obtaining the difference or error between two signals or generating the slope of a signal via a one-bit delay of \( e_d \) and a difference operation. The DM multiplier can serve as the basis for constructing a correlator. Although we can realize only discrete time delays, they take the very simple form of one-bit shift registers, since we must merely delay \( e_d \). These are only a few of the many potential uses of direct DM signal processing.

REFERENCES


A Novel Theorem with Potential Application to Automatic Equalization

SURENDRAPRA SAR

Abstract-When the signs of the alternate terms of a symmetric discrete time series are reversed and the newly created series is convolved with the original, the newly created time series will have alternate values equal to zero. This theorem is shown here to have a potential application in the design of a single-shot zero-forcing equalizer (of the adjust and freeze type). The system can be easily generalized to handle partial response signals.


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I. INTRODUCTION

Mereu [1] has shown that when the signs of alternate terms of a symmetric discrete time series are reversed and the newly created series is then convolved with the original series, the resultant time series will have alternate values equal to zero. In this note it is shown that this property of symmetric functions may be exploited to design a single shot (of the adjust and freeze type), zero-forcing equalizer which trades-off the adaptation software and convergence time with some increased hardware in the form of cascaded transversal filters. The approach presented here requires no polynomial division and no equations are required to be solved as in the conventional adaptive techniques. The method should be of interest in applications where the start-up time of the modem is required to be small; e.g., when the messages consist of only a few hundred bits. The time that must be allowed for the equalizer training is then usually the major delay in the start-up.

Performance of the new equalizer proposed here is also investigated through simulation studies. It is found that for the example channels, the peak distortion is practically equal to the minimum achievable value. Pending further investigations, this suggests that the proposed technique may serve as a possible alternative to the minimum 'peak distortion' equalizer in the form of cascaded transversal filters [31].

II. BASIC THEORY

Consider the 3-point, symmetric autocorrelation function \((x, 1, x)\) of a 2-point time series. It is easy to see that \((X, 1, x) * (-x, 1, -x) = (-x^2, 0, 1 - 2x^2, 0, -x^2)\), where \(\ast\) denotes the convolution operation. The reason zeros are inserted here may be easily seen if the above convolution is performed by multiplying their respective z-transforms:

\[
(xz^{-1} + 1 + xz)(-xz^{-1} + 1 - xz)
= (-x^2z^{-2} + 0, z^{-1} + (1 - 2x^2) + 0, z^{-2}x^2, z^2)
\]

When this operation is repeated on the newly created series, with signs of the alternate non-zero terms now reversed, more zeros are inserted into the output:

\[
(-x^2, 0, (1 - 2x9, 0, -x^2) \ast (x^2, 0, (1 - 2x^2), 0, -x^2, 1)
= (-x^2, 0, 0, (1 - 2x^2), 0, 0, 0, -x^2, 1)
\]

Continuing this operation \(N\) times, the number of zeros which are inserted in between the spikes (i.e., the non-zero terms) will equal \(2^N\). Thus, if \(N\) is increased from 1 to 7, the number of inserted zeros is \((1, 3, 7, 15, 31, 63, 127)\) respectively. Since \(ix I < 1\), the central spike is the largest, all others decreasing geometrically as one moves away from the center. The total length of the corresponding filters, however, also increases proportionally as more numbers of zeros are required to be inserted.

More generally, let the given input sample sequence be \(X\) and the desired output sequence of the equalizer be \(D\) (which may be the sequence \((0, 0, 0, 0, 1, 0, 0, 0, 0)\) with an appropriate number of zeros, or perhaps an appropriate partial response sequence which could be selected to save bandwidth [41]). The problem of designing the optimum (zero-forcing) equalizer can be stated as that of finding a filter impulse re-
Generalizing the steps of the above example, it can be easily seen that the filter \( H \) is given by

\[
H = H_o \ast H_1 \ast H_2 \ldots \ast H_N \ast D
\]  

(2)

where

\[
H_o = \text{Input sample sequence reversed in time}
\]

\[
H_i = (X \ast H_o) \text{, with signs of alternate terms changed}
\]

\[
H_i = (X \ast H_o \ast H_l) \text{, with signs of alternate non-zero terms changed.}
\]

\[
H_N = (X \ast H_o \ast H_l \ldots \ast H_{N-1}) \text{, with signs of alternate non-zero terms changed.}
\]  

(3)

An outline of the proof of (2) based on the example considered above is given below.

It should be noted that the first operation \( X \ast H_o \) generates the symmetric sample sequence (viz., the autocorrelation function of the input sequence) which is subsequently used for the zero-insertion procedure discussed above. In the special cases where \( X \) itself is symmetric, \( X \ast H_o \) can be replaced directly with \( X \). Thus we have:

\[
X \ast H \ast H_l = \text{a spike sequence, each spike separated from the others by one zero.}
\]

\[
X \ast H_0 \ast H_l \ast H_2 = \text{a spike sequence, each spike separated from the others by three zeros.}
\]

etc.

The filter \( H' \), given by:

\[
H' = H_o \ast H_l \ast H_2 \ldots \ast H_{N-1}
\]  

(4)

when operating on the output signal \( X \) yields an output signal having spikes separated by \( 2^N \) zeros, and thus approximates a delta function (since all the noncentral spikes are of much smaller magnitude than the central spike, for large \( H \)).

The filter \( H \) of (2) is obtained by cascading \( H' \) with a shaping filter with impulse response \( D \) to obtain the desired output sequence, in case it happens to be different from a delta function.

111. IMPLEMENTATION: ONE-SHOT EQUALIZATION

A. Operation of the Equalizer

We now consider the implementation of an equalizer using a single step adaptation based on an input training sequence. Two well known equalizer training methods involve transmitting a succession of isolated test pulses [21] or pseudo-random sequences [15]. The equalizer coefficients are adjusted in each symbol interval, or periodically, to minimize the mean square error or peak distortion. It has been shown [2] that when the peak distortion of the received signal (\( \ast \ldots \ast X \ast \ldots \ast X \ast \ldots \ast \)) is less than unity, the peak distortion in the equalized signal (with a given length \( 2M + 1 \) of the equalizer) is minimized when the equalizer forces \( M \) zeros on either side of the center. This requires the solution of the following equations (in the matrix form)

\[
x c = 1
\]  

(6)

where \( X \) is \( (2M + 1) \times 1 \) order square matrix whose \( i^{th} \) element is \( x_i \). \( C \) is the \( (2M + 1) \) column vector whose \( j^{th} \) element is \( C_j \), the \( j^{th} \) tap weight and 1 is the \( (2M + 1) \)-element column vector whose components are all zero except for the central element, which is unity.

In the implementation proposed here, it is assumed that a single isolated test pulse is transmitted and that the equalized pulse at the receiver is required to have \( M \) zeros on either side of the center, as required for the minimization of the peak distortion. Assuming that the effective duration of the auto-correlation function of the received sequence extends over \( n T \) seconds, \( n = 2m + 1 \), Fig. 1 illustrates a possible arrangement for carrying out the equalization for the case when \( m = 2 \).

The registers \( RO, R1, R2 \ldots \) etc. shown here store the coefficients of the filters \( H_o, H_l, \ldots \), respectively, while the shift registers \( S^1, S_o, S_2, \ldots \) etc. are directly excited by the outputs of the previous stages of filtering. During the start-up period, the filter coefficients are required to be stored in the coefficient registers \( RO, R1, R2 \ldots \) etc. This is achieved by keeping the switches \( SW1, SW2, SW3 \) etc. open and \( SW4, SW5, SW6 \) closed during the start-up period. The filter coefficients of any given stage are then obtained simply by shifting the output of the previous stage into a tapped delay line (\( T1, T2 \ldots \) etc.) and then storing the tap-outputs in the registers \( RO, R1, R2 \ldots \) etc. after appropriate sign reversals as shown. The tapped delay line \( Ti \) is disconnected from the previous stage as soon as all the coefficients of a subfilter have been obtained. This output is, however, now shifted through the upper shift registers (tapped delay line) to carry out the required convolution. This is achieved by incorporating appropriate delays (\( 2T, 3T, 5T \) etc.), as shown) cascaded in front of \( SO, S1, S2 \) etc. It is clear that the registers \( RO, R1, R2 \ldots \) etc. should remain disconnected (i.e., \( SW4, SW5, SW6 \) remain open) later in the equalization, while the switches \( SW1, SW2, SW3 \) are closed to bypass the start-up delays in front of various stages of filtering.

B. Remarks

It is seen from the above discussion that the conventional equalizer with a \( (2M + 1) \)-tap delay line and weighting network can be replaced by \( N \) cascaded stages of tapped delay line filters, with \( M = 2^N - 1 \). The overall length of the tapped delay line for this arrangement can be easily shown to be given by

\[
L = (2^N - 1)(n - 1) + N + m
\]  

\[
= (n - 1)M + \log_2(M + 1) + m
\]  

(7)

where, as explained earlier, \( n = 2m + 1 \), \( m \) being the number of interfering terms (equal to the effective length of the sampled impulse response of the channel) in the input sequence at the receiver. Some typical values of \( L \) as a function of \( m \) and \( M \) are summarized in Table I. The last term in (7) will, however, be absent when \( X \) is symmetric.
TABLE I
CHARACTERISTICS OF THE CASCADED EQUALIZERS

<table>
<thead>
<tr>
<th>No. of stages</th>
<th>No of taps of the equivalent zero-forcing equalizer</th>
<th>Overall filter length, ( L )</th>
<th>No. of non-zero terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>( 2M + 1 )</td>
<td>( n = 3 )</td>
<td>( n = 5 )</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>10</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>19</td>
<td>34</td>
</tr>
</tbody>
</table>

TABLE II
PERFORMANCE COMPARISON WITH THE PEAK DISTORTION MINIMIZING EQUALIZER

<table>
<thead>
<tr>
<th>Channel No.</th>
<th>Sampled impulse response of channel</th>
<th>Optimum, 7-tap Zero forcing equalizer weights</th>
<th>Peak distortion of optimum equalizer</th>
<th>Peak distortion of proposed equalizer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.165, 1.0165</td>
<td>-0.00563, 0.030501, -0.179824, 1.0589342</td>
<td>-56.1 dB</td>
<td>-55.6 dB</td>
</tr>
<tr>
<td>2</td>
<td>0.282, 1.0282</td>
<td>-0.032388, 0.114497, -0.373729, 1.210783, -0.0032288</td>
<td>-36.5 dB</td>
<td>-35.4 dB</td>
</tr>
</tbody>
</table>

It is worth mentioning here that each successive (and larger) filter in the sequence of cascaded filters \( H_0, H_1, H_2, \ldots \) contains a larger number of zeros in its weighting sequence. In fact, it is easy to verify that the number of non-zero terms in each filter is a constant equal to the number of points in the autocorrelation function. Thus, although the length of the overall delay-line is of the order of \( nM \) (eqn. (7)), the number of multipliers required to implement the filter is \( n \log_2 (M-1) \) — a much smaller value for large \( M \). Some typical values of this number are also shown in Table I, for comparison with the conventional, \( 2M + 1 \) tap equalizer.

Finally, it may be recalled that the implementation shown in Fig. 1 will get modified by the addition of a final stage with weights corresponding to the sequence "D", if this is different from a delta function.

C. Performance

As indicated in the above discussion, the most suitable performance criterion for a zero-forcing type of equalizer is the "peak-distortion" in the equalized pulse. Thus, if the output of the equalizer is denoted by the sequence \((-h_0, h_0, h_0, \ldots)\), then the peak distortion is given by

\[
D = \frac{1}{h_0} \sum_{i=0}^{\infty} |y_i|
\]

where \( h_0 \) is the value of the central spike of interest. Another performance factor of interest is the influence of sampling phase on the minimal achievable peak distortion. This factor, however, is difficult to evaluate and is presently under investigation.

A few preliminary results of computations on the peak distortion of two equalizers, based on two simple channel models are summarized in Table II. These results indicate that the proposed technique may serve as a possible alternative to the zero-forcing equalizer whose weighting coefficients satisfy (6).
Questions regarding the performance of the proposed technique with respect to peak distortion, mean-square error, start-up in noisy environment, influence of timing phase etc., however, are still open and need further investigations. Some of these questions are presently being looked into, and, it is hoped, some further results will be published later.

IV. CONCLUSION

A zero-forcing equalizer has been proposed which can be trained by a single isolated pulse transmission. The method can be easily generalized to handle partial response signals, by suitably altering the choice of \(D\) in (1).

REFERENCES


Correspondence

Comments on "Envelope Detector Performance for a Partially Coherent-Fading Sinewave in Noise"

W. A. GARDNER

Abstract—The basic premise in a recent paper* that the matched-filter square-law envelope detector is generally appropriate for fast-fading as well as slow-fading incoherent channels is questioned. Its inappropriateness for a fast-fading incoherent Gaussian channel is demonstrated by evaluation of probability of error for this receiver and for the optimum receiver. As a result, modifications to some of the conclusions drawn* regarding attainable performance gain with increasing observation time are proposed. In addition, it is pointed out that the performance predictor adopted*, v.i., deflection, exhibits anomalous behavior (for at least some incoherent fading channels), thereby rendering the other conclusions* questionable.

In a recent paper,” it is stated that

"The (matched-filter) square-law envelope detector appears to be a reasonable detector structure... (It yields) the correct basic physical results concerning random amplitude and phase channel performances... (for both slow and rapid fading channel fluctuations)."

The purpose of this note is to question this basic premise by illustrating the severe suboptimality of the proposed receiver for long signaling periods (7') over a Gaussian fast-fading incoherent channel. This suboptimality seems intuitively obvious if it is recognized that the proposed receiver is approximately equivalent to a narrow bandpass filter, with bandwidth inversely proportional to \(T\), followed by an energy detector; whereas the optimum receiver for a fast-fading incoherent Gaussian channel is approximately equivalent to a wide bandpass filter, with fixed bandwidth \((B)\) equal to that of the received signals \((S)\), followed by an energy detector. Thus, for long signaling intervals \((T \gg B^{-1})\), the suboptimum receiver rejects most of the signal \(S\).

To quantify this, we consider the slow-fading component \(S\) of a fast-fading signal \(S\):

\[
\hat{S}(t) \triangleq S \cos (\omega_0 t) + S \sin (\omega_0 t),
\]

where

\[
\hat{S}(t) \triangleq \frac{2}{T} \int_0^T \cos (\omega_0 t) S(t) dt
\]

and \(\omega_0\) is the frequency of the transmitted sinewave at the channel input. It can be shown that for \(T \ll B^{-1}\), the mean squared values of the coefficients (2) are

\[
E[\hat{S}^2(t)] \triangleq E[\hat{S}^2] \approx 2\Psi_1(\omega_0) / T,
\]

provided that the power spectral density (PSD) of the signal \(S\) contains no discrete component—a provision that is satisfied by a fast-fading channel; e.g., an incoherent Rayleigh fast-fading channel [11]. The output of the matched-filter square-law envelope detector is

\[
X = \left| \int_0^t \cos (w_0 t) Y(t) dt \right|^2 + \left| \int_0^t \sin (w_0 t) Y(t) dt \right|^2
\]