Excitation of Slow EM Waves in a Waveguide Having a Thin, Annular Plasma Sheet

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Abstract—A thin, annular plasma sheet in a waveguide slows down the phase velocity of electromagnetic modes in a plasma density window for frequencies below the electron cyclotron frequency via Cerenkov interaction. At lower densities, the plasma is not effective enough to reduce the phase velocity of the EM waves below c. At higher densities the plasma expels the radiation field leading to enhancement of phase velocity. The lowest mode having azimuthal number 1 = 0 is most unstable. The radial mode having amplitude maximum at r = a, viz the plasma boundary has the largest growth rate.

I. INTRODUCTION

Kuzelev et al. [1]-[3] have proposed an attractive configuration for the generation of microwaves (at 10 GHz) by relativistic electron beams. The interaction region is a cylindrical waveguide containing an annular plasma sheet produced by a low energy electron beam. The plasma is immersed in a strong axial magnetic field. The radius of the waveguide is 1.5 cm, the radius of the plasma is 0.7 cm, and the thickness of the plasma is 1 mm. This configuration supports electromagnetic waves with phase velocity slightly lower than c. When an annular electron beam propagates axially in the vicinity of the plasma, the slow mode is driven unstable, producing microwave radiation at a spectacularly high efficiency of 35%. Kuzelev et al. have given an elegant theory of slow waves supported by such a structure. In this paper, we present a simpler version of the theory modeling plasma thickness to be zero. We also assume an annular relativistic electron beam to propagate at the site of the plasma. The beam drives slow mode via Cerenkov interaction. It may be mentioned that recently Carmel et al. [4] have reported some stimulating results on the efficiency enhancement of a backward wave oscillator by introducing a plasma. At some specific plasma frequency, the efficiency of the device is dramatically enhanced from 5% to 35%. Others have reported efficient excitation of EM waves from a variety of plasma microwave sources [5], [7]. The plasma provides partial current and charge neutralization of the beam, raising the current density and density perturbation due to the plasma can be written as

$$J_s = \frac{-\omega_p^2}{\mu_0} E_z$$

(1)

$$n = \frac{me\omega_p^2}{\pi^2} E_z$$

(2)

For azimuthally symmetric modes, the wave equation for TM waves can be written as

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r} \frac{\partial E_z}{\partial r} - \alpha^2 E_z = \frac{-\omega_p^2}{\mu_0} a^2 \partial \phi(r - a)$$

(3)

where \(\alpha^2 = k^2 - \omega^2/c^2\) and \(\omega_p = (4\pi \rho_e/\mu_0)^{1/2}\) is the plasma frequency. The solutions in region I \((0 < r < a)\) and II \((a < r < b)\) are

$$E_I = C J_{1/2}(ar)$$

$$E_{II} = D J_{1/2}(ar) + D K_{1/2}(ar).$$

(4)

Multiplying (3) by \(rdr\) and integrating from \(a - \epsilon\) to \(a + \epsilon\) where \(\epsilon \to 0\), we obtain a jump condition across the plasma sheet,

$$\frac{\partial E_z^{II}}{\partial r} \bigg|_{r=a+\epsilon} - \frac{dE_I}{\partial r} \bigg|_{r=a-\epsilon} = \frac{-\omega_p^2}{\mu_0} a^2 \alpha^2 E_z^{II}(a).$$

(5)

Applying the continuity condition \(E_I = E_{II}^{ab}\) at \(r = a\) and the boundary condition \(E^s = 0\) at \(r = b\) in conjunction with the jump condition (5) we obtain a dispersion relation:

$$r_s(aa) - \frac{r_s(aa)K_s(ab) - L_s(ab)K_s(aa)}{L_s(aa) - \frac{r_s(aa)K_s(ab) - L_s(ab)K_s(aa)}{L_s(aa)}} = \frac{\omega_p^2}{\mu_0} a.$$

(6)
We have solved (6) numerically. We plot LHS as a function of $a a$ for $\ell = 0$ mode. We also plot the RHS as a function of $a a$ for values of $A = \frac{w_{p0}}{w^2}$ varying from 1 to 6. The points of intersection of the two curves give different values of $a a$ which satisfy the dispersion relation. This is shown in Fig. 2. In terms of $a a$ and $a a p_{0} = \frac{w_{p0}}{c}$, the dimensionalized wavenumber $K = k a$ can be written as

$$K^2 = (a a)^2 + \frac{\omega^2}{\Delta}. \tag{7}$$

Since $Q = \omega/c = \frac{w_{p0}}{\omega}(A)^{\frac{1}{2}}$ (by definition of $A$), one can obtain $Q$ vs. $K$ plot for different $p_{0}$ values as shown in Fig. 3.

### III. INSTABILITY ANALYSIS

Now we examine the role played by the beam. The current and density perturbation due to the beam electrons can be obtained as

$$J_{b_2} = -\frac{n_{a}e^{2}c_{g}}{m_{e}c^{2}(U - kv_{a})E_{z}}, \tag{8}$$

$$\zeta_{b_2} = \frac{n_{a}ek_{E_z}}{m_{e}c^{2}h_{a} - kv_{a}}. \tag{9}$$

Using (1), (2), and (8), (9) in the wave equation, we obtain

$$\frac{d\tilde{e}_{z}}{dr} + \frac{1}{r} \frac{de_{z}}{dr} - \alpha^2 \tilde{e}_{z} - \left[\frac{4\pi e^2}{\omega^2} \zeta + \frac{2}{7} \frac{e^2}{(w - kv_{a})^2}\right] a \delta(r - a) \tilde{e}_{z} \tag{10}$$

Expanding $D_2(u >, k)$ in Taylor series around $(w, k)$, we have

$$D_2(u >, k) = D_2(u >, k) + \frac{\partial D_2}{\partial u_{>}} \bigg|_{u_{>}} \delta,$$

where $u > p_{0} = \left(\frac{8}{3} w_{a0} e^{2}/m_{a}\right)^{\frac{1}{2}}$ is the beam plasma frequency. We multiply (10) by $r dr$ and integrate across the plasma sheet to obtain a jump condition

$$\frac{\partial E_{z}}{\partial r} \bigg|_{r=a+e} - \frac{\partial E_{z}}{\partial r} \bigg|_{r=a-e} = -\left[\frac{v_{>}}{w^2} \zeta + \frac{\alpha^2}{7} \frac{e^2}{(w - kv_{a})^2}\right] \tilde{e}_{z}. \tag{11}$$

Applying boundary conditions and jump condition (11), we obtain

$$D_2(u >, k) \frac{\omega_{p0} a a}{\gamma_{0}^2} \int_{\gamma_{0}}^{\infty} (u - kv_{a}) \tilde{e}_{z} \tag{12}$$

where

$$D_2(u >, k) = D_2(u >, k) - \frac{\omega_{p0} a a}{\gamma_{0}^2}.$$

For maximum growth rate of the unstable mode, the LHS of (12) must be zero simultaneously. We call this resonant frequency $u_{> r}$. Let there be a slight frequency mismatch due to finite beam density $(u|u > p_{0} > 0)$

$$w = kv_{a} + 6$$

where $D_2(u >, k) > 0$

Expanding $D_2(u >, k)$ in Taylor series around $(w, k)$, we have

$$D_2(u >, k) - D_2(u >, k) + \frac{\partial D_2}{\partial u_{>}} \bigg|_{u_{>}} \delta \sim \frac{8}{\gamma_{0}}.$$
Using these, the growth rate of the unstable mode can be expressed as

$$\gamma = \frac{1}{3} \left( \frac{\omega_{pe}^2 a a}{\omega_{pe}^2} \right)^{1/3}$$

(13)

where

$$\left. \frac{dD_2}{dm} \right|_{\omega_0} = \left. \frac{\partial D_1}{\partial \omega} \right|_{\omega_0} + \frac{2\omega_{pe}^2 a}{k^2 v_{ob}^2 \gamma_0} + \frac{\omega_{pe}^2 a}{k^2 v_{ob} c} \gamma_0$$

$$\approx \frac{\gamma_0 v_{ob}}{c^2} \left. \frac{dD_1}{da} \right|_{a=0, \gamma_0, v_{ob} = \omega_0} + \frac{2\omega_{pe}^2 a}{k^2 v_{ob}^2 \gamma_0}$$

The resonant frequency $C_{\rho} \equiv \gamma$ and the corresponding wave mode $K$ can be obtained by the intersection of beam mode with the slow EM mode from Fig. 3. This gives the resonant value of $\omega_{a,c} \approx k^2 a$. The slope in Fig. 2 at $a = a$, determines the term $k^2 a$. For beam current $J_1$, $\gamma_0 \sim 1$ KA, gamma factor $\gamma_0 \sim 2$, $v_{ob} \sim 2$ and $a \sim 2$ cm, we have shown the growth rate and the resonant frequency as a function of $r_{2\rho}$ in Figs. 4 and 5. Variation of growth rate with resonant frequency for $Q_{\rho} = 3$ is shown in Fig. 6. Fig. 7 shows mode structure as a function of $r/a$ for the fundamental mode.

IV. DISCUSSION

A waveguide having a thin, annular plasma sheet supports slow EM modes at frequencies $\omega < c$ with $c$. A moderately relativistic electron beam excites them via cerenkov instability. In this configuration, plasma density and beam energy are seen to be important parameters. From Fig. 4, we see that for $4 \sim 1$ KA, $\gamma_0 \approx 0.85$, a change in $r_{2\rho}$ value from 2 to 4 reduces normalized growth rate $7a/c$ from 0.84 to 0.5. This is accompanied by rather drastic modification in the operating frequency $il$ as seen from Figs. 3 and 5. Experiments have reported a plasma density threshold below which no radiation is emitted [1]. This is confirmed from our theory and is depicted in Fig. 3. It may be added that above a certain plasma density, the mode acquires phase velocity greater than $c$, hence cannot be driven unstable via cerenkov interaction. Another parameter of interest is the beam energy $\gamma_0$. From Fig. 3, it is obvious that by increasing beam energy, the operating frequency of the device also increases. Further, plasma density threshold also increases by increasing beam energy.

Fig. 6 shows behavior of $\gamma a/c$ vs. operating frequency $\gamma t$ for $Q_{\rho} = 3$. A higher growth rate is obtained at high $il$, indicating that the device is promising for generating high frequency radiation. $l$ denotes the azimuthal wave number. We find that the lowest mode with $l = 0$ is most unstable.

The configuration over here should be relevant to a short wavelength free electron laser, where a plasma supported em mode could act as a wiggler. Using an electron beam of velocity $\gamma l$, greater than the phase velocity of the slow mode and smaller than the phase velocity of the FEL mode one can operate the device in an explosive mode [10], producing intense short laser pulses.

It may be added that the choice of infinitesimally thin layers for beam and plasma has been made to model the actual experiment [1] where plasma and beam thickness are much smaller than the scalelength of field variations. Incorporating finite thickness effects, we write down the solution of (3) in three regions:

i) Region I ($0 < r < r_0$).

ii) Region II ($r_0 < r < a$), and

iii) Region III ($a < r < b$), where a plasma lies within Region II.

Matching the appropriate boundary conditions at $r = r_0$, $r =$
\( r_o + a \) and \( r = b \) one obtains a dispersion relation. However, the interaction process remains essentially the same. Further, one must examine the process of space charge neutralization which remains an outstanding problem.

Typical parameters in an experiment involving injection of a solid relativistic electron beam in a plasma cylinder are: beam current \( I \approx 0.9 \) KA, beam energy \( E \approx 480 \) Kev, beam radius \( A_o \approx 0.5 \) cm, plasma density \( n_p \approx 5 \times 10^{12} \) - \( 5 \times 10^{13} \) cm\(^{-3}\).

In the actual experiment the preliminary plasma is produced by a low energy electron beam (Energy \( E \approx 0.5 \) Kev, current \( h \approx 10^4 \)). In addition a strong external guide magnetic field \( B_0 > 30 \) kG along \( z \) direction confines the plasma radially.

\[ B_0 > 30 \text{kG} \]

It may be further added that two stream instability is primarily the excitation of electrostatic waves via relative motion of the beam electrons with the plasma electrons. However, in a cerenkov interaction, the electromagnetic mode slowed down by the plasma or a dielectric couples with the beam mode. The basic difference in the two is the electromagnetic effects.

**REFERENCES**


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