FRACTURE BEHAVIOUR OF CREEPING MATERIALS UNDER BIAXIAL LOADING BY FINITE ELEMENT METHOD

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Abstract—A procedure for characterising the stress strain rate field for fracture of creeping materials using Norton's law by finite element method has been developed. C* has been found to be the characterising parameter. Time to crack initiation, \( t_\text{i} \), has been calculated with critical CTOD as the criterion for crack initiation. The stress and the strain-time relations and the equivalent stress and equivalent strain contours have been drawn. Biaxial load effects are studied in detail.

1. INTRODUCTION

ENGINEERING components possess pre-existing flaws. On the application of sustained load, at high temperature, stress concentration and strain accumulation occur at the tip of the flaws giving rise to predominant macroscopic crack. Coalescence of such cracks during creep deformation results in fracture. However, in many practical situations the fracture takes place due to initiation and propagation of a large pre-existing crack. Fu [1] presents an excellent review of creep crack growth of alloys at elevated temperatures. Various criteria have been adopted for crack initiation and propagation. Stress intensity factor approach based on LEFM has been chosen as criterion for brittle behaviour of metals by Floreen [2]. Extensive time dependent plastic deformation takes place during creep and so Landes and Begley [3] used an energy rate line integral (C*) as criterion for creep crack growth. COD is an attractive parameter and several investigators, Burdekin and Stone [4], Neale and Siverns [5], Haigh [6] and Vitek [7] have used it as criterion for crack initiation. Robinson [8] obtained CTOD experimentally by castings of creep cracks as a function of time. Taira and Ohtani [9] determined stress relaxation and strain accumulation by FEM. Time to crack initiation has been obtained as a function of fracture strain, fracture process zone size and the incremental COD. Barnby [10, 11], To [12] Purushothaman and Tien [13] and Nikbin et al. [14] have suggested several models for initiation and propagation of crack growth but due to lack of experimental data, selection of particular criterion is not conclusive.

Yokobori et al. [15] experimentally found that the relative notch opening displacement (RNOD) at the instant of crack initiation at high temperature time dependent fracture, takes nearly the same value. They also found that RNOD at the crack initiation lies in the range of 0.25–0.38. Singh and Ramakrishnan [16] have found the CTOD to be constant within 7.5% in the range of biaxial load factor (\( k = -2 \) to 3 and \( \sigma / \sigma_y \) ratio of 0.3. Singh and Ramakrishnan [17] have used constant CTOA as criterion to determine energy parameters for stable crack growth simulation under biaxial loading using finite element method.

In this paper a sharp line crack in a plane stress centre cracked specimen has been taken up for detailed study of the behaviour of creeping materials at fracture. It is necessary to adopt an appropriate criterion for crack initiation. Critical CTOD has been chosen for this purpose. Creep strain rate has been obtained by using standard Norton's law. External load has been applied so as not to cause plastic prestrain. The energy rate line integral (C*), time to crack initiation (\( t_\text{i} \)) and the stress and the strain variation with time have been obtained. Biaxial load effects have been studied.
2. THEORY AND SOLUTION PROCEDURE

The rheological model (Zienkiewicz and Cormeau [18]) shown in Fig. 1 has been utilised for the analysis of fracture behaviour of creeping material. The plastic prestrain has not been considered. Thus, the dash pot in Fig. 1 is assumed to be physically absent and creep strain occurs at all levels of applied load. This is accomplished by assuming the yield stress to be zero in the yield function.

2.1. Mathematical formulation

Total strain ($\epsilon$) experienced by the above mentioned rheological model consists of initial ($\epsilon^0$), elastic ($\epsilon^e$) and creep ($\epsilon^{\text{creep}}$) components of strain, i.e.

$$ \epsilon = \epsilon^0 + \epsilon^e + \epsilon^{\text{creep}}. $$

The elastic strains ($\epsilon^e$) are computed from the stress–strain relation

$$ \epsilon^e = D^{-1}\sigma $$

where $D$ is the appropriate elasticity matrix and $\sigma$ is the stress vector. The creep strains ($\epsilon^{\text{creep}}$) are obtained from creep strain rate ($\dot{\epsilon}^{\text{creep}}$) as follows:

$$ \dot{\epsilon}^{\text{creep}} = \gamma \left< \phi(F/F_0)^n \right> \frac{\partial Q}{\partial \sigma} $$

where $\gamma = \text{fluidity parameter}$, $F = \text{yield function}$, $F_0 = \text{any suitable value of } F$ to make $F/F_0$ non-dimensional, $n = \text{creep exponent}$ and lies between 3 and 7 for most metals, $Q = \text{plastic potential}$ and $\sigma = \text{stress vector}$.

For von-Mises material the yield function ($F$) is given as

$$ F = \sqrt{3} \sqrt{J_2} - \sigma_y = Q $$

and with $\sigma_y = 0$ in the above equation,

$$ F = Q = \sqrt{3} \sqrt{J_2} $$

and

$$ \frac{\partial Q}{\partial \sigma} = \frac{\sqrt{3}}{2} \frac{1}{\sqrt{J_2}} M^1 \sigma $$

where $M^1$ is the square matrix given as

$$ M^1 = \begin{bmatrix} 2/3 & -1/3 & 0 & -1/3 \\ -1/3 & 2/3 & 0 & -1/3 \\ 0 & 0 & 2 & 0 \\ -1/3 & -1/3 & 0 & 2/3 \end{bmatrix} $$
Fracture behaviour of creeping materials

and $J_2$ is second invariant of stress tensor defined as

$$ J_2 = \frac{1}{2}(S_x^2 + S_y^2 + S_z^2) + \tau_{xy}^2 $$

(8)

with

$$ S_x = \sigma_{xx} - \sigma_m, \quad S_y = \sigma_{yy} - \sigma_m, \quad S_z = -\sigma_m $$

and

$$ \sigma_m = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}). $$

For uniaxial case, $F^u = (\sigma / \sqrt{3})^n$ and $\partial Q / \partial \sigma = \sqrt{3}$ and

$$ \dot{\varepsilon}^{\text{creep}} = \gamma \sigma^n. $$

(9)

For three-dimensional case, assuming isotropy and incompressibility to hold good

$$ \dot{\varepsilon}^{\text{creep}} = \frac{\sqrt{3}}{2} \frac{F^u}{J_2} M' \sigma $$

(10a)

$$ = \gamma J_2^{(n-1)/2} M' \sigma. $$

(10b)

This equation is recognised as the well established metallic creep law known as Norton's law. The parameter, $\gamma$ includes all other constants.

2.2. Numerical algorithm

The analysis is carried out using the Eulerian time marching scheme. The solution procedure at any particular time instant is described as follows.

(a) Starting from known values of $\sigma$, $a$, $\dot{\varepsilon}^{\text{creep}}$ and $F$ at a time instant, $t$, the rate of creep strains are computed using eq. (10b)

$$ \dot{\varepsilon}^{\text{creep}} = \gamma J_2^{(n-1)/2} M' \sigma. $$

(11)

Here $a$ and $F$ are respectively displacement and right hand side force vectors.

(b) The creep strain increment is approximated in the time interval, $\Delta t$, as

$$ \Delta \varepsilon_i^{\text{creep}} = \dot{\varepsilon}_i^{\text{creep}} \cdot \Delta t. $$

(12)

(c) The creep strain at time $t + \Delta t$ is given by

$$ \varepsilon_i^{\text{creep}} = \varepsilon_i^{\text{creep}} + \Delta \varepsilon_i^{\text{creep}}. $$

(13)

(d) The right hand side vector, $V$, at time $t + \Delta t$ is obtained by the equation

$$ V_{t+\Delta t} = F + \int_{\Omega} B^T D \varepsilon_i^{\text{creep}} \cdot \cdot d\Omega. $$

(14)

(e) The changed displacement is then calculated as

$$ a_{t+\Delta t} = K^{-1} V_{t+\Delta t}. $$

(15)

(f) The total strains are then calculated as

$$ \varepsilon_{t+\Delta t} = B a_{t+\Delta t}. $$

(16)

(g) The stresses are then calculated as

$$ \sigma_{t+\Delta t} = D(\varepsilon_{t+\Delta t} - \dot{\varepsilon}_{t+\Delta t} - \varepsilon_0). $$

(17)

The displacements, strains and stresses obtained in this step, serve as starting values for the next time step and the whole procedure from (a) to (g) is repeated for any desired number of time steps.

A two-dimensional non-linear general purpose finite element program developed by the first author has been used in the analysis. The program uses eight noded quadratic isoparametric elements with $2 \times 2$ Gaussian integration.
The analysis proceeds till the attainment of critical CTOD and/or steady state condition. The critical incremental time step $\Delta t_c$ has been obtained from equation (Cormeau [19])

$$\Delta t_c = 4 \left( \frac{1 + \nu}{3E'} \right) \frac{1}{\eta F'' - 1}. \quad (18)$$

The actual $\Delta t$ has been adopted so as not to cause large differences in incremental stress during any time interval. The stress relaxation is very fast in the beginning with exposure of time. So a smaller time interval $0.8\Delta t_c$, has been used in the analysis.

3. CRITERION FOR CRACK INITIATION

As already mentioned, adoption of an appropriate criterion for crack initiation is absolutely essential. The macroscopic criterion based on macroscopic properties of the grains has been arrived at for the purpose. The distance between the primary crack and void ahead of the crack tip is of similar order to that of the COD and so the creep damage during crack initiation is considered localised. COD is an attractive parameter because it gives actual crack profile, can be easily visualised and is not limited to small scale plastic deformation. Several investigators have used it for correlating creep crack growth rate. Present investigation also uses critical COD as criterion for crack initiation.

The critical CTOD has been obtained from the characteristic macroscopic dimensions of the particles and particle spacing. Commercial alloys may contain three classes of particles. They are small (up to 500 Å), intermediate (500–5000 Å) and large (0.5–50 μm). While the intermediate size particles are responsible for final separation by void coalescence, the larger ones are conceived as the most important with respect to fracture toughness, (Broek [20]). It is assumed here that the particle spacing ($D$) is twice the particle diameter ($d$) in keeping with the recommendation of Paul [21]. McMeeking [22] presents relations between critical CTOD, $d$ and $D$ for various materials. For aluminium alloy, CTOD/$D = 0.647$ corresponding to $D/d = 2$. Thus, assuming the particle diameter equal to 30 μm, the CTOD = 0.00388 cm. This value of CTOD is of the same order of magnitude as that of Robinson and Tetelman [23] and has been used as critical CTOD in the present analysis.

4. MATERIAL PROPERTIES

The material taken up for study is aluminium. The modulus of elasticity, $E = 7.757 \times 10^5$ kg/cm$^2$, $\sigma_y = 1038$ kg/cm$^2$ and $\nu = 0.3$. The creep coefficient and the exponent in the standard Norton's law, eq. (10b), have been taken as $\gamma = 3.93 \times 10^{-13}$ in kg, cm, hr unit and $n = 3.5$. These are the values as obtained by Penny and Hayhurst [24]. The time function has been assumed as unity because of dominance of secondary stage creep.

5. SPECIMEN GEOMETRY, DISCRETISATION, THE LOADING AND THE BOUNDARY CONDITIONS

The plane stress centre-cracked finite geometry specimen, originally used by Lee and Liebowitz [25] and by the authors in several publications, consists of a rectangular plate (30 × 75 cm) having 0.16 cm thickness with a sharp central crack (15 cm). The first quadrant of the specimen has been discretised into 74 quadratic isoparametric elements with 259 nodes. Figure 2(a)–(c) presents the finite element mesh discretisation and the $C^*$ paths. The smallest element just ahead of the crack tip has its side 1% of initial half crack length. The elements are fine in the crack tip region and coarse in the region away from it. The aspect ratios have been kept close to 1.25 for good accuracy.

External loads have been applied at $\sigma/\sigma_y = 0.065, 0.070$ and 0.075 with biaxial load factor ($k$) as 0, 1 and $-1$. While $k = 0$ represents uniaxial external loading, $k = 1$ and $-1$ represent tension–tension and tension–compression respectively of equal magnitude.

The boundary conditions for $k = 1$ are as shown in Fig. 3 and are appropriately changed according to biaxial load factors ($k = 0, -1$).
Fig. 2. (a) Finite element mesh near crack tip (enlarged details at A). (b) Finite element mesh near crack tip (details at B). (c) Finite element mesh (reduced).
6. FRACTURE BEHAVIOUR

The $C^*$ parameter has been used to characterise the stress–strain rate field and the time to crack initiation has been obtained. The stress and the strain variation with time has been obtained and the stress contours plotted.

6.1. $C^*$ parameter

Landes and Begley [3] determined energy rate line integral, $C^*$ defined as

$$C^* = \int_r \left( W^* \, d\gamma - T_r \frac{\partial u_i}{\partial x} \, ds \right)$$

where

$$W^* = \int_{\text{creep}} \sigma_{ij} \, d\epsilon_{ij}^{\text{creep}}$$

is the strain energy rate density associated with point stresses $\sigma_{ij}$, and the strains rate, $\epsilon_{ij}$. $C^*$ is a modified form of the $J$ integral in which the strain and the displacement vectors are replaced by their rates. The $C^*$ paths are the same as those for $J$-integral paths shown in Fig 2(a)–(c). Goldman and Hutchinson [26] described $C^*$ as a single parameter characterising the near tip stress–strain rate field for material governed by power laws for creep. Nikbin et al. [14] determined this parameter and named it as $J$. Harper and Ellison [27] suggested a method for determining $C^*$ through limit analysis for von-Mises material following associated flow rule. $C^*$ retains the path independence characteristic like $J$ and uniquely characterises the crack tip stress and strain rate field for materials following non-linear steady state creep law.

6.2. Time to crack initiation

Jones and Tetelman [28] determined time to rupture ($t_f$) from the concept of fracture strain ($\epsilon_f$) and steady state creep state ($\dot{\epsilon}_s$) such that $t_f = \epsilon_f / \dot{\epsilon}_s$. The steady state creep rate was indirectly
obtained in terms of $\delta = d(COD)/dt$ and the process zone size obtained as three times the grain diameter ($d$). They took $\epsilon_f$ from experimental results of Simmons and Cross [29]. Thus the time to rupture was given as

$$t_f = \epsilon_f \cdot 3 \frac{d(COD)}{dt}. \quad (20)$$

This investigation determines $t_f$ from the critical CTOD ($\delta_c$) mentioned in Section 3. At a particular load the incremental time step can be summed up over entire iterations till the critical CTOD is achieved. As is well known, such problems require a large amount of computing time. Achievement of critical CTOD thus becomes rather uneconomic. However, achievement of the steady state of the solution helps in extrapolating the time to crack initiation from the CTOD time curve for a particular applied load. Thus, with $d(COD)/dt$ known during steady state such that $\delta_c = d(COD)/dt$, the $t_f$ is obtained as

$$t_f = \delta_c / \delta_{\sigma}. \quad (21)$$

7. RESULTS AND DISCUSSION

Results are discussed in the following four parts,

(i) variation of CTOD with time,
(ii) study of $C^*$ parameter,
(iii) the equivalent stress variation with time and the equivalent stress contours and
(iv) the equivalent strain variation with time and the equivalent strain contours.

Biaxial load effects on CTOD, $C^*$, equivalent stress, equivalent strain and their contours have been studied and discussed along with those for uniaxial loading cases for the reasons of brevity and completeness.

7.1. Variation of CTOD with time

Semi CTOD for all the time steps at various $\sigma/\sigma_y$ ratios and $k$ values have been computed. Figure 4 presents CTOD–time curve for $\sigma/\sigma_y = 0.075$.

It has been found that CTOD increases with $\sigma/\sigma_y$ ratio. CTOD–time relation starts deviating linearly for various $k$ values with time exposure during steady state creep. As would be expected CTOD increases faster with time exposure in the case of $k = -1$. The $t_c$ is obtained by extrapolating the appropriate CTOD–time curve up to the value of $\delta_c$ from eq. (21). As a matter of fact $t_c$ can be extrapolated for any critical value of CTOD after steady state is reached. With the critical CTOD value in Section 3, the critical time for crack initiation has been obtained as shown in Table 1. The $\sigma/\sigma_y$ ratio vs log $t_c$ relations for various $k$ values are shown in Fig. 5. It is observed that the $t_c$ decreases with increasing applied load, is minimum for $k = -1$ and maximum for $k = 1$. 

![Fig. 4. Semi CTOD vs time curve for various $k$ values at $\sigma/\sigma_y = 0.075$.](image)
Table 1. Values of $t_c$ for various loads and biaxial load factors

<table>
<thead>
<tr>
<th>$\sigma/\sigma_y$</th>
<th>$t_c$ (hr)</th>
<th>$k$</th>
<th>$t_c$ (hr)</th>
<th>$\ln t_c$</th>
</tr>
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<tbody>
<tr>
<td>0.065</td>
<td>0.05</td>
<td>-1</td>
<td>19.438</td>
<td>2.9672299</td>
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<tr>
<td></td>
<td></td>
<td>0</td>
<td>32.630</td>
<td>3.4852321</td>
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<tr>
<td></td>
<td></td>
<td>1</td>
<td>42.214</td>
<td>3.7427519</td>
</tr>
<tr>
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<td>0.04</td>
<td>-1</td>
<td>14.337</td>
<td>2.6628785</td>
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<tr>
<td></td>
<td></td>
<td>0</td>
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<td>3.1855258</td>
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<td></td>
<td></td>
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<td>24.990</td>
<td>3.2184757</td>
</tr>
<tr>
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<td>0.03</td>
<td>-1</td>
<td>11.585</td>
<td>2.4497728</td>
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<tr>
<td></td>
<td></td>
<td>0</td>
<td>16.220</td>
<td>2.7862450</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>19.680</td>
<td>2.9796028</td>
</tr>
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Table 2. Path independence of $C^*$ parameter

<table>
<thead>
<tr>
<th>$\sigma/\sigma_y$</th>
<th>$C^*$ path</th>
<th>$t = 1.72$ hr</th>
<th>$t = 1.47$ hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.070</td>
<td>1</td>
<td>0.026975</td>
<td>0.034333</td>
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<tr>
<td></td>
<td>2</td>
<td>0.026050</td>
<td>0.033133</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.025975</td>
<td>0.033333</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.025250</td>
<td>0.032100</td>
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<td>0.025025</td>
<td>0.031833</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.025152</td>
<td>0.032033</td>
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<tr>
<td></td>
<td>7</td>
<td>0.025225</td>
<td>0.032133</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0.025350</td>
<td>0.032266</td>
</tr>
<tr>
<td></td>
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<td>0.034266</td>
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<tr>
<td></td>
<td>10</td>
<td>0.027550</td>
<td>0.035066</td>
</tr>
</tbody>
</table>

Average $C^*$

<table>
<thead>
<tr>
<th>(C$^*$)</th>
<th>Absolute maximum difference from $C^*_{%}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025945</td>
<td>6.18</td>
</tr>
<tr>
<td>0.03049</td>
<td>6.10</td>
</tr>
</tbody>
</table>

As would be expected, the tension-compression loading is most critical from the point of view of crack initiation and the present approach has given a quantitative measure of the reduction in the useful life of a particular specimen, on the basis of the critical CTOD hypothesis. However, detailed experimentation under biaxial loading would be of great help in substantiating the validity of CTOD criterion proposed in this investigation.

Fig. 5. Correlation between load and time to crack initiation.

Fig. 6. $C^*$ vs time curve for various $\sigma/\sigma_y$ ratios.
7.2. *Study of the C* parameter*

Values of $C*$ have been evaluated at all time steps for all the ten integration paths. Table 2 shows a sample value of $C*$ computed along different paths at various load levels at specified time instants.

Path independence of $C*$ is verified within 6.18% error from the average. The variation of $C*$ with time is shown in Fig. 6 for various load levels. It is observed that the $C*$ values are high immediately after load application but decrease with time and become constant after the steady state is reached. At higher load the value of $C*$ at crack initiation is found to be higher.

7.3. *Equivalent stress and the equivalent stress contour*

Though several attempts have been made to characterise the stress relaxation rate, the same could not be clearly defined due to experimental and analytical complexities (Jones and Tetelman [28]). This investigation attempts at solving these complexities by giving stress-time curves and the equivalent stress contour at a particular time instant.

Stresses which develop after application of load, relax due to local creep deformation with time exposure. The equivalent stress has been obtained at all Gaussian points, but for brevity and completeness has been plotted against time for the Gaussian point nearest and just ahead of the crack tip for $\sigma/\sigma_0 = 0.070$ at different $k$ values as shown in Fig. 7. It is observed that stress relaxation changes from sudden and non-linear during primary stage to gradual and linear during steady state creep for all $\sigma/\sigma_0$ ratios and $k$ values. While there appears to be little effect of external equibiaxial tensile loading over the uniaxial one, the equibiaxial tension-compression has pronounced effect. The effect of biaxial loading could have been more prominent at higher $\sigma/\sigma_0$ ratio which was not applied to avoid plastic prestrain.

Equivalent stress contours in the vicinity of the crack tip have been drawn at $\sigma/\sigma_0 = 0.075$ with $k = 0$ at 1.56 hr of elapse time, as shown in Fig. 8. Stress contours have been plotted at intervals of 25 kg/cm$^2$. Closely spaced high stress contours show stress concentration in the immediate vicinity of the crack tip while sparse and lower stress contours show reduced stresses away from the crack tip. On comparing the equivalent stress contour for $k = 0$ and $k = -1$, it is observed that almost every point in the specimen is subjected to higher stress due to $k = -1$ than that due to $k = 0$.

7.4. *The creep curve and the equivalent strain contour*

The equivalent strain at varying elapsed time has been computed for the Gaussian point closest and just ahead of the crack tip for $k = -1, 0$ and 1 for all $\sigma/\sigma_0$ ratios. Figure 9 presents equivalent strain variation at $\sigma/\sigma_0 = 0.070$. The equivalent strain increases for various $k$ values at all $\sigma/\sigma_0$ ratios in the primary creep range and stabilises with time exposure, as steady state approaches. Obviously, the strain increases with $\sigma/\sigma_0$. At a particular $\sigma/\sigma_0$ ratio, the variation of equivalent strain with time is rather complicated in the presence of biaxial load. In the primary creep range, this curve corresponding to $k = 0$ lies below that for $k = 1$. In the steady state range the strain for
$k = -1$ is clearly larger than that for the other two. For $k = 0$ and 1, in this range, the strain is quite close. The curve for $k = 0$, in the beginning of secondary creep range, is below that for $k = 1$ and then with further time exposure during steady state range, it is above that for $k = 1$. While there is little effect of $k = 1$ on creep strain, $k = -1$ has pronounced effect. It is apparent that the maximum strain is caused due to $k = -1$. 

![Creep curve for $\sigma/\sigma_c = 0.070$ for various $k$ values.](image)
Equivalent strain contour is shown in Fig. 10. Strain concentration occurs in the vicinity of the crack tip. The strain decreases with increase in distance from the crack tip.

7.5. Effect of the creep exponent

It has been observed that a higher value of creep exponent, reduces the critical time step, CTOD and equivalent strain becomes more and stress relaxation is faster. Steady state approaches faster with appropriate value of $\Delta t$.

8. CONCLUSIONS

(i) With critical CTOD ($\delta_c$) as criterion for crack initiation, time to crack initiation ($t_c$) has been computed using steady state crack tip opening displacement rate ($\delta_{\infty}$). As would be expected $t_c$ decreases with increasing load and creep exponent.

(ii) While the tension–compression biaxiality reduces $t_c$, the tension–tension increases it. Thus the useful life of the structure is reduced in the former case rather than in the latter one.

(iii) The $C^*$ is path independent within $\pm 6.18\%$ from the average value. At higher loads $C^*$ is found to be higher at crack initiation.

(iv) The CTOD, the equivalent stress, the equivalent strain and the stress contours are considerably affected by tension–compression biaxiality.

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REFERENCES


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