Abstract

As the chip dimensions are reduced, the performance of integrated circuits is limited by the delays associated with the interconnections. For the purpose of propagation delay estimation in digital circuits, these interconnections can be modelled by leaky RC tree/line/mesh. Signal delay in a nonleaky RC tree can be estimated by a tree algorithm, in this paper a modified tree algorithm (linear order) is presented for estimating signal delay in a leaky RC tree with nonzero initial conditions.

I Introduction

Interconnections in an integrated circuit have distributed resistances, capacitances and grounded conductances. As the chip dimensions are reduced, the performance of integrated circuits is limited by the delays associated with the interconnections. For the purpose of propagation delay estimation in digital circuits, these interconnections can be modelled by leaky RC tree/line/mesh. Penfield[1] and Wyatt[2] derived the waveform bounds for estimating signal delays in a nonleaky RC tree[1] and nonleaky RC mesh[2]. Lin and Mead[3] and Chan[4] developed the scheme for computing signal delays in nonleaky RC mesh having nonzero initial conditions. Recently Marinov[5] proposed the delay computation method for a rcg line. Marinov and Neittaanmaki[6] investigated the qualitative properties and numerical solutions of a circuit consisting of resistive multiport with r-c-g exactly modeled as distributed elements connected to its terminals.

This paper deals with the signal delay estimation in a leaky RC tree. Based on a modification of Elmore's delay and properties of the mutual resistances, a signal delay computation scheme is presented for a leaky RC tree having nonzero initial conditions. Special cases of leaky RC tree and their respective computational requirement is also presented.

II Signal Delay

A leaky RC mesh is an RC network with a capacitor between every node and ground but no coupling capacitors between any other nodes. There is at least one grounded conductance. An RC mesh having nonzero initial condition, the voltage response may be nonmonotonic[3]. Lin[3] and Chan[4] modified the Elmore's definition of delay to include the nonzero initial conditions.

In this section the Elmore's delay is redefined to obtain the estimate of signal delay in leaky RC mesh having nonzero initial conditions.

The KCL of an RC mesh with grounded conductances at each node and multiple source vector e(t) = [e_1(t), e_2(t) ... e_n(t)]^T can be written as

\[ \frac{dV}{dt} + G V = D e(t) \]  (1)

where

\[ V \in \mathbb{R}^n \; ; \; \text{the node voltage vector} \]

\[ G, C, D \in \mathbb{R}^{n \times n} ; \; G \; \text{is the node conductance matrix} \]

\[ D = \text{diag} [g_1 ... g_m] ; \; \text{the source conductance matrix.} \]

\[ C = \text{diag} [C_1 C_2 ... C_n] ; \; \text{the node capacitance matrix and} \]

\[ e_1(t), e_2(t) ... e_n(t) \; \text{are the voltage sources at the nodes 1, 2, ... n.} \]

The signal delay (T_{De}) definition of [3] at any node e can be extended to the case of a leaky RC mesh having nonzero initial conditions, as follows[7]:

\[ v_e(t) - v_e(0) \]

\[ T_{De} = \int_0^\infty [v_e(t) - v_e(0)] \, dt \]  (2)

for any \( v_e(0) \).
It can be observed that this definition of delay $T_d$ is consistent with Elmore's delay for monotone $v_e(t)$, and $v_e(0) = 0$.

He will now use this definition of delay to compute the signal delay in leaky RC mesh with multiple non unitstep excitations.

Let
\[ T = [T_{d1} T_{d2} \ldots T_{dn}]^T \]  \hspace{1cm} (3)
and
\[ \mathbf{T} = [T_{d1} \ldots T_{dn}] \]  \hspace{1cm} (4)

where
\[ T_{di} = [V_i(00) \ T_{di}] \]  \hspace{1cm} (5)

A convenient way to express delay for all the nodes in the circuit is to use the vector notation, i.e.: $d$.

\[ T = V(00) - V(t) \]  \hspace{1cm} (6)

By taking Laplace transform of (6) and taking their limits as $s$ approaches zero, we obtain

\[ T_s = \lim [u(s) V(\infty) - v(t)] \]  \hspace{1cm} (7)

When all the capacitors are charged to $V(\infty)$, from (1)

\[ Q V(\infty) = D \cdot e(\infty) \]  \hspace{1cm} (8)

Applying the Laplace transform on both sides of (1), and simplifying [7] we obtain

\[ G T_s = C [V(\infty) - V(0)] \]  \hspace{1cm} (9)

Note that the R.H.S. of (8) and (9) are constant vectors. To compute $T_s$ first $V(\infty)$ are obtained from (8) and substituted into (9). For a sparse conductance matrix $G$, the LU decomposition method can be used. Only once the LU decomposition of the $G$ matrix is performed to solve the equations (8) and (9). When there are no grounded conductances (a nonleaky RC mesh) i.e. $V(\infty) = L$, the equation (8) need not be solved.

The equations (8) and (9) can be further simplified using the superposition principle [7], i.e; the signal delay at any node $i$ can be decomposed into the delays at the node $i$ due to the individual excitations. Therefore we consider the computational aspect of signal delay due to only one excitation.

The KCL of an RC network with only one excitation $e(t)$ can be written as

\[ \frac{dV}{dt} + G V = d e(t) \quad t \geq 0 \]  \hspace{1cm} (10)

where $d$ is the source conductance vector.

The equations (8) and (9) reduces to

\[ G T_s = C [V(\infty) - V(0)] \]  \hspace{1cm} (11)

\[ d e(\infty) Lim [u(s) e(s)] \]  \hspace{1cm} (12)

Further for a step input (i.e. $e(s)/e(\infty) = u(s)$)

\[ G T_s = C [V(\infty) - V(0)] \]  \hspace{1cm} (13)

Without loss of generality, we assume that the excitation is at the node $n$ i.e.

\[ d = g_{n(\infty)} [1 \ldots 1]^T \]  \hspace{1cm} (14)

and the signal delay at any node $i$

\[ T_{pi} = \sum_{j=1}^{n} g_{n(\infty)} e(\infty) \]  \hspace{1cm} (15)

The intrinsic factor $Lim [u(s) - e(s)/e(\infty)]$ in the delay computation of $T_{pi}$ is constant for all the nodes. Thus the computation of $T_{pi}$ due to a nonunitstep excitation is similar to that of the computation of delay due to a unit step excitation. Note that as a special case of leaky RC mesh, a nonleaky RC mesh would have $V_j(\infty) = 1$ and $T_{pi}$ will reduce to $2 r^i C_j$ for a unit step excitation as shown by Wyatt[2].

Given the resistance matrix of a leaky RC mesh, the signal delay at any node can be computed from (15). When the resistance matrix is not available, the delay can be obtained by first solving (11) and then (12). In the following sections we present the signal delay computational scheme for a leaky RC tree.

**III. Leaky Resistor Tree**

In general, for computing signal delay in leaky RC mesh either the resistance matrix is required or two sets of linear
equations are required to be solved. However, for a leaky RC tree because of the restrictive nature of the network, these difficulties can be over come.

Leaky resistor tree is a resistor tree with grounded conductances, in which there is a unique path from the source node towards any node \( i \) without passing through the ground. For such a resistor tree network (e.g., Fig. 1) define

\[ p(i) \] - the adjacent node of the node \( i \) towards the source node. Further \( p'(i) \) \( \subseteq \) \( P\{P^{i+1}(i)\} \), \( P^w(i) \) \( \subseteq \) \( P \).

\[ R_i \] - the resistance between the nodes \( p(i) \) and \( i \).

\[ G_R(i) \] - the equivalent conductance between the node \( i \) and the ground, looking into the node \( i \) towards the terminal nodes including the grounded conductances at the node \( i \).

Therefore, \( G_R(i) \) can be computed starting from the leaf nodes towards the source node using the following equation

\[
G_R(i) = g_{io} + \sum_{k=P(k)}^{G_R(k)} \frac{G_R(k)}{k} \quad (16)
\]

where

\[
G_R(k) = 1 + R_i G_R(k) \quad (17)
\]

\( G_R(i) \) - the equivalent conductance between the node \( i \) and the ground, looking into the node \( i \) towards the source, including the grounded conductance at the node \( i \) but excluding the conductances of the branch having node \( k \); \( p(k) = i \). (The source node is assumed to be grounded for computing \( G_R(i) \). Therefore \( G_R(i) \) can be computed by starting from the source node towards the leaf nodes using the following equation

\[
G_R(i) = g_{io} + \frac{G_R^i(p(i))}{\beta_i} + \sum_{w=P(w)}^{G_R(w)} \frac{G_R(w)}{a_w} \quad (18)
\]

where

\[
B_i = 1 + R_i G_R^i(p(i)) \quad (19)
\]

and

\[
G_R^{(n+1)} = 0 \quad (20)
\]

Note that the total conductance at a node \( i \) will be

\[
G_T(i) = G_R(i) + \frac{G_R^i(p(i))}{\beta_i} \quad (21)
\]

The resistor subnetwork of a leaky RC tree is a leaky resistor tree, which can be viewed as an \( (n+1) \) port resistor network. Let the resistance matrix of the leaky resistor tree be \( R = [r_{ij}] \).

The following properties of the elements of the resistance matrix of the above resistor tree network are required to prove the modified tree algorithm for computing signal delay in a leaky RC tree:

**Lemma 1 [7]**

For any nodes \( i, j \) of a leaky resistor tree and any nonnegative integer \( q \leq n \)

\[
G_{l+1}(i) = G_l(i) + \frac{G_{q+1}^{(p(i))}}{\beta_i} \quad (22)
\]

\[
R_{lj} = \frac{r_{lj}}{\beta_j} \quad (23)
\]

where, the resistances in (22), (23) are elements of the resistance matrix \( R \).

**Corollary 1 [7]**

For any nodes \( i, j \) of a leaky resistor tree and any nonnegative integer \( q \)

\[
R_{lm} = \frac{r_{lm}}{\beta_l} \quad (24)
\]

and

\[
R_{lj} = \frac{r_{lj}}{\beta_j} \quad (25)
\]

where

\[
R_{lj} = \frac{1}{G_T(i)} \quad (26)
\]
and the source at the node \( n+1 \) is connected to the node \( n \) through the source conductance \( g_{n(n+1)} \).

IV Leaky RC Tree

Exploiting the relationship of elements of the resistance matrix of the resistor subnetwork of a leaky RC tree (Corollary 1), we obtain an algorithm for computing the signal delays at all the nodes of the leaky RC tree. This algorithm is a generalization of the tree algorithm of nonleaky RC tree proposed by Penfield[1] for zero initial condition and Lin[3] for nonzero initial conditions.

Theorem 3.2 [7]

The Signal Delay \( T_{Dj} \), at any node \( i \) in a leaky RC tree having a unit step excitation is

\[
T_{0i} = \frac{V_{j}(0)}{G_{R}(j)}
\]

where

\[
R_{ji} = \frac{R_{j}}{a_{r}}
\]

and

\[
C_{i} = \left\{ \begin{array}{ll}
-L & \text{if } j \in P(k) \text{ the leaf nodes} \\
-C_{i} - \frac{C_{j} + \sum_{l=p(k)}^{j-1} C_{l}}{1-\rho(k)} & \text{if } l < P(k) \text{ the internal nodes} \\
-C_{i} & \text{if } j = 0 \text{ the source node} \\
-C_{i} \left[ 1 - \frac{V_{j}(0)}{V_{i}(0)} \right] & \text{if } j \in P(k) \text{ the leaf nodes} \\
C_{i} & \text{if } j = 0 \text{ the source node} \\
\end{array} \right.
\]  

(26)

The computation of signal delay based on (26) is summarised in the following steps called as modified tree algorithm:

Modified Tree Algorithm

1. Compute \( G_{R}(j) \), \( C_{j} \) and \( R_{j} \) from the leaf nodes towards the source node.
2. Compute \( V_{j}(w) \) from the source node towards the leaf nodes.
3. Compute \( C_{j} \) from the leaf nodes towards the source node.
4. Compute \( T_{Dj} \) from the source node towards the leaf nodes.

Note that, initially when all the capacitors are at zero potential, \( V_{j}(0) \) is not required.

In general, a leaky RC tree need not have grounded conductances at all the nodes.

The computational requirement of signal delay in such a circuit can be reduced[7]. Let \( n_{\text{tail}} \) is the number of nodes towards leaf nodes at which grounded conductance are presented. Beyond these nodes there are no grounded[7] conductances. The following special cases can be enumerated

(i) If each node has a grounded conductance, then the total number of multiplications/divisions (\( N_{t} \))

\[
N_{t} = \begin{cases} 
10n - 3n_{\text{tail}} & \text{nonzero initial condition} \\
8n - 3n_{\text{tail}} & \text{zero initial condition}
\end{cases}
\]

(ii) An RC line terminated by a resistor, the

\[
N_{t} = \begin{cases} 
8n - 3 & \text{nonzero initial condition} \\
6n - 3 & \text{zero initial condition}
\end{cases}
\]

V Conclusions

This paper deals with signal delay estimation in leaky RC tree/mesh. Based on a modification of Elmore's delay, a signal delay computation scheme is developed for leaky RC mesh having multiple excitations (nonunit step) and nonzero initial conditions. It is shown that the signal delay depends upon both the intrinsic factors (resistor, capacitor values and initial charge on the capacitors) and the extrinsic factors (Laplace Transform of the voltage excitations). In computing the signal delay final voltages at all the nodes are needed. These can be computed easily when the resistance matrix is known. However, in practice only the conductance matrix of a resistor network is available. It is shown that two sets of simultaneous linear equations are to be solved to obtain the signal delays; one for computing the final voltages and another for signal delays. A sparse matrix techniques can be adopted to solve these equations. Since the system matrix \( G \) is same in both the equations, only once the LU decomposition of \( G \) matrix is done.

Using superposition property of voltage response in a linear circuit, the signal delay at a node due to multiple excitations can be decomposed into those due to single excitations in a leaky RC mesh.

In general, for computing signal delay in a leaky RC mesh, either the resistance matrix is required or two sets of linear equations are required to be solved. However, for a leaky RC tree because of the restrictive nature of the resistor subnetwork
these difficulties can be overcome. First, the properties of mutual resistances i.e. entries in the resistance matrix are established using equivalent conductance between a node and the ground, looking either towards the leaf nodes or towards the source nodes. With the help of these properties a signal delay computation scheme called as Modified tree Algorithm is obtained. The computation method and order of complexity (linear order) is same as that of the tree Algorithm for nonleaky RC tree.

Acknowledgement
The work of the first author is partially supported by the Department of Electronics (National Microelectronics Council) Govt. of India.

References


