Analysis of Diffused Planar and Channel Waveguides

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Abstract—A simple analytical method is presented which improves upon the Hermite-Gauss field proposed by Korotky et al. for diffused waveguides by taking into account the evanescent field in the cover region. The method is used to analyze the propagation characteristics of diffused planar and channel waveguides.

I. INTRODUCTION

The analysis of diffused channel waveguides plays an important role in designing several integrated optical devices, e.g., directional couplers, interference filters, optical switches, etc. These waveguides have a two-dimensional graded index profile and the scalar wave equation for these waveguides does not have closed form field solutions. Therefore, to study the propagation characteristics of such waveguides, either direct numerical methods or approximate methods are used. Although, the numerical methods based on finite difference [1] or finite element [2] methods are capable of giving high accuracy; these involve extensive computations and do not lead to simple analytical forms for the modal fields. This has led to the development of a number of approximate methods [3]-[5], which are based on the variational principle. Accuracy of the methods based on variational principle depends on the closeness of the assumed trial field to the exact modal field of the guiding structure. The three approximate methods [3]-[5] basically differ only in the form of the trial modal field and in all these methods, it has been assumed that the scalar modal field is separable in its dependence along the width and the depth. We have recently developed a numerical method [6] which is based on the variational principle and gives the best scalar results under the assumption of separable fields.

Of the three approximations, the most commonly used and the simplest is the one developed by Korotky et al. [4]. The field is assumed to be Gaussian along the width and the first-order Hermite-Gaussian along the depth with the field in the cover totally neglected. The only two parameters involved are the widths of the field along these two directions. However, the accuracy of this model is limited due to the assumption of vanishing field in the cover region. In the present paper, we give a simple analytical modification of this model to improve on this aspect at the same time maintaining the simplicity of the model. This modification reduces the error in the normalized propagation constant at least by an order of magnitude and improves considerably the shape of the field, particularly in the cover region. In order to illustrate the principle of our modification, we first apply it to asymmetric diffused planar waveguides in Section II and, then, discuss its application to diffused channel waveguides in Section III.

II. DIFFUSED PLANAR WAVEGUIDES

We consider asymmetric graded index planar waveguides with a refractive index distribution given by

\[ n^2(y) = n_s^2 + 2n_r A_n g(y) \quad y > 0 \]
\[ = n_s^2 \quad y < 0 \]

(1)

where \( n_s \) is the substrate index, \( n_r \) is the cover index, \( A_n \) is the peak index change, and \( g(y) \) is profile function. We have considered three types of profile functions

\[ g(y) = \begin{cases} \exp(-y/D) & \text{if } y > 0 \\ \exp(-(y^2/D^2)) & \text{if } y < 0 \\ \text{erfc}(y/D) & \text{if } y = 0 \end{cases} \]

(2)

where \( D \) is the depth of diffusion.

The stationary variational expression for the propagation constant \( \beta \) of the fundamental mode of the waveguide is given by

\[ \beta^2 = \frac{\int \int \frac{\exp(-y/D)(\nabla \psi)^2}{\psi^2} dy}{\int \int \frac{\nabla \psi^2}{\psi^2} dy} \]

(3)

where \( \langle \psi, \nabla \psi \rangle \), is the variational trial field assumed for the modal field of the waveguide. It contains a certain number of variational parameters with respect to which the expression for \( \beta^2 \) is maximized. The higher the number of variational parameters, the better are the results, but only at the cost of increased computational complexity. In terms of the dimensionless parameters, (3) can be written as

\[ \beta^2 = \frac{\int \int_\Omega (n_s^2 + 2n_r A_n g(y)) (\nabla \psi)^2 \psi^2 dy}{\int \int_\Omega (\nabla \psi^2)^2 \psi^2 dy} \]

(4)

where \( n_s^2 = n_r^2 + 2n_r A_n \) is the maximum refractive index of the waveguide which occurs at the surface of the waveguide \( y = 0 \). \( U = D(k_0 h)^{1/2} \) is the normalized propagation constant and the standard dimensionless normalized propagation constant is obtained from

\[ b = 1 - U^2 \lambda^2 \]

(5)

where \( V = f_0 D V(2n_r A_n) \) represents the normalized frequency.
One of the trial fields proposed for such waveguides [4, 7] is the Hermite-Gauss (HG) field which contains only one variational parameter and is given by

$$\phi(y) = (y/d) \exp(-y^2/2d^2) \quad y \geq 0$$

$$= 0 \quad y \leq 0 \quad (6)$$

This model is uncomplicated and requires comparatively less computational effort for obtaining the propagation constant. However, it neglects field in the cover region and therefore the accuracy may be improved.

Another trial field is the cosine-exponential (CE) model proposed by Mishra and Sharma [7]. This model gives much better results than HG model but trial field is relatively more complex and contains three variational parameters; hence, computational effort involved is relatively larger.

We propose a modified form of Hermite-Gauss trial field which does not neglect the field in the cover region and still contains only one variational parameter. The method involves two steps. The first step is exactly the same as the Hermite-Gauss method which involves minimization of $U^2$ in (4) with respect to $d$ appearing in (6). This gives a maximum value of $\beta$ and the corresponding value of $d$. The second step involves construction of a new trial field based on these values. The philosophy of our method is as follows. If we consider a symmetric waveguide such that it matches with the profile given in (1) in the $y > 0$ region, the HG field in the $y > 0$ region closely represents its first antisymmetric mode. We can also construct the corresponding first symmetric mode as $\exp(-y^2/2d^2)$. A linear combination of these two modes can be made in such a way that the resultant field in the $y > 0$ region has a single maxima and a nonvanishing field at $y = 0$, while the field in the $y < 0$ region is substantially small though it would not decay exponentially. Now, replacing the field in the $y < 0$ region by an evanescent field and using the continuity conditions at $y = 0$, we obtain the evanescent Hermite-Gauss (EHG) field given as:

$$\psi(y) = a \exp(-y^2/2d^2) \quad y \geq 0$$

$$= a \exp([W, y]) \quad y \leq 0 \quad (7)$$

where $W = \sqrt{V(0^2 - k^2)}$, $j^3$ being obtained in the first step using the HG field. This new trial field is then substituted in (4) to obtain a new value of $U$ (and hence of $\beta$) without any optimization. The value of $\beta$ thus obtained is much more accurate than the value obtained using the HG method. The results obtained using the evanescent Hermite-Gauss (EHG) method for various profiles given in (2) are given in Table I which also contain exact [7, 8] results as well as results obtained using other methods for comparison. The improvement using our technique is evident from this table. It should be noted that the cosine-exponential (CE) method [7] involves three variational parameters with respect to which $U^2$ must be minimized, and, therefore, it requires considerable amount of computational effort in comparison to the present method. The expression for $U^2$ for different profiles using the EHG field are given in Appendix A.

### III. DIFFUSED CHANNEL WAVEGUIDES

The method described in Section II can be directly used for the analysis of the 3-D diffused channel waveguides. The refractive index distribution of such waveguides is given by

$$n(x, y) = n_2 + 2n_c \text{Anf}(x)g(y) \quad y > 0$$

$$= n^2_c \quad y < 0 \quad (8)$$

where $\text{Anf}(x)$ and $g(y)$ are the profile functions along the width and the depth, respectively. A number of different profile functions have been used in the literature, but we will illustrate our method for the one used by Korotky et al. [4]:

$$\text{erf} \left( \frac{x + W/2}{\sqrt{2D}} \right) - \text{erf} \left( \frac{x - W/2}{\sqrt{2D}} \right)$$

$$= \frac{2}{\sqrt{\pi}} \exp \left( -\frac{x^2}{2D} \right)$$

$$g(y) = \exp \left( -\frac{y^2}{2d^2} \right) \cdot \text{erf} \left( \frac{x - W/2}{\sqrt{2D}} \right)$$

$D$ is the depth of diffusion and $W$ is the initial metal strip width. In the first step, separable trial field is assumed to be Gaussian in the symmetric $x$ direction and Hermite-Gauss in the asymmetric $y$ direction [4]:

$$\psi(x, y) = \psi(x) \phi(y)$$

$$\psi(x) = 1/\sqrt{\pi w/2} \exp(-x^2/w^2)$$

$$\phi(y) = 2/\sqrt{\pi d} \exp(-y^2/2d^2)$$

Stationary variational expression is given by

$$V^2 = \left( \frac{V^2}{2} \right) \left[ \int \phi^*(y) \phi(y) - \phi(x, y) \right] dx \cdot dy + \int \psi^*(x) \psi(x) dx \cdot dy$$

$$\phi(x, y) = \phi(x) \phi(y)$$

$$\psi(x) = 1/\sqrt{\pi w/2} \exp(-x^2/w^2)$$

$$\phi(y) = 2/\sqrt{\pi d} \exp(-y^2/2d^2)$$
Fig. 1. Normalized propagation constant \( b \) as a function of the normalized frequency \( V \) for a diffused channel waveguide with waveguide parameters: \( n_2 = 2.203, n_1 = 1.0, X = 1.3 \) \( \text{um} \), \( D = 3.35 \) \( \text{pm} \), and \( W = 6.0 \) \( \text{nm} \). Metal strip thickness \( T \) is varied to change \( V \). Dashed curve corresponds to [6], solid curve to the EHG trial field and dash-dot curve corresponds to the HG trial field.

Trial field in (11) is substituted in (12) and the expression is minimized with respect to variations in parameters \( w \) and \( d \). Optimized values of \( w, d, \) and \( /3 \) are obtained. In second step of optimization, field in \( y \) direction is modified to

\[
0/(y) = A/\chi + W/(y) \exp \left(-y^2/2d^2\right) \quad y \geq 0
\]

\[= A \exp (W/(y)) \quad y \leq 0 \quad (13)\]

where \( W = V/(3/2 - k/d) \), \( /3 \) being obtained above. \( y/\chi(x) \) remains unchanged. \( A \) is the normalization constant. The EHG field has no additional adjustable parameters and hence, when used in the stationary expression, gives directly a much improved value of \( /3 \) without minimization. Expressions for the EHG field are given in Appendix B.

For numerical illustration, we consider waveguides [4] with \( n_2 = 2.203, X = 1.3 \) \( \text{um} \), \( n_1 = 1.0 \) and

1) \( D = 3.35 \) \( \text{um} \), \( W = 6.0 \) \( \text{um} \), with thickness of metal strip \( T \) varied to change \( V \).

2) \( D = 5.08 \) \( \text{pm} \). \( T = 720 \) \( \text{A} \), with \( W \) varied to change \( V \).

The values of \( b \) for some values of strength parameters \( S \), are given in Table II where \( S \), is defined as [4]:

\[5, = 2n, dn/dc \ W Kl\]

and is related to \( V \) as

\[ V^2 = S,/(W/4D)/y/(2/\pi) \ \text{erf} (W/2n/2 \ D)\]

The variation of \( b \) versus \( K \) is given in Figs. 1 and 2 which show very clearly that application of present EHG trial field results in values very close to those of [6], in which field is obtained numerically in both \( x \) and \( y \) directions. Thus, we obtain a trial field which yields much better results for diffused channel waveguides without increasing any computational efforts.

IV. SUMMARY AND DISCUSSION

We have presented a simple analytical method which improves upon the Hermite-Gauss field proposed by Kotoky et al. [4] for diffused waveguides by taking into account the evanescent field in the cover region. The method is applied to analyze the propagation characteristics of diffused planar and channel waveguides. Results obtained by using the EHG field are much better as compared to HG field [4].

We would like to add here, that although it is possible to choose in the first place a trial function with an evanescent field in the cover of the asymmetric diffused waveguide, it increases the number of variational parameters from one to two. Furthermore, the results obtained by using such a trial function showed hardly any noticeable improvement as compared to the one in (7). Thus, the results did not improve significantly whereas the computational
effort was increased manifold. Another improvement in the chosen field could be to use an exponential tail to account for the slow variation of the field into the substrate. However, one has to choose judiciously the point where the change-over to the exponential tail has to begin. The choice of this transition-point depends on the \( V \) number of the waveguide and is useful only when the mode is near cutoff. Therefore, it has to be treated as an additional variational parameter and makes the field quite complicated, particularly for a diffused channel waveguide. Further, the integrations involved have to be evaluated numerically in all the cases. The results do improve marginally, particularly for a near cutoff mode, but at the cost of significant addition in the computational effort. Our aim in this paper has been to present a method which improves the results without any significant increase in the computational effort or complexity of the modal trial field.

APPENDIX A

Expression for \( U^2 \) for planar diffused waveguides using EHG trial field:

\[
U^2 = \frac{V^2 e^{2W_C} + V^4 \text{Int} - V^2/4 + \text{Int}}{IN}
\]

where

\[
V_C = \text{ftbDV}(ng - \omega)
\]

\[
I_D = \frac{\text{det}}{2(1 + W^2_d/2 + 2W_d/d^2 K)}
\]

\[
I_I = D^2 W_d^2 + \frac{\pi}{2} W_7^2/2(l/2 < t^2 + 3W_d^2/4)
\]

\[
I_N = I_D + l/2W_C
\]

1) For a Gaussian profile:

\[
I_N = \frac{1}{l/2} q (1 + W^2_d/2q^2 - 1W_dq^2/lr)
\]

where \( q = V(l/2D_C - \sqrt{d^2}) \)

2) For an exponential profile:

\[
I_N = -1/2D_C \left( 1 - \text{erf}(2/d2D) \right)
\]

\[
\left[ 2 - W^2_d d^2/4 + d^2/2D^2 \right] - 2W_d d^2/4 W_d d^2/4 + W_d d^2
\]

3) For a complementary error function profile:

\[
I_N = \frac{d}{\sqrt{\pi}} \left[ \text{tan}^{-1} (D/d) + W^2 d^2/2Vir \right]
\]

\[
\left[ \text{tan}^{-1} (D/d) - (d/D) d^2/4 \right] + W^2 d^2/4 (d^2/4)
\]

APPENDIX B

Expressions obtained using EHG trial field for diffused channel waveguides:

\[
U^2 = \frac{W^2 d^2}{2w^2} - W^2 \text{Int}/4IN + V^2 \left[ 1 - \text{Int}/IN \right]
\]

where

\[
I_N = l/2W_C + dV_7/2(1 + W^2_d/2 + 2W_d/d^2 - Jv)
\]

\[
I_I = W^2_d/2 - V_7 t/4^2 - W^2_C dV_7/8
\]

\[
I_G = (j_j c - \rho^2 AW^2 An + V^2/2\text{Int}) \left[ 1 + W^2 d^2/4 + 2W_d/d^2 K \right] \text{ where } r = s(1/d^2 + 1/2D^2)
\]

\[
I_c = \text{erf}(1/V/(w/W)^2) + \text{erf}(W/2-j2D)
\]