Modeling and optimal design of ground air collector for heating in controlled environment greenhouse

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Abstract

A mathematical model is devised to study the thermal behavior of a greenhouse while heating with a ground air collector (GAC). A computer program based on MatLab software has been used to predict the plant and room temperatures as a function of various design parameters of the ground air collector. Extensive experiments have been conducted during December 2000 to March 2001 for an even span greenhouse of effective floor area of 24 m$^2$ with a GAC and having a brick north wall. The model was validated experimentally in the climate of Delhi for the winter season. A parametric study involves the area of the GAC, mass flow rate and heat capacity. The predicted plant and room temperatures show fair agreement with the experimental values.

Keywords: Greenhouse; Controlled environment; Solar energy; Ground air collector

1. Introduction

A greenhouse is essentially an enclosed structure, which traps short wavelength solar radiation and stores long wavelength thermal radiation to create a favorable micro-climate for higher productivity. Thus, greenhouses are known as controlled environment greenhouses.

Heating and cooling of agricultural greenhouses are the utmost important activities to extend its application throughout the year for crop production. A controlled environment greenhouse can be thermally heated by various methods. Connellan [1] used warm water as a heat source to
Nomenclature

\[ A \quad \text{area (m}^2) \]
\[ C \quad \text{specific heat (J/kg °C)} \]
\[ e \quad \text{root mean square of percent deviation (\%)} \]
\[ F \quad \text{fraction of solar radiation} \]
\[ FR \quad \text{heat removal factor for ground collector} \]
\[ h \quad \text{radiative and convective heat transfer coefficient (W/m}^2 \text{ °C)} \]
\[ I \quad \text{solar radiation (W/m}^2 \) \]
\[ K \quad \text{thermal conductivity (W/m°C)} \]
\[ I \quad \text{thickness (m)} \]
\[ \dot{m}_A \quad \text{mass flow rate of air in ground collector (kg/s)} \]
\[ M \quad \text{mass (kg)} \]
\[ N \quad \text{number of air changes per charge} \]
\[ q \quad \text{heat transfer flux (W/m}^2 \) \]
\[ q_u \quad \text{rate of useful thermal energy supplied by ground collector to greenhouse (W)} \]
\[ r \quad \text{coefficient of correlation} \]
\[ T \quad \text{temperature (°C)} \]
\[ U \quad \text{over all heat loss or gain (W/m}^2 \text{ °C)} \]
\[ v \quad \text{wind velocity (m/s)} \]
\[ V \quad \text{volume of greenhouse (m}^3 \) \]

Greek letters

\[ \alpha \quad \text{absorptivity} \]
\[ \tau \quad \text{transmissivity} \]
\[ \rho \quad \text{reflectivity} \]
\[ \rho o \quad \text{transmittance-absorptance product} \]

Subscripts

\[ a \quad \text{air or ambient air} \]
\[ b \quad \text{bottom of ground air collector} \]
\[ c \quad \text{ground air collector} \]
\[ g \quad \text{ground or floor of greenhouse} \]
\[ h \quad \text{horizontal surface} \]
\[ i \quad \text{number of walls and roofs of greenhouse (1, 2, \ldots, 6)} \]
\[ n \quad \text{north wall} \]
\[ o \quad \text{wind} \]
\[ p \quad \text{plants} \]
\[ r \quad \text{room air} \]
\[ t \quad \text{top of ground air collector} \]
\[ y=0 \quad \text{surface of floor of greenhouse} \]
\[ z=0 \quad \text{surface of north wall} \]
\[ oo \quad \text{ground at higher depth} \]
heat the greenhouse. Grafiadallis [2] used water as a storage medium and found a 5 °C difference of temperature. Al-Amri [3] studied the effect of a solar water heater for greenhouse heating. Blackwell and Garzoli [4] studied a rock bed storage for greenhouse heating. Santamouris et al. [5] tested the performance of the north wall and reported that the temperature inside the greenhouse was 10 °C higher than the ambient. Kurata and Tatakura [6] studied underground storage. A buried plastic pipe transfers the stored heat of the ground inside the greenhouse. Santamouris et al. [7] found the heating effect by using the buried pipe in the ground. Singh and Tiwari [8] studied the effect of a thermal storage north wall integrated with a ground air collector (GAC) on the plant and room temperatures and reported that the temperature inside the greenhouse was 20 °C higher than the ambient during the day time.

In this study, an even span greenhouse of effective floor area of 24 m$^2$ with GAC and a north wall has been considered for experimentation. The observations have been recorded during December 2000 to March 2001. A computer program based on MatLab software has been prepared to predict the plant and room air temperatures. The GAC parameters are optimized by using the results of the developed mathematical model in quasi-steady state condition. It is inferred that the room air temperature of the greenhouse increases with the increase of area and mass flow rate of the GAC. The proposed model is validated by the experimental observations for a typical set of parameters. The optimum area of the GAC was obtained as 17.55 m$^2$ at 100 kg/h mass flow rate and 4190 kJ/°C heat capacity. The optimum mass flow rate and heat capacity were evaluated as 200 kg/h and 12570 kJ/°C, respectively, for the given climatic condition and design parameters.

2. Design of experimental greenhouse

A roof type even span greenhouse [9] with an effective floor area of 6 x 4 m has been considered for experiment (Fig. 1a). The orientation of the greenhouse was east-west. The greenhouse was installed to study the thermal load leveling (TLL) under the composite climate of India (Delhi). There was a brick north wall to reduce the thermal losses from the greenhouse during the winter, as suggested by Santamouris [10] and Singh and Tiwari [8], as shown in Fig. 1a. The central height and height of north wall and south wall were 3 and 2 m, respectively. A provision for a cooling pad size of 3 x 1.15m was kept on the west wall for cooling the greenhouse in summers. Thus, this area was subtracted for computation of the solar energy from the west wall (Fig. 1a).

A GAC was constructed along the south face along the length of the greenhouse. The GAC was made of PVC tube in a sand bed at 0.20 m depth in 6 m x 2 m area. The PVC tube, of 0.06 m diameter, was laid in the sand bed of 0.10 m thick in a serpentine manner [11]. The length and width of the serpentine tubes were 5.85 and 0.20 m, respectively with 11 numbers of turns. The top of the sand bed was colored black and covered with UV film (Fig. 1b). Thus, the whole structure worked like a flat plate solar collector with serpentine tube as suggested by Abdel-Khalik [11]. A positive displacement of air (twin lobe compressor) type blower of 100 kg/h mass flow rate was attached at the inlet of the GAC and installed inside of the greenhouse. The outlet of the ground air collector was left open in the center of the greenhouse (Fig. 1a). The design and operating parameters are given in Table 1.
Fig. 1. (a) Experimental greenhouse showing the GAC and north wall. (b) Photograph of experimental greenhouse with GAC. (c) Cross-sectional view of experimental greenhouse with north wall and GAC showing various heat fluxes.

Table 1
Input parameters used for computation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
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</thead>
<tbody>
<tr>
<td>$A_p$</td>
<td>$100 , \text{m}^2$</td>
</tr>
<tr>
<td>$C_a, C_p$</td>
<td>$1005.8, 4190 , \text{J/kg°C}$</td>
</tr>
<tr>
<td>$F_p$</td>
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<td>$h_{w,c}, h_{g,c}, h_{pc}$</td>
<td>$5.7, 5.7, 13.6 , \text{W/m}^2\text{°C}$</td>
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<td>$0$</td>
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<tr>
<td>$U_b, U_t$</td>
<td>$18, 6 , \text{W/m}^2\text{°C}$</td>
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<tr>
<td>$V$</td>
<td>$60 , \text{m}^3$</td>
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<tr>
<td>$\nu$</td>
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<tr>
<td>$s, \Delta z$</td>
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<tr>
<td>$q_n$</td>
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</table>
3. Experimental observation

Hourly observations were taken from December 2000 to March 2001 for 24 h at one week intervals. Room temperature was recorded in the center of the greenhouse with the help of a calibrated mercury bulb thermometer. The plant temperature was taken with the help of a non-contact type (laser ray) thermometer on the plant leaf. Solar radiation was recorded with calibrated solarimeters on the floor and north wall of the greenhouse and outside the greenhouse. However, the experimental validation has been done for typical dates (clear sunny days) of observation (for 20/12/2000, 27/12/2000, 10/01/2001 and 17/01/2001). The total radiation on each wall and roof was computed by using the Liu and Jordan [12] formula to determine the total solar energy received by the greenhouse and used as the input value for computation of the plant and room temperatures.

4. Thermal analysis

The energy balance equations for the different components of the greenhouse system, as shown in Fig. 1c, are written on the following assumptions:

(i) absorptivity and heat capacity of air are negligible,
(ii) storage capacity of the wall and roof materials is negligible,
(iii) heat flow is one dimensional and in a quasi-steady state condition,
(iv) radiative exchange within the walls and roofs of the greenhouse is neglected,
(v) thermal properties of plants in the greenhouse are the same as those of water, and
(vi) greenhouse is east-west oriented.

The energy balance equations for the various components like the north wall, plant mass, floor and room air of greenhouse can be written as follows:

(a) for north wall

\[
a_n(1 - p_Q)F_Q Y^\alpha I^\alpha i = [Kr(T_{\alpha x} - T_i) + h_{\alpha u}(T_{\alpha x} - T_a)]A_Q
\]

(b) for plants

\[
F_p op[(1 - F_n) + p_QF_Q] Y^\alpha I^\alpha i = M^P C^P \delta^P + M^P T_r A_p
\]

(c) for floor

\[
a_g(1 - F_p)(1 - F_m) + p_d F_d Y^\alpha h AM = [h_{\alpha g}(T_{\alpha y}=0 - T_r) + h_{\alpha oo}(T_{\alpha y} = T_A)]A_n
\]

(d) for air [13]

\[
[(1 - a_g)(1 - F_p) + (1 - a_p)F_p] [(1 - F_n) + p_QF_Q] \sum I^i A^i \delta^i t^i + h_{\alpha m}(T_{\alpha z}=0 - T_r)A_n
\]

\[
+ h_{\alpha g}(T_{\alpha y}=0 - T_r)A_g + h_{\alpha p}(T_{\alpha r} - T_r)A_p + \dot{Q}_a
\]

\[
= \sum U^i A^i (T_r - T_a) + 0.33 NV\partial T_r - T_P \delta V
\]
4.1. Useful energy gain

The useful energy gain of the air collector can be evaluated by [11]:

$$\dot{Q}_u = A_c F_k [(T_c a_c) I_h - U_L (T_r - T_a)]$$

(5)

where $UL$ is the total loss coefficient and the collector is exposed to ambient from top and bottom, both sides. In the case of the GAC, where the collector is (tube buried in sand bed type) below the ground surface, the $UL$ ($= U_t + U_b$) would be the sum of the top and bottom loss or gain coefficients, and the useful energy gain can be written as:

$$\dot{Q}_u = A_c F_k [(T_c a_c) I_h - U_t (T_r - T_a) - U_h (T_r - T_g)]$$

(6)

Here, the term $U_b (T_r - T_g)$ would be a loss if $T_r > T_g$ and gain if $T_r < T_g$.

Eqs. (1)-(4) and (6) can be solved for determining the temperature of the greenhouse ($T_r$) and plants ($T_p$). With the help of Eqs. (1) and (3), by eliminating the $T_{\zeta=0}$ and $Z T_a$ in Eq. (4), the expression of $T_r$ can be written as:

$$T_r = \frac{I_{d \Phi R} + H p I_{d \Phi N} + H t I_{d \Phi G} + A_c F_k [(T_c a_c) I_h + U_t T_g] + h_p A_p T_p + Z T_a}{Z + h_p A_p + A_c F_R U_b}$$

(7)

by substituting the expression of $T_r$ from Eq. (7) into Eq. (2), we get

$$\frac{dT_p}{dt} + \frac{[Z + A_c F_k U_h] H_T}{M_p C_p} = \frac{H p I_{d \Phi R} + H t I_{d \Phi G} + H I_{d \Phi S} + H I_{d \Phi S} + A_c F_k [(T_c a_c) I_h + U_t T_g] + Z T_a}{M_p C_p}$$

(8)

This is the form of a first order differential equation

$$\frac{dT_p}{dt} + a T_p = F(t)$$

(9)

where

$$a = -\frac{Z + A_c F_k U_h}{M_p C_p}$$

(10)

and

$$F(t) = \frac{I_{d \Phi R} + H p I_{d \Phi N} + H t I_{d \Phi G} + A_c F_k [(T_c a_c) I_h + U_t T_g] + Z T_a}{M_p C_p}$$

(11)

The analytical solution of Eq. (9) can be written as

$$T_p = F \delta t + \frac{F \delta t p_0}{p_0} e^{-a t}$$

(12)

where $T_{p0}$ is the temperature of the plants at $t = 0$, $F \delta t$ is the average value of $F \delta t$ for the time interval between 0 and $t$ and $a$ is constant during the time. Once the numerical value of $T_p$ is determined, then the room temperature ($T_r$) can be evaluated from Eq. (7). The expressions of the various derivations used in Eqs. (7) and (8) are presented in Appendix A.
4.2. Computation of thermal load leveling

Since the room temperature ($T_r$) is a function of time, the fluctuation in room temperature plays a vital role for plant health. TLL gives an idea about the fluctuation of temperature inside the greenhouse. The TLL for a greenhouse can be defined as:

$$\text{TLL} = \frac{T - T_{\min}}{T_{\max} - T_{\min}}$$

For a given temperature difference, the denominator should be a maximum for plant growth due to heating in the winter, and therefore, TLL should be a minimum for the winter condition and a maximum for summer conditions [8,14]. Therefore, TLL is an important index for optimizing the heating parameter.

5. Computational procedure and input parameters

The mathematical model has been solved with the help of a computer program based on MatLab software. The input values of the parameters used for experimental validation are given in Table 1. The experimental validation has been done for four clear sunny days (20/12/2000, 27/12/2000, 10/01/2001 and 17/01/2001) for the plant and room temperatures. Optimizing the GAC parameters was done for the observations of a single day (10/01/2001). For optimizing the GAC parameters like area of the collector ($A_c$), mass flow rate ($m_a$) and heat capacity ($C_p$), a single parameter was changed while the other two were kept constant. The ranges of parameters that have been optimized are given in Table 2. The heat removal factor ($F_R$) for the ground air collector has been determined by using the procedure given by Abdel-Khalik [11] for a flat plate solar collector with serpentine tube. The heat removal factor ($F_R$) is a function of the area of the collector ($A_c$) and mass flow rate ($m_a$). The heat removal factor decreases with an increase in the area of the collector and increases with an increase in mass flow rate.

For varying the parameters of the GAC like area of the collector, the number of turns of tube ($N$) has been varied as 3, 7, 11, 15 and 19 with the length and width of the serpentine tube being 5.85 and 0.20 m, respectively. Thus, the varying area of the ground air collector varied as 3.5, 8.19, 12.87, 17.55 and 22.23 m$^2$, respectively. The heat removal factor for the respective areas of ground air collector was computed as 0.5641, 0.4367, 0.3002, 0.2365 and 0.1819, respectively. Similarly, for the varying mass flow rate as 25, 50, 100, 200 and 400 kg/h, the heat removal factor was.

<table>
<thead>
<tr>
<th>S. no.</th>
<th>Parameter to be optimized</th>
<th>Range of parameters</th>
<th>Mass flow rate (kg/h)</th>
<th>Heat capacity (kJ/°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Area of ground air collector</td>
<td>3.51, 8.19, 12.87, 17.55, 22.23</td>
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<td>4190</td>
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<tr>
<td>2</td>
<td>Mass flow rate</td>
<td>12.87</td>
<td>25, 50, 100, 200, 400</td>
<td>4190</td>
</tr>
<tr>
<td>3</td>
<td>Heat capacity</td>
<td>12.87</td>
<td>100</td>
<td>4190, 8380, 12570, 16760</td>
</tr>
</tbody>
</table>
computed as 0.0909, 0.1819, 0.3002, 0.4913 and 0.6369, respectively. The heat capacity in the greenhouse includes all the isothermal masses like mass of the plants, water tank, pots etc.

The experimental error has been determined in terms of percent uncertainty (internal and external) (Appendix B) [15]. The closeness of the predicted and experimental values of temperature has been presented with the coefficient of correlation (r) and root mean square of percent deviation (e) [16].

6. Results and discussion

6.1. Hourly variation of plant and room temperature

The plant and room temperatures were computed with the help of Eqs. (12) and (7), respectively. Hourly observations on the plant and room temperatures were taken for days of the

![Fig. 2. Hourly variation of plant and room air temperatures in greenhouse with ground collector.](image-url)
months of December and January. The experimental and computed hourly plant and room temperatures along with ambient temperatures are plotted in Fig. 2 (for 20/12/2000, 27/12/2000, 10/01/2001 and 17/01/2001, respectively). The closeness of the predicted and experimental values of temperature has been presented with the coefficient of correlation (r) and the root mean square of percent deviation (e). The best possible agreement of the predicted and experimental values will be at values of the coefficient of correlation and the root mean square of percent deviation of one and zero, respectively. The predicted temperatures $T_p$ and $T_r$ show good agreement with the experimental observations with coefficient of correlation ranges from 0.9835 to 0.9142 and root mean square of percent deviation from 5.6597% to 13.1567%. It can also be observed from these figures that there is a significant effect of the GAC on the plant and room temperatures during the night. The stored thermal energy of the ground collector gives a rise of temperature of 6-7 °C from the ambient during the night (Fig. 3). The percent uncertainty of the plant and room temperatures were found to be 23.6501 and 23.6112, respectively.

6.2. Optimization of ground air collector parameter

6.2.1. Effect of area of ground air collector

The effect of the change of area of the GAC on the plant and room temperatures is presented in Fig. 4a and b, respectively. It is worth mentioning here that with an increase of area of the GAC ($A_c$), the heat removal factor ($F_R$) decreases for the same mass flow rate ($m_{\text{a}}$). Thus, the overall impact of the increase of area of the ground collector from 8.19 m$^2$ onwards on the plant and room temperatures is not effective. These figures also show the plant and room temperatures without GAC, which are lower than those with the ground air collector during the night and higher during the day hours. This clearly indicates that the present design of GAC is effective to increase the plant and room temperatures during the night hours due to the storage of thermal energy and to reduce the temperatures during the day hours due to the higher losses. The graphical representation of the effect of area of the ground air collector on TLL, as shown in
Fig. 4c, provides the optimum area of the ground collector as 17.55 m² for minimum TLL in winters for given mass flow rate and heat capacity.

6.2.2. Effect of air mass flow rate

The effect of mass flow rate in the GAC on the plant and room temperatures is shown in Fig. 5a and b, respectively. It is evident that the plant and room temperatures increase with an increase of the mass flow rate from 25 to 400 kg/h, especially during the night hours, whereas, there is essentially no variation in $T_p$ and $T_r$ at the varying mass flow rate during the day time due to the poor design of the GAC. It also indicates that there is no significant effect of mass flow rate beyond 200 kg/h. The effect of mass flow rate on TLL is presented in Fig. 5c. The TLL in room air temperature decreases with an increase in mass flow rate. The optimum mass flow rate on this figure comes at 200 kg/h, where the TLL is a minimum as desired in winters.
6.2.3. Effect of heat capacity

Fig. 6a and b present the effect of heat capacity ($M_p \times C_p$) on the plant and room temperatures, respectively. Higher heat capacity lowers the plant and room temperatures during the day time. There is not much effect of the change of heat capacity on the plant and room temperatures during the night hours. The TLL decreases linearly with the increase of heat capacity (Fig. 6c). Therefore, the required heat capacity for minimum TLL from Fig. 6c comes at 20950 kJ/°C.

6.3. Evaluation of threshold solar intensity

It is important to define the threshold intensity for the ground air collector, which is the effective solar intensity at which the ground air collector gives $Q_{\text{ut}}$ P 0. Thus, the threshold intensity for the GAC can be obtained by putting $Q_{\text{ut}} = 0$ in Eq. (6) as
At the condition when $T_r < C T_g$, the $I_{th}$ would be negative. At such condition, the GAC would work as a buried pipe ground storage and provide a temperature higher than the room air temperature in the greenhouse. This is an example of heating the greenhouse at low ambient temperature during off sunshine hours (night) with buried pipe ground storage. At $T_r > T_g$ and when the available solar intensity is less than the threshold, the ground collector cools the room air temperature, which is a loss to the greenhouse. In this condition, it is advisable to operate the ground air collector during night hours only.

The threshold solar intensity ($I_{th}$) was evaluated for the single day (10 January 2001) of observation of room and ambient air temperatures for the given top and bottom losses ($U_t$ and $U_b$).
The threshold and available solar intensity are graphically presented in Fig. 7. The threshold intensity for an ideal ground air collector is also presented in this figure. It clearly indicates that the available solar intensity was not sufficient to supply useful energy to the greenhouse from the GAC. Thus, it can be inferred from this result that for the given climatic condition of winter, the design of the GAC was poor. However, for an effective GAC having the transmittance-absorbance product of 0.756, the threshold solar intensity has been computed with the help of Eq. (14), and the threshold solar intensity for such ground collector is lower than the available solar intensity.
6.4. Effect of ground air collector on room temperature

The effect of an efficient design of GAC on room air temperature has been analyzed for the optimum area and mass flow rate of the GAC obtained through the modeling. For an effective GAC having the transmittance-absorptance product of 0.756, the threshold solar intensity for such ground collector is lower than the available solar intensity. In such a case, the ground air collector will supply air at a higher temperature than that of the room air. The predicted effect of the GAC on the room air temperature has been shown in Fig. 8. It indicates that the room air temperature in the greenhouse is 5-7 °C higher with using the GAC than that without the ground collector. This reveals that an efficiently designed GAC can be used for heating the greenhouse effectively during the day and night in the winter season.

7. Conclusion

A mathematical model has been developed to study the thermal behavior of a greenhouse using a GAC (serpentine tube type flat plate collector). This model has been solved with the help of a computer program based on MatLab software. The GAC parameters like area of the GAC, mass flow rate and heat capacity have been optimized against the plant and room air temperature behaviors and TLL. The predicted plant and room air temperature show good agreement with experimental values with an average coefficient of correlation of 0.9377 and root mean square of percent deviation of 8.3538. The optimum area of the GAC, mass flow rate and heat capacity were obtained as 17.55 m², 200 kg/h and 20950 kJ/°C, respectively, for the given size and shape of greenhouse and climatic condition. The stored thermal energy of the ground collector gives a rise of temperature of 6-7 °C from the ambient during the night. The percent uncertainties of the plant and room temperatures were found to be 23.6501 and 23.6112, respectively. A GAC with efficient design can be effectively used for heating the greenhouse in the winter season.

Acknowledgement

The financial support for this research work from the Indian Council of Agricultural Research (ICAR), New Delhi, is thankfully acknowledged.

Appendix A

\[ f_{fN} = \alpha n (1 - q_n) P F_n X i A_i s_i \]
\[ f_{effP} = F_p a_p [(1 - F_n h q p F_Q) X i A_i s_i \]
\[ f_{effG} = a_g (1 - F_p) [(1 - F_Q) + p_Q F_Q] Y, \]
\[ f_{em} = [(1 - a_g) (1 - F_i) + (1 - O p) F_p] [(1 - F_n) + p_Q F_Q] J \hat{T} - T, \]

(A.4)
\[ H_n = \frac{h_{nr}}{h_{nr} + h_{na}} \quad (A.5) \]

\[ H_l = \frac{h_{gr}}{h_{gr} + h_{gl}} \quad (A.6) \]

\[ UA = \left[ \frac{1}{h_{w}^{2} a_{n}} + \frac{1}{h_{n}^{2} a_{n}} \right] \]

\[ UA_{goa} = \left[ \frac{1}{S_{5}} + \frac{1}{S_{1}} \right]^{-1} \quad (A.8) \]

\[ Z = UA_{ra} \cdot UA_{gl} \cdot \sum_{i} A_i F_i U_i \cdot \sum_{i} U_i Ai \sim 0.33 N \delta \quad A.9 \]

\[ H_p = \frac{h_{pc} A_p}{Z + h_{pc} A_p + A_c F_R U_c} \quad (A.10) \]

\[ h_o = h_{in} = 5.7 + 3.8 \nu \quad (A.M) \]

\[ h_{na} = \left[ \frac{l_n}{K_n + h_o} \right]^{-1} \quad \delta A:12 \]

### Appendix B

#### B.1. Procedure for calculation of experimental percent uncertainty (U)

\[ U_1 = \frac{\sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_N^2}}{TV} \quad (B.1) \]

where \( r \) is the standard deviation, expressed as

\[ \sigma = \frac{1}{N} \sum_{i} (X_i - \bar{X})^2 \quad (B.2) \]

where \( X_i - \bar{X} \) is the deviation of observation from the mean and \( N \) and \( N_o \) is the number of set and number of observations in each set, respectively.

The percent uncertainty, therefore, could be expressed as

\[ \text{% Internal uncertainty} = \frac{U_1}{\text{mean of the total observations}} \times 100 \quad \delta B:3 \]

The external uncertainty is taken from the list count of the measuring instruments.
B.2. Procedure of calculation of coefficient of correlation (r)

\[ r = \sqrt{b_{XY} b_{YX}} \]  \hspace{1cm} (B.4)

where

\[ b_{XY} = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (Y_i - \bar{Y})^2} \]

and

\[ b_{YX} = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2} \]

B.3. Procedure of calculation of root mean square of percent deviation (e)

\[ e = \sqrt{\sum \left( e_i \right)^2 / N_o} \]  \hspace{1cm} \text{(B.5P)}

where \( e_i = \text{abs} \left[ \frac{X_{\text{predicted}} - X_{\text{experimental}}}{X_{\text{predicted}}} \right] \times 100 \)

References