Scalar Modes and Coupling Characteristics of Eight-Port Waveguide Couplers

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Abstract—In this paper, we have given the scalar modes and discussed the coupling characteristics of an eight-port waveguide coupler consisting of four parallel single-mode waveguides. The analysis is based on a rectangular waveguide model which has been known to give accurate results in the case of two waveguide directional couplers. A classification of its various scalar modes is also given.

INTRODUCTION

MULTI-PORT waveguide couplers have interesting applications in dividing optical power from one channel to many channels and in determining the in-phase and quadrature components of a signal in coherent communication systems and fiber-optic sensors. Although the minimum number of power measurement ports to determine the in-phase/quadrature components of a signal is three, the signal processing is easier with a suitable four detector multiport. Travis and Carroll [1] have suggested an eight-port coupler, using four parallel optical fibers, to realize such a four-detector in-phase/quadrature measuring multiport device. In order to understand the coupling characteristics of such multiport junctions, their modal characteristics should be known. Recently, modes of a 3 x 3 fiber coupler, using three parallel fibers, have been reported by Black et al. [2] and Stevenson and Love [3]. In the case of a 4 X 4 fiber coupler, while Travis and Carroll [1] have discussed some basic features of guided modes of such couplers, no formal method is reported to obtain the coupling characteristics of these couplers. Here, we present a simple method to understand the coupling characteristics of an eight-port coupler which can be obtained either by a) fusing four identical parallel fibers, b) polishing the surfaces of two dual-core fibers and cementing them together, or c) using integrated optical channel waveguides with square cross sections. The analysis is based on a rectangular-core waveguide model (i.e., replace the circular cores by square cores as shown in Fig. 1(a)). The dielectric constant distribution of the rectangular-core waveguide model (hereafter referred to as structure 1) is given by

\[ n(x, y) = \begin{cases} n_0, & d < |x| < (a + d) \\ n_0, & d < |y| < (a + d) \\ n_1, & \text{otherwise.} \end{cases} \]

where \( n_0 \) stands for \( x \) or \( y \).

The above structure can be taken as a perturbed form of a pseudowaveguide coupler (hereafter referred to as structure 2), whose dielectric constant distribution is given by (see Fig. 1(b)):

\[ n'(x, y) = n_0(x) + n_0(y) - n_1 \]

where

\[ n_0(|\xi|) = \begin{cases} n_1, & d < |\xi| < (a + d) \\ n_2, & \text{otherwise} \end{cases} \]

where \( |\xi| \) stands for \( x \) or \( y \).

Since the dielectric constant distributions of the two structures differ only in the corner regions (shown shaded in Fig. 1(b), where the fractional modal power is rather small) by a small amount \( (n_0 - n_1) \), the modes of structure 1 should be nearly the same as that of structure 2. Further, since the dielectric constant distribution of structure 2 is separable in \( x \) and \( y \) coordinates, the scalar modes of structure 2 can be obtained analytically. As a result, the propagation constants of various guided modes of structure 1 can be obtained by applying a first-order correction to the propagation constants of the corresponding modes of structure 2.

Analysis

An ideal eight-port coupler consists of four identical fibers, with their centers making the four corners of a square, each side of which represents the center to center distance between two nearby fibers. In order to study the coupling characteristics of such a coupler, we use a rectangular-waveguide model (i.e., replace the circular cores by square cores as shown in Fig. 1(a)). The dielectric constant distribution of the rectangular-core waveguide model (hereafter referred to as structure 1) is given by

The propagation constants of various guided modes of the eight-port coupler are obtained and its coupling characteristics are discussed. A classification of its scalar modes is given and it is shown that a basic assumption regarding the propagation constants of the guided modes, made in [1], is not correct.
MODES OF STRUCTURE 2

We assume that the fibers used for making the coupler are weakly guiding, and hence, discuss only the scalar modes of the coupler which can be obtained by solving the following scalar wave equation:

$$\frac{d^2 \psi}{dx^2} + \frac{d^2 \psi}{dy^2} + \left( k_0^2 n^2_0(x, y) - \beta^2 \right) \psi = 0.$$  \hspace{1cm} (4)

For structure 2, since the refractive index profile is separable in x and y coordinates, the wave function $\psi(x, y)$ is of the form

$$\psi(x, y) = X(x) \cdot Y(y).$$  \hspace{1cm} (5)

where both $X(x)$ and $Y(y)$ satisfy the following one-dimensional equations:

$$\frac{d^2 X}{dx^2} + \left( k_0^2 n^2_0(x) - a_{x}^2 \right) X = 0.$$  \hspace{1cm} (6a)

and

$$\frac{d^2 Y}{dy^2} + \left( k_0^2 n^2_0(y) - a_{y}^2 \right) Y = 0.$$  \hspace{1cm} (6b)

where $a_{x}$ and $a_{y}$ are two separation constants such that

$$\beta^2 = \alpha_x^2 + \alpha_y^2$$  \hspace{1cm} (7)

represents the propagation constant of a guided mode of structure 2, and $x$ and $y$ stand for $x$ (or $y$) representing symmetric (or antisymmetric) mode in $x$ (or $y$), respectively. Physically, $X(x)$ (or $Y(y)$) represents the mode of a directional coupler consisting of two parallel slab waveguides with boundaries parallel to $y$ (or $x$) axis, whose refractive indices are given by (3), and $a_{x}$ (or $a_{y}$) represents the propagation constants of their guided modes. Equations (6a) and (6b) can be solved analytically and $a_{x}$ ($x$ stands for $x$ or $y$) can be obtained by solving the following simple transcendental equations [9].

For the mode symmetric in $x$:

$$\frac{\mu_{x1}}{\mu_{x2}} \tanh \left( \delta \mu_{x1} \right) = \frac{\tan \left( \alpha_x / a_x \right) \cdot \left( \alpha_y / a_y \right)}{1 + \left( \mu_{x1} / \mu_{x2} \right)^{1/2} \alpha_x \delta \mu_{x1}}$$  \hspace{1cm} (8)

For the mode antisymmetric in $x$:

$$\frac{\mu_{x1}}{\mu_{x2}} \coth \left( \delta \mu_{x1} \right) = \frac{1 - \left( \mu_{x1} / \mu_{x2} \right)}{1 + \left( \mu_{x1} / \mu_{x2} \right)} \tan \left( \alpha_x / a_x \right)$$  \hspace{1cm} (9)

where

$$\mu_{x1} = \alpha_x^2 - k_0^2 n_0^2$$

$$\delta = 2 \alpha_x a_x$$

and

$$\alpha_x = \sinh \left( \delta \mu_{x1} \right), \quad S_x = \sinh \left( \delta \mu_{x1} \right), \quad C_x = 1.0$$

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For the unperturbed structure, the propagation constants can be obtained after incorporating the difference in the dielectric constants of the two structures in the corner regions through the first order perturbation approach and are given by

$$tfj = Pf + kl(n_x - n_y)Y$$  \hspace{1cm} (10)

where

$$\Gamma = \Gamma_1 \cdot \Gamma_2$$

with

$$Y_k = (1 + A)/(\sqrt{A + B})$$

$$A = (2/R)[(t_1 \cos (dn_{t2})) + \left( \frac{\mu_{x1} / \mu_{x2}}{S_x \sin \left( dp_{x2} \right)} \right)]^2$$

$$B = \frac{\delta_{x1}}{R \mu_{x2}} \left[ f \cdot 2a_{n2} + \sin \left( 2a_{n2} \right) \right]$$

$$- \left( \frac{\mu_{x1}}{S_x} \right) \left[ \left( 2a_{n2} \sin \left( 2a_{n2} \right) \right) \right]$$

$$+ \left( 2a_{n2} \sinh \left( 2a_{n2} \right) \right)$$

$$R = 2C_2 \cdot a_{n2} + \sinh \left( 2a_{n2} \right)$$

$$t_1 = \cosh \left( dx_{t1} \right), \quad S_x = \sinh \left( dt_{x1} \right), \quad C_x = 1.0$$

$$t_2 = \sinh \left( dx_{t2} \right), \quad S_x = \cosh \left( dt_{x2} \right), \quad C_x = -1.0$$
and \( k \) stands for 5 (or a). It is obvious that modes \( LP_{sa} \) and \( LP_{gs} \) are degenerate (i.e., \( |\beta_{sa}| = |\beta_{gs}| \)). It should be mentioned that \( |\beta_{j}|, j \not\in 2 \), and \( |\beta_{4}| \) of [1] correspond to our \( \{\beta_{ss}, \beta_{a}, \beta_{as}, \beta_{aa}\} \), respectively. Further, since \( LP_{ss} \) is antisymmetric in \( x \) as well as in \( y \), it would be a mode of higher order than \( LP_{sa} \) or \( LP_{as} \) and hence, \( |\beta_{4}| \) should be less than \( |\beta_{2}| \) contrary to the assumption made in [1].

COUPLING CHARACTERISTICS

Light energy propagating in any channel would be coupled to the other channels through evanescent field coupling which can be understood as follows.

If at the input end of the coupler, power is injected only into waveguide 1 (say with an amplitude of 4) it will excite all the four modes discussed above, each with an amplitude 1. The amplitudes and the relative phases in various channels for different modes are shown in Fig. 2. Since different modes propagate with different propagation constants, after a distance \( z \), the total amplitude due to all the four modes in different channels would be different. The field in channel 1 would be given by

\[
\begin{align*}
|E_1| &= \exp(-i0_Mz)[1 + 2 \exp\{-i(0_M - 0_a)z\} \\
&\quad + \exp\{-i(0_M - 0_{as})z\}floor \\
&= \exp(-i0_Mz)[1 + 2 \exp\{i(z/L_2)\} \\
&\quad + \exp\{i(z/L_4)\}]
\end{align*}
\]

(11)

where \( L_2 = x/(0_M - 0_a) \) and \( L_4 = x/(0_M - 0_{as}) \) are the coupling lengths between modes \( LP_{ss}, LP_{sa} \) and modes \( LP_{as}, LP_{aa} \) respectively. Similarly, the fields in channels 2, 3, and 4 would be given by

\[
\begin{align*}
|E_2| &= |E_3| = \exp(-i0_{ss}z)[1 - \exp\{i(z/L_4)\}] \\
|E_4| &= \exp(-i0_{sa}z)[1 - \exp\{i(z/L_2)\}]
\end{align*}
\]

(12) and

(13)

We now discuss some specific cases of practical interest:

a) \( z/L_2 = 2n \) and \( z/L_4 = 2m + 1 \)

\( m \) and \( n \) are integers.

Using equations (11)—(13), it can be seen that in this case the amplitude \( a_i \) in the \( i \)th waveguide (\( i \) stands for 1, 2, 3, or 4) would be given by

\[
\begin{align*}
a_1 &= 2, \quad a_2 = 3, \quad a_3 = 2, \quad a_4 = -2
\end{align*}
\]

i.e., the power will be equally divided in all the channels.

b) \( z/L_2 = In + 1 \) and \( z/L_4 = 2m + 1 \)

\[
\begin{align*}
a_1 &= -2, \quad a_2 = a_3 = a_4 = 2
\end{align*}
\]

I.e., power Will be equally divided in all the Channels again.

Fig. 3. Variation of the normalized propagation constant \( P \) of different scalar modes as a function of normalized frequency \( V \) for a coupler with \( n_1 = 1.4603, n_2 = 1.4573, a = 5.0 \text{ \mu m}, \) and \( d = 2.5 \text{ \mu m} \).

Fig. 4. Variation of coupling lengths \( L_2 \) and \( L_4 \) as a function of normalized separation \( d/ab\)w between the waveguides of the coupler with \( n_1 = 1.4603, n_2 = 1.4573, a = 5.0 \text{ \mu m} , \) and \( V = 2.3 \).

Fig. 6. Relative amplitudes and phases of various scalar modes of the coupler in different channels.
c) \( z L_2 = \ln 1 + 1 \) and \( z L_4 = 2m \)
\[ a_x = a_2 = a_3 = 0, \quad a_4 = 4 \]
i.e., the whole power is transferred to channel 4.
d) \( z L_2 = 2n \) and \( z L_4 = 2m \)
\[ \omega = 4, \quad a_2 = a_3 = a_4 = 0 \]
i.e., the incident power would come back in channel 1.

Thus, depending upon the values of \( L_2 \) and \( L_4 \) and the interaction length between the different channels, one can achieve signals of desirable amplitudes/phases in various channels. The values of \( L_2 \) and \( L_4 \) depend upon the \( K = k_x a / (n - n_f) \) for number of individual waveguides and also on the separation \( d \) between the waveguides. Thus, in order to satisfy specific conditions by \( L_2 \) and \( L_4 \), the values of \( K \) and \( d \) can be selected accordingly.

In Fig. 3, we have plotted the variation of the normalized propagation constants \( P_{ij} = (\beta_{ij} - k a) / (\beta_{ij} - n_f) \) of different modes as a function of \( V \) parameter. The numerical values of various parameters used in the calculations are given below:

\[ a_x = 1.4603, \quad n_2 = 1.4573, \quad a = 5.0 \mu m, \quad d = 2.5 \mu m. \]

This figure clearly shows that \( \beta_{11} \) (or \( \beta_{22} \)) is greater than \( \beta_{33} \). In Fig. 4, we have shown the variation of \( L_2 \) and \( L_4 \) as a function of normalized separation \( (d/a) \) between the waveguides. This figure shows that as \( (d/a) \) increases, the coupling lengths \( L_2 \) and \( L_4 \) also increase, which is due to decreasing interaction between the waveguides.

In summary, we have given a simple method to obtain the scalar modes and the coupling characteristics of an eight-port coupler consisting of four parallel optical waveguides in a cross-sectional square arrangement.

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REFERENCES