract—The problem of estimation for a class of distributed parameter systems is approached via maximum principle. As a consequence, the advantages of the "optimal control" point of view for tackling the estimation problem for distributed parameter systems are indicated.

1. INTRODUCTION

In adaptive control and also in various other situations, the problem of state estimation based on a distributed model of a system is assuming importance. In particular, considerable literature existing on the control of distributed parameter systems is accumulating [11, 12]. It is considered desirable to approach the estimation problem via optimal control [12].

This correspondence takes such a view and deals with a derivation of the Kalman-Bucy type filter for linear distributed parameter systems (DPS) utilizing the maximum principle. Although the final filter equations obtained by this procedure are the same as those obtained by Tartakovsky and Xing [3], and similar to those of Thau [4] etc., it is felt that this procedure based on an "optimal control" point of view treats the problem more directly and is also capable of tackling the more general problem of nonlinear estimation.

11. PROBLEM FORMULATION

Let \( O \) be an open, simply connected subset of \( \mathbb{R} \)-dimensional Euclidean space \( E \)., and let \& denote its boundary, and let \( x = (x_1, \ldots, x_k, \ldots, x_n) \) be a generic point in \( n \). Let \( T = [t,0,T] \) denote a fixed segment.

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