A MODEL-BASED SLIDING MODE CONTROLLER FOR ROBOT MANIPULATORS

M.HANMANDLU
Department of Electrical Engineering
Indian Institute of Technology, Delhi
New Delhi 110 016, India

S.R.PANDIAN
Department of Mechanical Engineering
Ritsumeikan University, Kyoto
Kyoto 603, Japan

ABSTRACT

A generalized model-based sliding mode controller for robot trajectory tracking is proposed, which incorporates the full-order, nonlinear, uncertain actuator dynamics in control law design. The problem of chattering is alleviated by replacing the discontinuous sliding mode control in the vicinity of the sliding mode by a high gain, continuous control. Independent joint control, dual mode control in the vicinity of the sliding mode by a high gain, continuous control, and online feedforward control laws are derived. Simulation results are presented for the first three links of the PUMA 560 arm to illustrate their effectiveness.

1 Introduction

A well-known approach to the control of interconnected systems is to consider the interactions between subsystems as disturbances, and to reject their effects by disturbance rejection methods [1]. In this paper, we develop model-based sliding mode controllers based on the above concept. We treat the stable (or locally stabilized) isolated subsystem as a model for the interconnected system, so that the stabilization of the latter becomes a problem of model following.

Robot control methods in literature are mostly based on the second-order manipulator dynamics, possibly combined with the second-order motor dynamics. However, the importance of full-order, nonlinear, uncertain actuator dynamics has recently been recognized by researchers, in the case of direct-drive robots, hydraulically actuated robots modeled by third- or fifth-order systems, pneumatically actuated robots modeled by high-order nonlinear dynamics, flexible robots, as well as indirect-drive robots in high-speed, high-payload applications (e.g., [2], [3]).

Here, we extend the model-based tracking control method to the case of incorporating the full-order, nonlinear actuator dynamics, and develop a class of sliding mode controllers with a built-in trade-off between simplicity of implementation and precision of tracking. The continuous control case of the proposed method is treated in [4], and is extended to the adaptive case in [5].

2 Generalized Model-based Robot Control

An JV-joint rigid manipulator with each joint powered by an actuator can be considered an interconnection of N actuator subsystems interacting through the manipulator link dynamics:

\[ \dot{x}_i = A_i x_i + b_i u_i + d_i \tau_i, \quad i = 1, \ldots, N \]  \hspace{1cm} (1)

where \( A_i \) is \( n_i \times n_i \), \( b_i \); and \( d_i \) are \( n_i \times 1 \), and \( (A_i, b_i) \) is a controllable pair. For a general actuator, the order of system (1) is \( n_r \geq 2 \). Extension to the case of nonlinear, time-variant actuator dynamics is straightforward. In (1), \( \tau \) is the torque at the i-th joint, given by the i-th equation of the Lagrange-Euler dynamics,

\[ \tau = H(q)\dot{q} + h(q, \dot{q}) + g(q) \]  \hspace{1cm} (2)

For the trajectory tracking problem, there exists a nominal control \( U(t) \in [0, T] \), such that the isolated subsystem in (1) tracks the nominal state trajectory \( x_i \). That is

\[ \dot{x}_i = A_i x_i + b_i u_i \]  \hspace{1cm} (3)

The isolated actuator subsystem is the model for the interconnected subsystem, so that the interactions are treated as a disturbance. Denoting the state tracking error as \( \epsilon = x - x_i \); the error dynamics is obtained from (1) and (3) as

\[ \dot{\epsilon} = A_i \epsilon + b_i u_i + d_i \tau_i \]  \hspace{1cm} (4)

where \( u_i = a_i \hat{q}_i + \tau_i \). \( u_i \) is a motor model-based control to help track the nominal trajectories, and \( \hat{q}_i \) is an error feedback control to compensate the effects of manipulator dynamics, parametric uncertainties, and payload variations.

The state vector \( \epsilon \) can be made invariant with respect to the disturbance \( \tau_i \) if the rank condition \( \text{rank}(b_i) = \text{rank}(b_i : d_i) \) is satisfied. Else we introduce a coordinate transformation to satisfy this condition [1]. To simplify the discussion, here we assume that \( (4) \) satisfies the invariance condition.

In the independent joint control (IJC) scheme, we choose the feedback law as

\[ f_i = k_i \epsilon_i. \]  \hspace{1cm} (5)

Here, neglecting \( T \); requires high feedback gains. So, in the offline feedforward compensation (OFF) scheme, tracking is improved along with a reduction in gains by augmenting the error feedback control (5) with offline feedforward compensation of interactions as

\[ X_i = A_i X_i + b_i(u_i + \tau_i) \]  \hspace{1cm} (6)

where \( t_i \) is calculated along the desired trajectories, using nominal parameter values. Tracking performance can be improved further, if the feedforward compensation is made on-line, then \( t_i = U^* + U \hat{q}_i \); \( \hat{q}_i = \hat{u}_i + \tau \) with \( \hat{u}_i = -f_i(x, \dot{x}_i) \). \( u_i \) is given by (3) and \( \tau \) has the same form as (5). This online feedforward compensation (ONF) law requires smaller gains than the OFF scheme, though at the cost of increased online computation.
In view of the trade-off between simplicity of control law implementation and precision of tracking, any of the three controller configurations above may be chosen, depending on the complexity of the tracking task and significance of actuator dynamics.

3 Dual Mode Control Law

We choose the linear, time-invariant sliding hyperplane

\[ \sigma(\xi) = \sigma^0(\xi) \]  

The disturbance \( T(X,X) \) in (4) is a global state and parameter dependent nonlinearity. So, for simplicity, we treat it as a piecewise-continuous, time-variant bounded disturbance. The error dynamics accordingly can be controlled by a variable structure control (VSC) law with a relay component as

\[ \ddot{\eta}_i = \sum_{j=1}^{n_b} \Psi_{ij} \eta_j - \Omega_i \sigma(\xi_i) \]  

where the error feedback gains \( \Psi_{ij} \) are switched according to

\[ \Psi_{ij} = \begin{cases} \alpha & \text{if } s_i > 0 \\ \beta & \text{if } s_i < 0 \\ 0 & \text{if } s_i = 0 \end{cases} \]  

The condition for the existence of a sliding mode on the hyperplane is given by \( s, \dot{s} < 0 \). Accordingly, the switching gains are selected to satisfy the following conditions:

\[ \sigma(\xi_i^{(1)}, \xi_i^{(2)}) \alpha_i > \| (\xi_i^{(1)})^{-1} \| \| \xi_i \| \]  

\[ \sigma(\xi_i^{(1)}, \xi_i^{(2)}) \beta_i < \| (\xi_i^{(1)})^{+} \| \| \xi_i \| \]  

\[ \sigma(\xi_i^{(1)}, \xi_i^{(2)}) \| A_i \| \| \xi_i \| > \| \eta_i \| \]  

\[ |\eta_i|\] is the upper bound on \( r_i \) and \( A_i^j \) is the \( j \)-th column of \( A_i \).

To minimize chattering and yet achieve effective tracking, we specify a dual mode control law in the form

\[ \ddot{\eta}_i = \sum_{j=1}^{n_b} \Psi_{ij} \eta_j - \Omega_i \sigma(\xi_i) + g_i \frac{d}{dt} \eta_i - k_i \dot{\eta}_i \]  

where \( g_i \) is a large scale factor, \( k_i \) is the integral feedback gain, and \( \xi_i \) is the sliding vector chosen as the proportional gain vector.

4 Simulation Results

The utility of the new sliding mode control algorithms is illustrated for the first three links of the PUMA 560 manipulator. To study the robustness of the control to parametric uncertainties and payload variations, a payload of 2.3 kg and a 10% increase in the link-3 moment of inertia are assumed, and offline and online compensation are based on the unloaded arm dynamics.

For a cycloidal reference trajectory, the position errors are shown in Figure 1. The nature of the control action can be inferred from Figure 2: the nominal control action accounts for a major part of total control (due mainly to gearing), and the chattering error control accounts for the error dynamics. Figure 3 shows the effectiveness of the sliding mode control in comparison with high gain control. Finally, from Figure 4 the dual mode control is shown to result in a trade-off between precision of tracking and minimization of chattering.

5 Conclusions

A new class of model-based sliding mode controllers is proposed, incorporating the full-order, nonlinear uncertain actuator dynamics in control law design. There is a trade-off between accuracy of tracking and complexity of implementation, and dual mode control is used to reduce chattering.

6 References