A Simple and Accurate Modal Analysis of Strip-Loaded Optical Waveguides With Various Index Profiles

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Abstract—A simple first-order perturbation approach has been developed to study the propagation characteristics of strip-loaded diffused waveguides with various refractive index profiles. Propagation constants of the guided modes of rib waveguides and strip-loaded waveguides with exponential and Gaussian refractive index profiles are obtained and it is observed that our results are in good agreement with the results reported in the literature using variational and numerical techniques. The technique also provides analytical expressions for the modal field profile that should be useful in the design of various integrated optical devices.

I. INTRODUCTION

Strip-loaded waveguides are promising integrated optical components because in such waveguides the modes are confined mainly under the dielectric strip and a lack of edge smoothness in the loading strip does not cause significant scattering losses [1]. Single mode operation over a wide frequency range can be obtained easily in these waveguides by choosing a suitable width for the loading dielectric strip [2]. The exact analytical modal analysis for such waveguides is not possible because of their complicated refractive index distribution and the presence of the corners. As a result, various approximate methods such as the effective index method [2]-[4], the variational method [5], [6], the method of lines [7], the mode matching method [8], and the finite element method [9] have been used in order to obtain the propagation characteristics of such waveguides.

The effective index method does not provide modal field patterns and runs into serious difficulty in the regions that are below cutoff because in such regions one cannot define the effective index. On the other hand, the accuracy of the variational method depends on the choice of the trial function and the numerical techniques [7]-[9] require large computational calculations.

In the present paper, we report a simple and accurate perturbation method to obtain the modes of such waveguides. The present method utilizes the fact that the modal field in the corner regions of such waveguides is quite small as compared to the region below the strip. As a result, corresponding to the refractive index profile of strip-loaded waveguides (shown in Fig. 1(a)) and a rib-waveguide (shown in Fig. 1(b)), one can write an approximate refractive index profile that is separable in x and y coordinates and differs from the actual profile only in the corner regions. The propagation characteristics of the approximate index profile can be obtained easily by solving two simple one-dimensional wave-equations, and the effect of the difference between the two profiles in the corner regions can be incorporated through the first-order perturbation approach. In contrast to the numerical methods, the present method also provides analytical expressions for the modal field patterns that should be useful in the optimum design of various integrated optical devices and in the calculation of coupling efficiencies between strip-loaded waveguides and optical fibers.

Using the above approach, we have obtained the propagation characteristics of some rib and strip-loaded diffused waveguides. A good agreement between the present results and those reported in the literature using various techniques has been observed.

II. ANALYSIS

A strip-loaded integrated optical waveguide (shown in Fig. 1(a)) can be characterized by the following dielectric constant distribution:

\[
n^2(x, y) = n_2^2(y) = n_2^2[l + 2Af(y)],
\]

for \(0 < x < \infty, \quad 0 < y < \infty\)
\[ f(y) = \begin{cases} 1 & \text{for } O < y < r \\ 0 & \text{for } r > y > 0 \end{cases} \]

where \( A = \frac{n^2_c - n^2_l}{2n^2_l} \). For a strip-loaded diffused waveguide, \( f(y) \) represents the shape of the refractive index profile. The most common diffusion profiles obtained in practice can be approximated by an exponential or a Gaussian function.

In order to analyze the structure given by (1), we consider an equivalent pseudo waveguide (say 2) shown in Fig. 2, whose dielectric constant distribution is given by

\[ n(x, y) = n_l(x) + n_b(y) - n^2_c \]

where

\[ n_l(x) = n^2_c, \quad \text{for } |x| < \frac{W}{2} \]

\[ n_l(x) = n^2_l, \quad \text{for } |x| > \frac{W}{2} \]

and

\[ n_b(y) = \begin{cases} n^2_c, & \text{for } O < y < \infty \\ n^2_l, & \text{for } -t < y < 0 \\ n^2_l, & \text{for } y > 0 \end{cases} \]

The dielectric constant distributions of the two waveguides (i.e., 1 and 2) differ only in the shaded (corner) regions. Since the modal power is expected to be small in these corner regions as most of the power is confined below the strip only, the propagation characteristics of waveguide 1 should be very nearly the same as those of waveguide 2, which is much simpler to analyze as the corresponding dielectric constant profile is separable in \( x \) and \( y \) coordinates. The modes of waveguide 2 can be obtained by solving the following equation:

\[ \left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_0^2 n^2_l(x, y) \right] \psi(x, y) = 0 \]

where \( k_0 = 2\pi/\lambda_0 \), \( \lambda_0 \) being the wavelength of the light, is the free space wave number and \( j \beta_0 \) represents the propagation constant of a mode of waveguide 2. Assuming \( \psi(x, y) = X(x) Y(y) \), it can be shown easily that \( X(x) \) and \( Y(y) \) satisfy the following one-dimensional equations:

\[ \left[ \frac{d^2}{dx^2} + k_0^2 n^2_l(x) \right] X(x) = 0 \]

and

\[ \left[ \frac{d^2}{dy^2} + k_0^2 n^2_b(y) \right] Y(y) = 0 \]

where \( k_0 \) and \( \beta \) are the two separation constants such that

\[ \beta = \beta_0^2 = \beta_0^2 + \beta_0^2 = k_0 n^2_c. \]

Equation (7) can be solved analytically and the solutions are given by

\[ X(x) = A_1 \cos \left( \frac{2\mu_1}{W} x - \theta_1 \right), \quad \text{for } |\psi| < W/2 \]

\[ = A_2 \exp \left( -2\mu_2 \frac{|\psi|}{W} \right), \quad \text{for } |\psi| > W/2 \]

where

\[ \mu_1 = \frac{W}{2} \left[ k_0^2 n^2_l - \beta^2 \right]^{1/2} \]

\[ \mu_2 = \frac{W}{2} \left[ k_0^2 n^2_b - \beta^2 \right]^{1/2} \]

and \( \theta = 0(\pi/2) \) for a mode symmetric (antisymmetric) in \( x \) and \( A_1, A_2, \beta_0, \beta_0^2 \) are constants.

The solutions of (8) depend on the functional form of \( f(y) \), and can be obtained analytically for many practical waveguides such as rib waveguides and the strip-loaded diffused waveguides with exponential profile [11]. For a waveguide with Gaussian profile also, one can obtain accurate analytical solutions by approximating it by a se-cant-hyperbolic profile [12].

In order to obtain the values of \( \beta_0^2 \) and \( |\beta_0| \) of the modes, we satisfy the boundary conditions for the dominant field vector [10]. For example, the \( E_{pq} \) mode
would approximately correspond to a TE mode in the x-direction and to a TM mode in the y-direction. Thus, by assuming \( E_x = \mathcal{V}l(x, y) \), the boundary conditions require the continuity of
\[
\frac{\partial \psi_x}{\partial x} \left|_{y = \pm W/2} \right. = \psi_x \left|_{y = \pm W/2} \right.
\]
and
\[
\frac{n_i n_f}{n_i} \frac{\partial \psi_x}{\partial y} \left|_{y = \pm W/2} \right. = -t \quad \text{and at } y = 0 (orr)
\]
for a strip-loaded diffused (or rib) waveguide. Similarly, an \( E_{pq} \) mode would require the continuity of
\[
\frac{\partial \psi_{pq}}{\partial y} \left|_{y = \pm W/2} \right. = \pm \psi_{pq} \left|_{y = \pm W/2} \right.
\]
and
\[
\frac{\partial \psi_{pq}}{\partial x} \left|_{y = 0 (orr)} \right. = \psi_{pq} \left|_{y = 0 (orr)} \right.
\]
for a strip-loaded diffused (or rib) waveguide.

Applying the above mentioned boundary conditions, one can easily obtain the eigenvalue equation determining \( \beta^2 \) for some strip-loaded waveguides for which analytical solutions of (8) can be obtained.

A. Rib Waveguides

For a rib waveguide (shown in Fig. 1(b)), \( f(y) \) is given by
\[
f(y) = \begin{cases} 
0, & \text{for } y > r \\
1, & \text{for } -t < y < r \\
0, & \text{for } y < -t
\end{cases}
\]
which corresponds to an asymmetric planar waveguide of core-thickness \( (f + r) \); the corresponding solutions of (8) are given by
\[
Y(y) = \begin{cases} 
A_c \cos \left( \frac{\nu_i y}{t} \right) + B_c \sin \left( \frac{\nu_i y}{t} \right), & \text{for } -t < y < r \\
A_c \exp \left( \frac{\nu_i y}{t} \right), & \text{for } y < -t \\
A_a \exp \left( \frac{\nu_i y}{t} \right), & \text{for } y > r
\end{cases}
\]
where
\[
\nu_i = \sqrt{\left( \frac{n_f^2 - n_a^2}{n_i^2} \right)^2 + \left( \frac{2}{\alpha_0 T} \right) \frac{n_f^2 - n_a^2}{n_i^2} \nu_i^2}
\]
and \( A_c, A_a, B_c, B_a \) are constants.

Applying the boundary conditions given by (11) and (12), one obtains the following eigenvalue equation for \( \beta^2 \):
\[
\tan^{-1} \left( \frac{\nu_i}{\nu_f} \right) + \tan^{-1} \left( \frac{D \nu_i}{\nu_f} \right) = 0
\]
where \( D \) and \( \alpha \) represent the diffusion depth and the refractive index at the surface \( y = r \) respectively. In this case one can also obtain analytical expressions for \( Y(y) \), which are given by
\[
Y(y) = \begin{cases} 
A_c \exp \left( \frac{\nu_i y}{t} \right), & \text{for } y < -t \\
A_c \cos \left( \frac{\nu_i y}{t} \right) + B_c \sin \left( \frac{\nu_i y}{t} \right), & \text{for } -t < y < 0 \\
A_c \exp \left( \frac{\nu_i y}{t} \right), & \text{for } y > 0
\end{cases}
\]
(20)

2) Gaussian Index Profile: In this case \( f(y) \) is given by
\[
f(y) = \exp \left( -\frac{y^2}{2D^2} \right)
\]
(23)
Although for this profile an exact solution of (8) does not exist, it has been shown [12] that a Gaussian profile can be very well approximated by a secant hyperbolic profile. As such, we have approximated $d(y)$ by $y' \exp \left( a y/D \right)$, where

$$J_0' = \frac{\pi}{\sin(\pi e^{-1/4})}.$$  

The value of the constant $d$ is selected in such a way that the area under the curves representing the Gaussian and secant hyperbolic functions is the same up to the diffusion depth $D$, and is given by $d = 1.0434$ [12].

The solution $Y(y)$ of (8) for $y' (y)$ is given by [12]

$$Y(y) = A_e \exp \left( a y/D \right) + B_e \exp \left( -a y/D \right),$$

for $-t < y < 0$

$$= A_e P_{pq}^\mu(l),$$

for $y > 0$

(25)

where

$$M = i/d, \quad f = \tanh(y/E),$$

$$\psi = \left[ \left( \left( f^2 + \frac{4}{1} \right) - 1 \right) \right].$$

$^T(CF)$ is the Associated Legendre polynomial, and other parameters have been defined in (20). The boundary conditions mentioned in (11) and (12) lead to the following eigenvalue equation for $J_0$ of the $E_{pq}$ mode:

$$\left[ \frac{T_{s1} + \frac{1}{2} T_{s2} + \frac{1}{2}}{T_{s1} \cdot T_{s2}} \right] \tan \left( (\nu - \mu) \pi/2 \right) - \frac{T}{d} = 0$$

(26)

where

$$s_1 = (\nu - n + 1)/2, \quad s_2 = s_1 + \mu$$

$T_x$ is the Gamma function and $T$ is defined in (22).

After obtaining $\mu_1$ and $\mu_2$ and hence $\beta_0$ (using (9)) the propagation constant $\theta$ of the guided modes of the given waveguide 1 can be written as

$$\theta' = 0@ + i\beta^2$$

(27)

where $\beta^2$ is the first-order perturbation correction to $J_3@$, which is given by

$$\beta^2 = \int_0^{\infty} \int_0^{\infty} \frac{L(x, \gamma)^2 b_{n_1}^2}{\gamma} \gamma^2 d\gamma dx$$

(28)

where $b_n^2$ is the difference between the dielectric constant distribution of the two waveguides, given as follows:

a) for rib waveguides:

$$b_{n_1}^2 = (n^2_c - n^2_a),$$

for regions (1) and (2)

$$= 0,$$

otherwise.

b) for strip-loaded diffused waveguides:

i) with exponential index profile

$$bn^2 = n_1^2 - nl,$$

for regions (1) and (2)

$$= 0,$$

otherwise.

ii) with Gaussian index profile

$$bn^2 = \left[ \frac{n_1^2 - nl}{s + n - n^2_c} \right]$$

for regions (2)

$$= s,$$

for regions (3)

$$= 0,$$

otherwise.

(30)

(31)

where

$$s = (y^2 - nl) \left[ \exp(-y^2) - \sech^2(y/D) \right].$$

Regions (1), (2), and (3) are shown in Fig. 2.

Thus, one can easily find out the propagation characteristics of $E_{pq}$ modes of the strip-loaded diffused and rib-waveguides. Propagation characteristics of $E_{pq}$ modes of the rib-waveguides can also be obtained easily by following a similar procedure.

III. RESULTS AND DISCUSSION

In order to see the applicability of the present approach, numerical calculations were carried out for various strip-loaded diffused and rib-waveguides and results are compared with those reported by others.

We first obtained the propagation constant of the guided modes of rib-waveguides and compared our results with that of the transverse resonance method [13], the method of lines [7], [8], and the mode matching method [8]. Fig. 3 illustrates the variation of the effective index ($J_3@$) of the fundamental scalar mode ($E_n$) of a rib-waveguide with the following values of various parameters: $n_e = 3.44$, $n_b = 3.35$, $n_d = 1.0$, $r = 0.2 \, \mu m$, $t = 0.8 \, \mu m$, and $X = 1.15 \, \mu m$ as a function of rib width ($W$). The solid circles and the solid curve correspond to the approximated and approximate eigenvalue equations, respectively, of the transverse resonance method as reported by Payne [13]. The crosses represent the results obtained by the present perturbation approach. A comparison of these results indicates a good agreement between them.

Fig. 4(a) and (b) shows the variation of the effective index of the first two $y$-polarized and $x$-polarized modes, respectively, of a rib-waveguide with $W/r = 6.0, 1.0, n_e = 1.742, n_b = 6.0$, and $n_d = 1.0$ as a function of the normalized parameter $e/\lambda$. The solid curves and circles correspond to the results reported by Diestel [7] using the method of lines and Yasuura et al. [8] using the mode-matching method, respectively. The results obtained by using the present method are shown by crosses. These figures show that the present results match very well with those obtained by numerical techniques [7], [8]. These figures also indicate that near cutoff, the agreement of the present results with other results is less. This is due to the fact that near cutoff, the modal field will be less...
confined in the central core region, and hence the accuracy of the first-order perturbation approach would decrease.

We now apply the present perturbation approach to strip-loaded diffused waveguides and compare the results so obtained with those reported by others. In Fig. 5, we have plotted the normalized propagation constant $B\equiv (\beta/\beta_0) - 1/(\epsilon_2/n_b^2)$ of the $E_x$ mode of strip-loaded diffused waveguides with exponential and Gaussian index profile, as a function of normalized frequency $V/\sqrt{k_D D(n_b^2 - n_l)^2}$. The values of various parameters used are given below: $W/D = 8.0$, $t/D = 1.0$, $n_a = 1.0$, $n_b = 1.506$, $n_c = 1.5432$, and $a = 1.0$.

Fig. 6 illustrates the variation of the normalized propagation constant $B$ of the $E_x$ mode as a function of the F-value for the strip-loaded diffused waveguide with the exponential index profile having $t/D = 1.0$, $W/D = 8.0$, and various values of asymmetric parameter $a$. The solid curves correspond to results of [5] obtained by variational analysis while the crosses correspond to the present results. The figure indicates that the present results also

![Fig. 3. Variation of the effective index $(\beta/\beta_0)$ of the scalar fundamental mode $(\beta_n)$ of the rib-waveguide having $n_a = 1.0$, $n_b = 3.35$, $n_c = 3.44$, $r = 0.2$, and $r = 0.8 \mu m$, as a function of width parameter $(W)$ at $X = 1.5 \mu m$.](image)

![Fig. 4. Dependence of the effective index $(\beta/\beta_0)$ of the $(a) E_x$, $(b) E_{y}$, $(c) E_{z}$, and $(d) E_{z}$ mode on the normalized parameter $k_0 r$ of the rib waveguide having $W/D = 6.0$, $t/D = 1.0$, $n_a = 1.742$, $n_b = 1.69$, and $a = 1.0$.](image)

![Fig. 5. Variation of the normalized propagation constant $B$ of the $E_x$ mode as a function of $V$ for a strip-loaded diffused waveguide with the exponential index profile having $t/D = 1.0$, $W/D = 8.0$, and various values of asymmetric parameter $a$. The solid curves correspond to results of [5] obtained by variational analysis while the crosses correspond to the present results. The figure indicates that the present results also](image)

![Fig. 6. Variation of $B$ of the $E_x$ mode as a function of $V$ for the strip-loaded diffused optical waveguides. The values of the parameters used in the analysis are: $W/D = 8.0$, $t/D = 1.0$, $n_a = 1.0$, $n_b = 1.506$, $n_c = 1.5432$, and $a = 1.0$.](image)
match excellently with those obtained by variational analysis.

We would like to mention here that the present method is applicable to the waveguides for which a suitable unperturbed dielectric constant profile (separable in the x and y coordinates and matching with the given profile in all the regions except corner regions) can be selected. For example, the present method cannot be applied for waveguides with two-dimensional diffusion or waveguides with nonrectangular cores such as trapezoidal. Further, even if the unperturbed profile is separable in the x and y coordinates, the present method is not recommended to the cases in which both the resulting one-dimensional wave equations are not analytically solvable because in such a case analytical expressions for field profiles would not be available for the calculation of first-order perturbation correction.

IV. CONCLUSIONS

A simple perturbation approach has been developed in order to study the propagation characteristics of rib waveguides and strip-loaded waveguides with exponential and Gaussian refractive index profiles. The proposed technique has been shown to give accurate results. The method also gives analytical expressions for the modal field pattern that should be useful in the design of various integrated optical devices such as couplers, etc..

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