Abstract: For networks containing resistors, independent sources, gyrators, ideal transformers, diodes and other saturating monotone piecewise linear resistors, a new condition is proposed to guarantee uniqueness of solutions. This is both necessary and sufficient and is easy to test. It requires that the characteristics should satisfy a condition with the null vectors of the matrix of the linear part of the network whenever it is singular. Diodes with infinite reverse resistance are also allowed in the formulation.

1 Introduction

Saturating characteristics commonly occur in electronic circuits. Diodes and transistors saturate in some region or the other. Uniqueness of solutions of resistive networks containing such characteristics is therefore an important problem in the study of digital circuits [1-3].

Desoer and Wu [2] studied this problem from a topological point of view when the network contains two terminal linear and nonlinear resistors and no controlled sources. Sandberg and Willson [1, 3] studied this problem from a mathematical point of view when the network contains linear resistors, independent sources, gyrators, ideal transformers, diodes and other saturating monotone piecewise linear resistors. A new condition is proposed to satisfy this condition. More generally, any network containing linear resistors, independent sources, ideal transformers and gyros and several types of diodes satisfy this condition [3]. Some networks containing controlled sources also satisfy this condition [4, 6].

Since the passive pair condition is the property of the linear part of such networks, it can be taken for granted in practice.

Theorem: Referring to the above notation, eqn. 1 has a unique solution for every \( y \) if, and only if, \( f_i(x) \neq f_i(x + a) \), for all \( x \) where \( a \) is a vector in the null space of \( B \) denoted by \( N(B) \).

Proof (sufficiency): Sufficiency is proved by contradiction. Consider a point \( y^* \). First, we assume that it has at least another solution. Then, \( f_i(x^*) \neq f_i(x^* + a) \), for all \( x \) where \( a \) is a vector in the null space of \( B \) denoted by \( N(B) \).

2 Uniqueness condition

Consider an equation of the form \[ P(x) \pm Af(x) + Bx = y \] (1)

where \((A, B)\) is a passive pair [1, 3] of matrices (a pair is said to be passive if, for all \( u \) and \( v \), such that \( Au = Bv \), \( u^T v > 0 \), and \( l(x) = r|f_1(x), \ldots, Ux| \), \( f_1 \) for \( i = 1, 2, \ldots, n \) are continuous piecewise linear and monotone (not strictly monotone) increasing. Equations of this type arise in resistor diode networks. More generally, any network containing linear resistors, independent sources, ideal transformers and gyros and several types of diodes satisfy this condition [3]. Some networks containing controlled sources also satisfy this condition [4, 6].

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the Jacobian of $F(x)$ in each region in the $x$ space is non-singular. If this is not true, there would be at least one $y$ which has infinitely many solutions. This violates the fact that $y$ has only one solution. Let $y^0$ be a point whose solution is required. Join $y^0$ and $y^1$ by a line, call it $L$. Trace its inverse image $L^{-1}$ from $F(x)$, Since every point on $L$ has a unique solution, Katzenelson's method works [5]. Even if $L$ hits a corner point it can always be extended into a new region [5]. It must therefore pass through the solution of $y^0$. Thus every $y$ has a unique solution. This proves sufficiency.

Proof (necessity): Given that $F$ is a homeomorphism, we wish to prove that $f(x) \neq (x + a)$ for all $x$ and $a \in \mathbb{R}$. If this is not true, let $f(x^{(1)}) = (x^{(1)} + a)$ for $a \in \mathbb{R}$. Let $x^0 = x^1 + a$. Then,

$$Af(x^{(TM)}) = 4/(x^{(1)} + a) = 4Ax^0$$

Further,

$$Bx^0 = Bx^0 + Ba = Bx^0$$

as $a$ is a null vector of $B$. Therefore,

$$Af(x^0) + Bx^0 = Af(x^0) + Bx^0$$

that is, $x^0$ and $x^1$ are two solutions of the same $y$ which is a violation. Hence the result.

Remarks:

(i) The above theorem is applicable to equations of the form $f(x) + H(x) = y$ also taking $a$ as null vector of $H$.

(ii) Consider the following piecewise linear function.

$$f(x) = \begin{cases} 1 & x < 0 \\ 0 & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

where

$$f(x_i) = \begin{cases} 1 & x_i < 0 \\ 0 & 0 \leq x_i \leq 1 \\ 1 & x_i > 1 \end{cases}$$

Note that $a \notin \mathbb{R}$ is of the form $a = [0, 0, 1]^T$. It is easily verified that $f(x) \neq f(x + \Delta x)$ for all $x$. Therefore there is a unique solution for every $y$ according to the theorem.

(iii) Sandberg and Willson [1, 3] studied this problem when the characteristics are nonlinear. They can be into mappings. In case the characteristics can be into. They need not be strictly monotone. Further zero slopes can be anywhere not necessarily for large values of the individual variables. However, the characteristics must be piecewise linear.

Consider the following equations

$$E\textbf{SH1} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

where

$$f(x) = \begin{cases} 0 & x_2 \leq 0 \\ 100x & x_2 > 0 \end{cases}$$

for $i = 1, 2$

$f(1) =$ region in which $y$ is bounded is $x_j < 0, x_2 < 0$

$N(B)$ is $x_2 = 0, N(B) n P(f) = \varnothing$.

Thus Sandberg and Willson's condition is satisfied. But $f(x) = (x + a)$ for

$$\begin{bmatrix} -t \mathbf{S} \\ -1 \end{bmatrix} = \begin{bmatrix} [-1] \end{bmatrix}$$

showing that the null space condition is not satisfied. Therefore, from the theorem, this function is not a homeomorphism whereas if we use Sandberg and Willson's theorem, we will be forced to conclude that this function has a unique solution for every $y$. Thus Sandberg and Willson's theorem is not applicable to the piecewise linear case.

Consider the equation $f(x) + Ox = y$ where $fix) = e^x$. $N(B)$ is the entire real line and $\mathbb{R}(l)$ is $x < 0$. Thus $N(B) n \mathbb{R}(l)$ is not null. But the null space condition is satisfied as $e^x$ is strictly monotone.

$P(f)$ refers to large values of $x$ as boundedness is tested for large values of $x$. Thus $N(B) n P(f) = 0$ is to be tested for vector $x$ as $||x|| \rightarrow \infty$ whereas in the null space condition, a vector with finite magnitudes should also be considered. Thus these arguments show that the null space condition and Sandberg and Willson's condition are different.

(iv) Note that the necessity part of the proof requires neither the passive pair property nor the monotonicity of the characteristics. Therefore all homeomorphic functions of the form $e^x$. I satisfy this. However, this alone is not enough to guarantee uniqueness of solutions as the following example shows. Consider the equations

$$U \begin{bmatrix} x+ \mathbf{L}^* -2JU \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

where $y(x) = x$, for $x < 0$, and $100x$, for $x > 0$, $i = 1, 2, x = 0$. Therefore, the null space condition is satisfied. But it is not a homeomorphism.

(v) The theorem will not be true if $(A, B)$ is not, see Reference 1 or 3 for the definition of $W_n$ pair as the following example shows.

Consider the piecewise linear equations

$$L_2 \begin{bmatrix} x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} JU \begin{bmatrix} y_2 \end{bmatrix}$$

where

$$f_2(x) = \begin{cases} 0 & x_2 < 0 \\ 100x & x_2 > 0 \end{cases}$$

The null space condition is satisfied. But it is easily verified that it is not a homeomorphism as the Jacobian is singular in some regions.

3 References


4 LIN, P.M.: 'Formulation of hybrid matrices of linear multipoles containing controlled sources', IEEE CAS., 1974, 21, pp. 169-175
