Application of optimal control strategy to automatic generation control of a hydrothermal system

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Abstract: The paper highlights the design of automatic generation controllers through optimal control strategy, for an interconnected hydrothermal system using a new performance index that circumvents the need for a load demand estimator. The dynamic performances of these controllers are analysed and compared with those obtained through the usual performance index as that used by Fosha and Elgerd, considering a step-load perturbation in either of the areas. Attempt is made to suitably design the new optimal controller that can provide safe generation rate and reasonably good response.

List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>/ = nominal system frequency</td>
<td></td>
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<tr>
<td>/ = subscript referring to area / (i = 1, 2)</td>
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<tr>
<td>( AP_i ) = incremental change in tie-line power</td>
<td></td>
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<tr>
<td>( AF_i ) = incremental frequency deviation</td>
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<tr>
<td>( AP_{gi} ) = incremental generation change</td>
<td></td>
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<tr>
<td>( AX_{gi} ) = incremental governor valve position change</td>
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<tr>
<td>( A^D_i ) = incremental load demand change</td>
<td></td>
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<tr>
<td>( AI_i ) = incremental change in speed changer position</td>
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<tr>
<td>( H_i ) = inertia constant</td>
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<tr>
<td>( D_i ) = load-frequency constant (( K_{pi} = l/D_i, \ T_{pi} = 2H_i/J(D_i) ))</td>
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<tr>
<td>( K_i ) = high-pressure turbine power fraction</td>
<td></td>
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<tr>
<td>( T_r ) = reheat time constant</td>
<td></td>
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<tr>
<td>( T_{2i} ) = synchronising coefficient</td>
<td></td>
</tr>
<tr>
<td>( P_{ri} ) = rated area power, ( a_{1i} = -P_{ri}/P_2 )</td>
<td></td>
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<tr>
<td>( T_s ) = time constant of steam turbine governor</td>
<td></td>
</tr>
<tr>
<td>( T_{c0} ) = steam-cylinder time constant (control valves to HP exhaust)</td>
<td></td>
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<tr>
<td>( Ri ) = self-regulation parameter for the governor of /th area</td>
<td></td>
</tr>
<tr>
<td>( Bj ) = frequency bias constant</td>
<td></td>
</tr>
<tr>
<td>( Pi ) = area frequency response characteristic (( D, +/l(Ri) ))</td>
<td></td>
</tr>
<tr>
<td>( P_{f(max)} ) = maximum tie-line power handling capability</td>
<td></td>
</tr>
<tr>
<td>( \phi_0 ) = nominal phase angle of voltage</td>
<td></td>
</tr>
<tr>
<td>( T_H, T_1, T_2 ) = time constants of the hydrogovernor</td>
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<td>( T_w ) = water starting time constant</td>
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1 Introduction

During the last two decades, considerable interest has been shown towards the application of optimal control theory for arriving at more efficient automatic generation controllers for interconnected power systems. Fosha and Elgerd [1] were the first to apply modern optimal control theory to the automatic generation control (AGC) problem for a two equal-area nonreheat thermal system. Carpentier [8] has presented an excellent critical review on the application of modern control theory to AGC. However, for the realisation of such a modern controller, it is necessary to have the knowledge of the new steady state which, in turn, is a function of the load demand. There is, thus, a problem of estimating the load demand through a load-demand estimator, which is a complicated and expensive proposition. Some researchers have designed optimal controllers with state and load-demand estimators. So far, no attempt has been made to design optimal controllers circumventing the load-demand estimator.

It is known [2] that, although the loads are time-variant, the variations are relatively slow. From minute to minute we have an almost constant load. A minute is a long time period as compared with the electrical time constants of the system, and thus permits us to consider the system operating in a steady state: a steady state that slowly shifts throughout the 24 hours of the day. Thus, the rate of change of load demand for a small duration considered in AGC problems can, for all practical purposes, be neglected, and hence a constant but unknown load demand may be assumed.

Through the use of a new performance index it is possible to circumvent the load demand estimator, thereby considerably simplifying the realisation of the linear optimal controller. The proposed new performance index, through a judicious choice of the weighting matrices, can control the rate of change of generation so important from the viewpoint of energy source dynamics.

Moreover, the works reported in the literature on AGC using optimal control strategy pertain to either thermal systems or hydro systems. There is no work on AGC for a hydrothermal system using optimal control strategy. In a mixed power system, it is usual to find an area regulated by hydro generation interconnected to another area regulated by thermal generation. The hydropower systems also differ from steam electric power systems, in that the relatively large inertia of the water used as the source of energy causes a considerably greater time lag in the response of changes of the prime-mover torque, due to a change in the gate position. With the hydro turbines there is again an initial tendency for the torque to change in a direction opposite to that finally
produced. In addition, the response may contain oscillating components caused by the compressibility of the water (and expansion of piping) or by surge tanks. In securing stable operation, these response characteristics, in conjunction with limitations on permissible surge pressures, make it necessary to use speed governors having very different characteristics. Usually, the speed governor has a relatively large temporary droop and long washout time.

For such hydrothermal systems of widely different characteristics surprisingly no work has been done to suitably design automatic generation controllers using modern optimal control strategy. The main objectives of this piece of work are the following:

(a) to design an optimal controller for an interconnected hydrothermal system through optimal control strategy using the Fosha-Elgerd [1] approach, and, hence, to study the dynamic responses for small perturbation in either the thermal or the hydro area

(b) to highlight the feasibility of designing an optimal controller through a new performance index (henceforth called new optimal controller), so that its realisation does not need any load demand estimator

(c) to compare the performance of the new optimal controller with that of the optimal controller designed through the Fosha-Elgerd approach

(d) to investigate the effect of varying weighting matrices in the new performance index on the rate of change of generation.

2 System investigated

The AGC system investigated comprises an interconnection of two areas, area I comprising reheat-type thermal system and area II comprising a hydro system. The nominal parameters of the system are given in the Appendix.

3 Transfer function model

Fig. 1 shows the transfer-function block diagram of a two-area small perturbation model of a hydrothermal system. The detailed transfer-function models of speed governors and turbines are discussed and developed in the IEEE Committee report on dynamic models for steam and hydro turbines in power-system studies [3].

4 Specifications for control

Questions about how well a system should be controlled have yet to be fully resolved. Minimum performance criteria [5] laid down by the North American Power Systems Interconnection Committee (NAPSIC) are constantly being reviewed, because of the developments, such as increasing costs associated with regulation and changes in generation mix, regulatory limitations on nuclear units, decreasing percentage of hydro capacity in some systems and the reduced response capability of the new steam units being installed. Minimum control requirements stated by Fosha and Elgerd [1], and as discussed therein [1] by Cohn, are either incomplete or incorrect. In view of Cohn’s discussion [1], the following qualitative specifications may be considered for design purposes:

(i) The steady-state frequency error following a step load change should vanish, provided the area in which the load change occurred can adjust its generation fully to accommodate this change. If it cannot, the system operating objective is to permit frequency deviation to persist to a degree sufficient to permit or cause other areas to provide an assistance to the area in need for the full duration of the need.

(ii) The static change in tie-power flow, following a step-load change in an area, must be zero, provided the area in which step-load change occurred can adjust its generation, in the steady-state sense, to accommodate the change. If this is not the case, the operating objective is to permit such flow to persist, so that other areas will provide sustained assistance to the area in need, for the full duration of the need.

(iii) The transient frequency and tie-power errors should be small. Time error and inadvertent interchange should also be small.

(iv) An automatic generation controller providing a slow monotonic type of response is preferred in order to reduce wear and tear of the equipment.

5 Dynamic model in state variable form

The general linearized state-space model for the AGC system in controlled mode is written in the form

$$X = AX + BU + TP$$

(1)

where $X$, $U$ and $P$ are the state, control and disturbance vectors, respectively. $A$, $B$ and $T$ are real constant matrices of compatible dimensions and depend on system parameters and the operating point.

6 Analysis

6.1 Synthesis of optimal controller

First, an optimal controller is derived following the approach of Fosha and Elgerd [1]. The linear state-space model of the system (eqn. 1) is obtained by defining the vectors $X$, $U$ and $P$ as follows:

$$X = \left[ \begin{array}{c} \dot{AP}_{11} \\ \int AF_1 \\ \dot{AP}_{13} \\ \int AP_{13} \end{array} \right]$$

(2)

$$U = \left[ \begin{array}{c} \dot{AF}_2 \\ \dot{AP}_{21} \end{array} \right]$$

(3)

$$P = \left[ \begin{array}{c} \int AP_{21} \end{array} \right]$$

(4)

Redefine the state, control and disturbance vectors in terms of their final steady-state values, i.e.
\[ X = X - X^\wedge \] (5)
\[ U = U - U_{SS} \] (6)
\[ P = P - P_{SS} \] (7)
Eqn. 1 thus takes the form of
\[ \dot{X} = AX + BU \] (8)
where
\[ X(0) = X_{ss} \] (9)
because \( X(0) = 0 \), where
\[ \dot{X} = [0 0 0 \ AP_{910} \ AP_{920} \ AP_{210} \ AP_{220}] \] (10)

Therefore, from eqn. 5,

\[ F = [J \ AP_{31} \ dt J \ AF_{X} \ A \ AF((AP_{910} - AP_{910}) \ x \ (AP_{910} - AP_{920}X_{X1} - A X_{1} \ J \ J \ AF_{2} \ dt \ AF_{2} \ x \ (AP_{2} - AP_{920}X_{X2} - A X_{2} \ AP_{2} - AP_{220}))] \] (11)
The final steady-state control vector:
\[ V = [AP_{310} \ AP_{210}] \] (12)

Therefore, from eqn. 6,

\[ (F = [(AP_{310} - AP_{210}X_{X2} - AP_{220})] \] (13)

In eqn. 7, \( \dot{p} = 0 \) because \( p = p_{ss} \) for all time, corresponding to the given step-load perturbation. For a step-load perturbation:
\[ AP_{910} = AP_{910} = \Delta X_{E10} = AP_{910} = AP_{920} \] (14)
\[ \Delta P_{920} = \Delta P_{920} = \Delta X_{E20} = \Delta P_{920} = \Delta P_{220} \] (15)
The control vector \( \dot{U} \) which minimises the quadratic cost function
\[ J = \frac{1}{2} \int_{0}^{\infty} (X Q X + t P R U) \ dt \] (16)
is given by
\[ \dot{U} = -K_{F} X \] (17)
where \( K_{F} = R^{-1}B^{T}P \), \( P \) is the symmetric positive definite solution of the algebraic matrix Ricatti equation:
\[ A^{T}P + PA - PBR^{T}P + Q = 0 \] (18)

Examining eqn. 17, it may be seen that the control vector is a linear function of the state \( X \), which is again dependent on the load demand, thereby requiring a load-demand estimator.

The weighting matrices \( Q \) and \( R \) are chosen in a manner similar to that considered by Fosha and Elgerd [1]. Accordingly, the nonzero elements of \( Q \) are
61: \( i = 10 \), \( Q_{E2} = 1 + (2nT_{E2})^{2} \) \( Q_{E2} = -(2nT_{E2})^{2} \),
63: \( i = 10 \), \( Q_{E2} = -(2nT_{E2})^{2} \) \( Q_{E2} = -(2nT_{E2})^{2} \),
\[ c_{E2} = 1.0 \] and \( R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \)

where \( P_{2} \) is the maximum power-handling capability of the tie line, \( \theta_{L1} \) and \( \theta_{L2} \) are the nominal phase angles of the voltages at the ends 1 and 2 of the tie line. Solving the algebraic matrix Ricatti equation 18 by the approach given in Reference 9 and, using expression for \( K_{F} \), the feedback gain matrix \( K_{F} \) is obtained as
\[ K_{F} = \begin{bmatrix} -0.125 & 1.409 & 1.663 & 4.536 & -1.132 & 0.267 \\ -0.983 & -0.06 & 0.147 & 0.435 & -0.111 & 0.025 \\ -0.006 & 0.569 & 2.183 & 5.321 & 0.0631 & 0.025 \\ -0.119 & -0.252 & 0.207 & 8.176 & -0.545 \end{bmatrix} \] (19)

6.2 Dynamic performance with the optimal controller
With the controller design given in Section 6.1, analysis is now made to understand the system performance under a step-load perturbation. Fig. 2 shows the dynamic response for 1% step-load perturbation in either of the areas. It is seen that the maximum transient frequency and tie-power deviations are higher for step-load perturbation in the hydro area than in the thermal area. In particular, the maximum tie-power deviation is several times more (around 3.5 times for the system investigated) when the perturbation occurs in the hydro area than in the thermal area. The settling time is also considerably more for step-load perturbation in the hydro area than in the thermal area.

Examining the generation responses (Fig. 3) for step-load perturbation in the thermal area, it is noted that the maximum generation rate realised in the thermal area is about 75% per minute, while that in the hydro area is about 2% per minute. Moreover, there is practically no generation assistance from the hydro area, because the maximum generation rate realised in the thermal area is about 55% per minute, while that in the hydro area is around 4% per minute. The thermal area provides adequate generation assistance from the hydro area.
assistance during the period when the hydro generation is slowly trying to pick up to its desired steady-state value. Analysis clearly reveals that the optimal controller

![Fig. 3 Generation responses with $K_F$ for 1% step-load perturbation in the thermal area](image)

![Fig. 4 Generation responses with $K_F$ for 1% step-load perturbation in the hydro area](image)

demands, in general, an extremely high rate of thermal generation which the system cannot withstand. The rate of hydro generation, on the other hand, remains sufficiently below the permissible value.

6.3 Synthesis of a new optimal controller

A linearised minimal order state-space model of the hydrothermal system may be written in the form

$$X = AX + BU + TP$$

$$Y = CX$$

(20)

(21)

where

$$TC = [AF_1 AP_{12} AP_{ii} A] A^* E_1 AP_{i2} AF_2$$

$$f^p = [AP_{12} AP_{i2}]$$

$$p^T = [AP_{12} AP_{i2}]$$

(22)

(23)

(24)

The real constant matrices $A$, $B$ and $T$ are functions of the operating point and system parameters. Considering area control errors as the outputs of the system, the output vector $Y$ is defined as

$$Y = \begin{bmatrix} ACE_1 \\ \Delta P_{t1} + B_1 A F_1 \\ \Delta P_{t2} + B_1 A F_2 \end{bmatrix}$$

(25)

Thus, matrix $C$ in eqn. 21 is defined as

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(26)

An optimal control law to be determined is the one that ensures asymptotic stability and output regulation, i.e. $\dot{X}, Y \to 0$ as $t \to \infty$. Following an approach suggested by Smith et al. [6], an optimal controller can be determined which is not an explicit function of the external load disturbance vector $p$.

Define the vectors $Z$ and $V$ as follows:

$$Z^T = [X^T Y^T]$$

$$V = U$$

(27)

(28)

Transform eqns. 20 and 21 by differentiation. The result is

$$\dot{Z} = AZ + BV$$

(29)

$$Z(0) = \begin{bmatrix} A & B & n \\ C & o & o \end{bmatrix} \begin{bmatrix} X(0) \\ U(0) \\ p(0) \end{bmatrix}$$

(30)

where

$$A = \begin{bmatrix} \hat{A} \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} \hat{B} \\ 0 \end{bmatrix}$$

It may be noted that as the problem is defined in its minimal order, the matrix

$$\begin{bmatrix} YA & B & n \\ C & o & o \end{bmatrix}$$

is of full rank and $Z(0)$ may lie anywhere in its 11-dimensional state space.

Thus, joint requirements $\dot{X}, y \to 0$ as $t \to \infty$ may now be restated, the origin of $Z$ space must be asymptotically reachable from the entire space. This will be so if, and only if, the pair $(A, B)$ is stabilisable. This requirement means that

(i) the pair $(A, B)$ is stabilisable

(ii) the matrix

$$\begin{bmatrix} A & S^- \end{bmatrix}$$

is of full row rank.

It is extremely important to note that the dynamics of the AGC system have now been described in the standard form in eqn. 29. Thus, a stable linear state feedback controller may be designed by any suitable means, e.g. by pole assignment, optimal linear quadratic control or inverse Nyquist techniques.

Assume a new performance index (cost function):

$$J_{\text{new}} = \int_0^\infty (X^T Q_1 + Y^T Q_2 Y + t F R t) dt$$

(31)

The performance index is new because in eqn. 31 $J_{\text{new}}$ consists of derivatives of both the state and control vectors, in addition to the output vector, unlike the usual quadratic function depending only on the state and control vectors. Noting that $Z^T = [X^T Y^T]$ and $V = u$, eqn. 31 can be written in the compact form as

$$J_{\text{new}} = \frac{1}{2} \int_0^\infty (Z^T Q Z + V^T R V) dt$$

(32)

where

$$Q = \begin{bmatrix} Q_l & 0 \\ 0 & Q_2 \end{bmatrix}$$
is a positive-semidefinite matrix and $R$ is a positive-definite matrix. The optimal control law which minimises $J_{	ext{con}}$ is a linear function of the transformed state vector $Z$, i.e. the optimum value of $V$ is

$$V^* = -K_N Z$$

(33)

$K_N$ is a 2 x 11-dimensional constant control gain matrix, and can be evaluated as

$$K_N = R\beta^T P$$

(34)

In eqn. 34, the matrix $P$ is the unique positive-definite symmetric matrix obtained as the solution of the matrix Riccati equation:

$$A^TP + PA - PBR^{-1}B^TP + Q = 0$$

(35)

This new form of the cost function allows the possibility of penalising the rates of change of state variables.

Let us partition the feedback gain matrix $K_N$ in accordance with the form of $Z$. Thus,

$$K_N = [A_N, A_S]$$

(30)

where $A_N$ is a 2 x 11 matrix, $A_S$ is a 2 x 9 matrix and $K_2$ is a 2 x 2 matrix.

The control law, on transformation into the original co-ordinates, becomes

$$U = K_2 Y$$

(37)

On integrating eqn. 37,

$$V = K_N X + K_2 \int_0^t Y(x) dx + C$$

(38)

$C$ is zero because $U(0) = 0$, $X(0) = 0$ and $\int_0^t Y(x) dx = 0$ at $t = 0$, i.e.

$$U = K_N X + K_2 \int_0^t Y(x) dx$$

(39)

The control law is thus a function of linear combination of the state variables in $X$ and an integral of area control errors. The state variables being deviations from the nominal values are independent of the load demand. Thus, the realisation of the optimal controller does not need a load-demand estimator.

As the selection of weighting matrices $Q$ and $R$ plays a significant role in the design process, an attempt is therefore made to choose weighting matrices judiciously. Conceptually, it is felt that relatively large weightings should be attached to $AP_{el}$ and $AX_{el}$ to contain the thermal generation rate to a low value, while $AP_{gl}$ and $AX_{gl}$, in view of high permissible generation rate of the hydro area, may be allowed to change freely. Relatively low weightings may be attached to $AF_{gl}$, $AF_{el}$, $ACE_{gl}$ and $ACE_{el}$. In view of this, let $Q$ and $R$ be chosen to be of the following form:

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix}$$

(40)

$$R = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix}$$

(41)

It is intended to vary $g$ over a wide range to study its effect on generation rates. For each chosen $Q$, an optimal state feedback control law is determined by solving the algebraic matrix Riccati equation 35, and the corresponding dynamic responses are obtained by solving state equation 20, using Kutta-Merson technique for a step-load perturbation in either of the areas.

Table 1 shows the variations of the maximum rate of change of generations realised for 1 % step-load perturbation in the thermal area with $q$ on the basis of considering the proposed new optimal controller. It is again seen that the maximum generation rate realised in the thermal area has considerably decreased, i.e. from 56% min$^{-1}$ to 18% min$^{-1}$ by increasing $q$ from 0 to 30. It is also worth mentioning that the maximum generation rate in the hydro area has increased from 15% min$^{-1}$ to 24% min$^{-1}$ (a desirable feature due to the inherent slow response characteristics of the hydro area, and also due to the permissible rate of change of generation, i.e. about 270% min$^{-1}$ for raising and 360% min$^{-1}$ for lowering the generation).

Comparing the maximum generation rates realised for any value of $q$ for 1 % step-load perturbation in the thermal area and in the hydro area (Tables 1 and 2), it may be observed that the $AP_{el}$, $AX_{el}$, $ACE_{el}$ and $ACE_{el}$ is a positive-semidefinite matrix and $R$ is a positive-definite matrix. The optimal control law which minimises $J_{\text{con}}$ is a linear function of the transformed state vector $Z$, i.e. the optimum value of $V$ is

$$V^* = -K_N Z$$

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As the selection of weighting matrices $Q$ and $R$ plays a significant role in the design process, an attempt is therefore made to choose weighting matrices judiciously. Conceptually, it is felt that relatively large weightings should be attached to $AP_{el}$ and $AX_{el}$ to contain the thermal generation rate to a low value, while $AP_{gl}$ and $AX_{gl}$, in view of high permissible generation rate of the hydro area, may be allowed to change freely. Relatively low weightings may be attached to $AF_{gl}$, $AF_{el}$, $ACE_{gl}$ and $ACE_{el}$. In view of this, let $Q$ and $R$ be chosen to be of the following form:

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix}$$

(40)

$$R = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix}$$

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It is intended to vary $g$ over a wide range to study its effect on generation rates. For each chosen $Q$, an optimal state feedback control law is determined by solving the algebraic matrix Riccati equation 35, and the corresponding dynamic responses are obtained by solving state equation 20, using Kutta-Merson technique for a step-load perturbation in either of the areas.

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Comparing the maximum generation rates realised for any value of $q$ for 1 % step-load perturbation in the thermal area and in the hydro area (Tables 1 and 2), it may be observed that the $AP_{el}$ for any value of $q$ for 1% step-load perturbation in either area is more or less equal. It may also be seen that the generation rate realised in the hydro area, i.e. $AP_{el}$ for any value of $q$, is more for 1% step-load perturbation in the hydro area than what it is for a similar perturbation in the thermal area.

It is thus construed that the low generation rate in the thermal area can be realised by heavily penalising $AP_{el}$ and $AX_{el}$. However, the reduction in $AP_{el}$ is marginal for the values of $q$ beyond 30. Hence, $Q$ may be chosen as

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \end{bmatrix}$$

(42)

Let the feedback gain matrix corresponding to the chosen $Q$ be called $K_N$, where

$$K_N \approx \begin{bmatrix} 0.924 & 11.72 & -4.19 & 4.647 & -0.606 \\ 3.56 & -1.20 & 0.027 & 1.023 \\ 0.744 & 2.087 & 6.325 & -0.046 & 1.324 \\ 0.438 & 2.069 & 14.50 & -0.679 & 0.495 \\ 1 & 0.15 & 0.24 & 1.80 & 1.323 \\ \end{bmatrix}$$

6.4 Dynamic performance with the new optimal controller

Fig. 6 shows the generation responses for 1% step-load perturbation in the thermal area, with feedback gain matrix $K_N$. It is seen that the generation rate realised is about 18% min$^{-1}$ (which is about a quarter of the generation rate attained with $K_F$; compare Figs. 3 and 6). The maximum generation rate of the hydro area is found to be around 11.25% min$^{-1}$ with $K_N$, while it is only 2% min$^{-1}$ with $K_F$. With the use of the new controller $K_N$ the rate of hydro area generation is increased, compared to that with $K_F$. This is a desirable feature, as the hydro area provides more assistance to the deficient area. The maximum generation rate of the hydro area with $K_N$ is found to be around 11.25% min$^{-1}$ with positive assistance to the thermal area, where the generation rate is around 2% min$^{-1}$ with $K_F$, and that, also, withdrawing the assistance to the thermal area because the generation remains negative over entire transient (Fig. 3).

Fig. 5 shows the dynamic responses in either area with state feedback gain matrix $K_N$. It is seen that the dynamic responses with $K_N$ are practically the same, irrespective of the step-load perturbation in either area, unlike that experienced with $K_F$ (Fig. 2).

Fig. 7 shows the generation responses with perturbation in the hydro area and feedback gain matrix $K_N$.

The maximum generation rate realised in the thermal area is about 18% min$^{-1}$, which is only around 30% of the corresponding value of the maximum generation rate in the thermal area with $K_F$ (compare Figs. 4 and 7). The maximum generation rate in the hydro area is about 24% min$^{-1}$ with $K_N$, which is quite high as compared with $K_F$, being thus a desirable feature for providing assistance to the other area.

The above analysis clearly demonstrates that the optimal controller, based on the new performance index, not only obviates the need for a load-demand estimator, but also provides a much better transient response, from the viewpoints of containing the thermal generation rate and, simultaneously, providing better assistance from the hydro area to the regulation process.

7 Conclusions

The following significant contributions are made in the paper:

(i) A linear state-space model for the AGC of a hydro-thermal system, considering reheat-type thermal unit in the thermal area and detailed representation of the governor in the hydro area, has been developed for the first time.

(ii) Optimal automatic generation controllers considering two different performance indices, i.e. one based on the Fosha-Elgerd approach and the other using a new performance index, are synthesised and their performance compared.

(iii) Use of a new performance index obviates the need for a load-demand estimator.

(iv) The performance of the new optimal controller is found to be much superior to the one based on the conventional optimal control strategy used by Fosha and Elgerd. The use of a new performance index for the design of an optimal controller reveals that the thermal area generation rate can be contained to safe permissible limits, by heavily penalising the rate of change of thermal
area generation and opening and closing of the thermal area valves. Moreover, the new controller also exhibits an improved performance of the hydro area in the regulation process.

8 References

5 'Current operating problems associated with automatic generation control', *ibid.*, 1979, PAS-98, pp. 88-96
8 CARPENTIER, J.: 'State of the art review, "To be or not to be modern" that is the question for automatic generation control (point of view of a utility engineer)', *Int. J. Electr. Power & Energy Syst.*, 1985, 7, pp. 81-91

9 Appendix

The nominal parameters of the system are:

\[
\begin{align*}
\bar{f} &= 60 \text{ Hz}, \\
T_1 &= 0.08 \text{ s}, \\
T_2 &= 10 \text{ s}, \\
T_3 &= 0.513 \text{ s}, \\
\beta &= 30^\circ, \\
B_1 &= B_2 = p = 0.425 \text{ p.u. MW/Hz}, \\
\delta &= 0.31, \\
P_{\text{max}} &= 2000 \text{ MW}, \\
T_{1r} &= 0.3 \text{ s}, \\
T_{2r} &= 5 \text{ s}, \\
R_i &= R_2 = 2.4 \text{ Hz/p.u. MW}, \\
P_{(\text{max})} &= 200 \text{ MW}, \\
T_{G1} &= 0.2 \text{ s}, \\
H_1 &= H_2 = 5s, \\
K_r &= 0.5, \\
T_i &= 48.7 \text{ s}, \\
T_w &= 1.0 \text{ s}, \\
D_\pm &= D_2 = 8.33 \times 10^{-3} \text{ p.u. MW/Hz}, \\
a &= 0.04.
\end{align*}
\]