Ranking of line outages in an AC-DC system causing overload and voltage problems

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Abstract: In the paper an efficient noniterative method for ranking line outage contingencies in an AC-DC system based on overload and voltage deviation performance indices is developed. The method is based on the calculation of complex line outage distribution factors using a coupled AC-DC load flow Jacobian matrix. The proposed method is further extended for the calculation of line outage distribution factors which could be very useful for reducing the computational burden in security constrained optimisation problems. A mathematical model for handling bus-type switching resulting from reactive power limit violations is also developed. Results for a modified IEEE 14-bus system have been obtained with the proposed method and are compared with those obtained by rigorous AC-DC load flow.

1 Introduction

Online steady-state security analysis of power systems requires evaluation of the effects of a large number of contingencies in order to assess the security of the system. Contingency analysis is an important component of the security function which is considered to be an integral part of the modern energy management system at energy control centres. Because of the high speed of solution required for online processing of a large number of contingencies, approximate noniterative techniques are most often used.

It is well known that contingencies that cause line flow problems may not necessarily cause bus voltage problems and vice versa [10]. Hence, two separate ranking lists are required, one for line flow problems, and the other for bus voltage problems.

Sufficiently fast and accurate methods for overload contingency ranking are available. The methods based on voltage deviations are also equally important because major power disruptions can be caused by voltage collapses. Some of the recent methods developed for voltage problems are discussed here. Brandwajn and Lauby [1] have extended their efficient bounding technique based on the MW flow violation detection method to the bus voltage limit violation problem. A subnetwork with significant voltage shifts is circled based on approximate Q-V iteration method is applied on this subnetwork to obtain the voltage and angle change vectors. The method appears to be efficient, but it is based on heuristics. Chen and Bose [2], have developed a method known as the direct ranking algorithm for contingency selection taking into consideration voltage security aspects. The performance index is calculated directly without estimating the post contingency voltages at individual buses. Two basic limitations of such an approach are

(i) It is not possible to filter critical contingencies causing limit violations; i.e. it is difficult to avoid a masking effect.
(ii) The effect of reactive limit violation and consequent bus status change cannot be taken into account.

Mohamed and Jasman [3] have proposed a technique that uses the DC formulation with the compensation method to compute the post outage angles. The super decoupled load flow formulation and compensation method are used for calculating post outage voltages. Ejebe et al. [4] have proposed methods to form a subnetwork with voltage sensitive buses only. Although the method is efficient, it is based on heuristics.

It has been recognised that a stand alone Q-V model for voltage security analysis is unreliable [9]. Since the effect of angle change is important for computational efficiency it is desirable to develop a unified contingency simulation procedure to produce separate rankings according to overload and voltage deviation performance indices.

To the best of the authors' knowledge there are to date no papers in the literature on contingency ranking in AC-DC systems. The only method available is a first iteration of AC-DC load flow. This, however, requires refactorisation of the Jacobian matrices for every topology change, which is quite time consuming. In this paper, therefore, we develop a noniterative technique for contingency ranking in an AC-DC system based on the compensation approach.

1.1 Line outage distribution factors

Distribution factors are widely used in real time contingency ranking owing to the speed with which this analysis can be performed once these factors are evaluated and stored. Moreover, such problems as security constrained optimisation (e.g. pre- and post-contingency corrective rescheduling) are computationally very demanding and require generation shift factors and line mismatches at buses and incremental reactive losses in branches. The impact of contingency on the bus voltage profile is then evaluated by truncating the encircled area.

The Q-V iteration method is applied on this subnetwork to obtain the voltage and angle change vectors. The method appears to be efficient, but it is based on heuristics. Hence, we develop a noniterative technique for contingency ranking owing to the speed with which this analysis can be performed once these factors are evaluated and stored. Moreover, such problems as security constrained optimisation (e.g. pre- and post-contingency corrective rescheduling) are computationally very demanding and require generation shift factors and line
outage distribution factors to reduce the complexity of the problem.

Although distribution factors are quite common in overload contingency ranking, a literature survey reveals that, to the best of the authors’ knowledge, similar distribution factors have not yet been developed for the ranking of contingencies for overload and voltage problems simultaneously. Hence, this paper will show a simple and efficient way to calculate the approach distribution factors.

2 Line outage simulation

For noniterative line outage simulation, the compensation technique is ideal, since it avoids the need to refactorise the Jacobian for every simulation.

While using the compensation approach there is a difficulty in using the conventional power mismatch version of the Newton load flow. Although the system can be considered to be lossless from the real power point of view, it cannot be considered lossless from the reactive power point of view, because of significant reactive power loss in the power system network elements. This makes calculation of dissimilar compensations at the two ends difficult. If a current mismatch version of the Newton load flow is used, this difficulty is obviated because, discounting the effect of line charging, the currents at both ends of a line are same.

For line outage simulation in the AC-DC system, in the coupled Newton load flow model, the DC equations for the convertor proposed by Arrillaga et al. [8], are appended to the AC equations. In the proposed model, ground is used as a reference. All buses are considered to be of the $P-Q$ type. It has been found that removal of shunt effects and off-nominal taps gives better results. The removal of $Q-V$ equations for all voltage controlled buses (including slack) and the PS equation for a slack bus, is simulated by adding a very large term $(BSH = \frac{1}{X_{sh}})$ to the main diagonal of these equations. This amounts to the masking of these equations, or shorting these buses to the reference bus as far as these equations are concerned. This type of Jacobian structure not only permits generalisation of compensation calculations for line outage simulation irrespective of the type of buses to which it is connected, but it also facilitates treatment of bus status changes following reactive limit violations.

Although two variables per convertor are sufficient in order to have simpler representation of control specifications, five DC variables per convertor are used [8]. In the light of the above considerations, the linearised load flow equations for the AC-DC system are as shown in eqn. 1.

\[
\begin{align*}
\Delta V & = \begin{bmatrix} A^V \\ A^x \end{bmatrix} \\
\Delta x & = \begin{bmatrix} A^x \\ A^x \end{bmatrix} \\
\Delta p & = \begin{bmatrix} A^p \\ A^p \end{bmatrix} \\
\Delta q & = \begin{bmatrix} A^q \\ A^q \end{bmatrix} \\
\Delta p - \Delta q & = \begin{bmatrix} A^p - A^q \\ A^p - A^q \end{bmatrix}
\end{align*}
\]

where, $A^p$ and $A^q$ are real and reactive current mismatches at AC buses.

\[
A_{c_{term}} = \text{real current mismatch at the AC convertor terminal bus}
\]

\[
A_{d_{term}} = \text{reactive current mismatch at the AC convertor terminal bus}
\]

DC mismatch vector $R$ is calculated using the following equations for each convertor.

\[
R_1 = V_d \cos \beta - \cos \delta \\
R_2 = V_d \sin \beta \cos \delta + (V_n) \lambda_0 \\
R_3 = V / (\delta, \delta)
\]

where

\[
\begin{align*}
\alpha & = \text{firing delay angle} \\
f_1 & = \text{power factor angle of the convertor} \\
f_2 & = \text{direct current} \\
f_3 & = \text{commutation reactance} \\
f_4 & = \kappa \frac{2}{\pi} \\
f_5 & = \kappa \frac{2}{\pi} \\
f_6 & = \text{transformer off nominal tap ratio} \\
f_7 & = \text{voltage update vector for purely AC buses} \\
f_8 & = \text{voltage update vector for converter terminals} \\
f_9 & = \text{DC variable update vector}
\end{align*}
\]

The following set of variables permits a simple relationship for all normal control strategies [8].

\[
x = [x_1, x_2, x_3, x_4, x_5, x_6]^{T}
\]

The angle, voltage and DC variable updates can be calculated from the following equation

\[
\begin{align*}
\Delta V &= \begin{bmatrix} A_1 \ A_2 \ A_3 \ A_4 \ A_5 \ A_6 \end{bmatrix} \Delta x \\
\Delta x &= \begin{bmatrix} A_1 \ A_2 \ A_3 \ A_4 \ A_5 \ A_6 \end{bmatrix} \Delta x
\end{align*}
\]

Where the matrix on the right-hand side of eqn. 2 is the inversion of the Jacobian in eqn. 1.

The line outage is simulated by connecting a fictitious line across the line to be outaged with an impedance equal to its negative. The pre-outage network is represented by the Thevenin equivalent seen from the buses of the outaged line. Such an equivalent is obtained from the sensitivity matrix used in Newton power flow. As explained in Appendix 8, the compensating current to simulate line outage across buses $k-m$ is calculated from eqn. 15 as follows:

\[
\begin{align*}
V_{km} &= \begin{bmatrix} R_{km} & -X_{km} \\ X_{km} & R_{km} \end{bmatrix} \begin{bmatrix} A_{km} \\ A_{km} \end{bmatrix} \Delta x + R_{km} + L_{km} \\
&= \begin{bmatrix} \begin{bmatrix} c_{km}^{P} \\ c_{km}^{Q} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} d_{km}^{P} \\ d_{km}^{Q} \end{bmatrix} \end{bmatrix} \end{bmatrix}
\end{align*}
\]
where the current \((c_f + jdf)\) is the post-outage flow in the fictitious line. This compensating current is injected at buses \(k\) and \(m\) with negative and positive signs, respectively, because the line current at a bus and injection have opposite sign conventions. The voltage, angle and DC variable update vectors are calculated from eqn. 2. The mismatch vector will have only four nonzero terms as explained above.

It has been recognised that failure to model reactive power limit violations can give unreliable results [9]. In view of this, an extension of the proposed method to handle such situations will now be explained.

### 3 Treatment for reactive power limit violation of PV buses

With the new voltages obtained in the previous step, reactive powers for all PV buses are calculated and the reactive power limit violations are checked. If some violations are detected, bus-type switching is performed and consequent voltage and angle changes are calculated using the following two-step procedure.

In the first step, the specified reactive powers at switched PV buses are set equal to violated limits and voltage and angle updates are obtained by solving eqn. 2. The current mismatch vector for this calculation will be

\[
\begin{bmatrix}
0 & \cdots & 0 & -Ad_{PV} & 0 & \cdots & O
\end{bmatrix}^T
\]

where,

\[-Ad_{PV} = \text{d}P_{PV}\text{ limit}
\]

+ dPV3 base : iPV3 (switched PV buses)

Let the new voltages so obtained be denoted by \([V_k]^{PV}\]^T. The real part of the current, \(k\rightarrow m\) with a negative sign and at bus \(m\) with a positive sign. Similarly, the imaginary part of the current, \(c\rightarrow m\) is injected at buses of \(m\) with a positive sign.

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The voltage and angle update vector is calculated using eqn. 2. The injections at the appropriate buses are negatives of the compensations calculated from the previous equation. Final voltage and angle updates are obtained by summing these from the previous two steps.

### 4 Line outage distribution factors

It will now be shown that, in the process of line outage simulation and calculation of post-outage voltages and angles, most of the calculations that depend only on the network operating condition can be performed separately and stored as factors. Using these factors the post-outage state update vector of the power system can be obtained with a couple of multiplications and addition of simple scalar quantities. The procedure for calculation of line outage distribution factors is as follows.

On expanding eqn. 16, we have

\[
c_L = B_{TH}V_{in} + G_{TH}V_{in}
\]

\[
d_L = G_{TH}V_{in} + B_{TH}V_{in}
\]

The real part of the current, \(c_f\) from eqn. 5, is injected at bus \(k\) with a negative sign and at bus \(m\) with a positive sign. Similarly, the imaginary part of the current, \(d_f\) from eqn. 6, is injected with similar signs at the buses of the outage line. All other injections are zero. The angle, voltage and DC variable update vectors can be obtained using eqn. 2 as follows:

\[
\Delta S = \begin{bmatrix}
-A_{2TH}B_{TH} & A_{2TH}G_{TH}V_{in} + L_{A2TH}\phi_{in}V_{in} \\
-A_{2TH}G_{TH} & A_{2TH}B_{TH}V_{in}
\end{bmatrix} V_{in}
\]

where

\[A_{2TH} = A_{1TH} - A_{2TH}
\]

\[A_{2TH} = A_{2TH} - A_{2TH}
\]

or

\[
\Delta q_i = \phi_i(V_{in}^2 + B_iV_{in})
\]

where

\[V_i = -A_{2TH}B_{TH} - A_{2TH}G_{TH}
\]

\[p_i = -A_{2TH}B_{TH} - A_{2TH}G_{TH}
\]

Similarly, the voltage change at bus \(i\) is given by

\[AV_i = lA_{2TH}B_{TH} - A_{2TH}G_{TH}u_{km} + lA_{2TH}G_{TH} - A_{2TH}B_{TH}V_{in}
\]

or

\[\Delta V_i = p_iB_{km} + p_iV_{in}
\]

The elements of the matrix are picked up from the relevant portions of the inverted Jacobian matrix as explained in Appendix 8. \([X_{Bkm}^2]^{PV}\) are the external shunt reactances put earlier at respective switched buses to simulate elimination of corresponding \(-d\rightarrow V\) equations.

The voltage and angle update vector is calculated using eqn. 2. The injections at the appropriate buses are negatives of the compensations calculated from the previous equation. Final voltage and angle updates are obtained by summing these from the previous two steps.
where
\[ f(i) = -A_{i0}B_{i0} - A_{i0}B_{i0} \]

Similar factors for all other AC and DC variables can be obtained. Thus, for simulating NL line outages a total number of \([ANL x JV]\) + \([IOVL]\) such factors need to be stored, where \(N\) is number of buses.

After updating the voltages and angles ranking based on voltage and overload performance, indices are obtained and outage cases causing limit violation are ranked in order of severity.

The voltage performance index \(PI\) chosen to quantify the system abnormalities reflected in the bus voltage profile is obtained from

\[ PI = \frac{n}{i=1} \left[ \frac{V_{\text{new}} - V_i^2}{V_{\text{max}}^2} \right]^{\alpha} \]

where,
- \(V_{\text{new}}\) = post-outage voltage at bus \(i\)
- \(V_i\) = voltage at bus \(i\) from base case load flow
- \(W_i\) = real nonnegative weighting factor
- \(V_{\text{max}}\) = maximum voltage change allowed, taken as 0.03 for all buses
- \(n\) = exponent of penalty function (taken as one in present studies) for improving accuracies.

This performance index takes into account the voltage magnitude constraints at all load buses in the system. The permissible voltage deviation \(V_{\text{max}}\) represents the threshold beyond (above and below) which bus voltage magnitudes are outside their limits, yielding a large value of index \(PI\). When all deviations from the specified bus voltage magnitudes are within their corresponding \(V_{\text{max}}\), the performance index \(PI\) is small.

Similarly, the performance index for quantifying the extent of line overloads may be defined as follows:

\[ PI_L = \frac{n}{i=1} \left[ \frac{P_{\text{new}} - P_i}{P_{\text{max}}} \right]^{\beta} \]

where,
- \(P_{\text{new}}\) = post-outage real power flow in the line \(i\)
- \(P_i\) = real nonnegative weighting factor used to reflect the importance of lines
- \(P_{\text{max}}\) = real power flow limit for the line \(i\).

Following a contingency, any line which is overloaded will make normalised flow greater than unity, whereas a line whose flow is below its respective limit will make normalised flow less than unity. Squaring the non-normalised flows further increases the contributions due to overloads; but it decreases the contributions from nonoverloaded circuits.

It may be noted that the weighting factors \(W_i\) and \(W_j\) can be regarded as tuning parameters. The factors are selected on the basis of experience with the system and on the relative importance placed on various limit violations. For the study made here, these parameters are all set to 10.

It must be emphasised that, once the post-outage state variables are known, any other performance index can as well be used, if it can reduce the masking effect.

5 Results

The proposed algorithm is used for ranking single line outage contingencies of the IEEE 14-bus system with a DC transmission line [7]. The line between buses 6 and 8 is changed to a DC link by converting these buses to inverter and rectifier buses respectively. Ranking results obtained with the proposed method are compared with the corresponding results obtained with rigorous AC-DC load flow. The DC variables obtained by the two methods are also compared. The generators are assumed to have sufficient reactive generation capability so that no limit violations take place following the outages.

Results for two different control strategies of DC variables are obtained. In the first case, the inverter DC voltage \(V_f\), the cosine of the firing angle \(cos\) \(of\), the cosine of the extinction angle \(cos\) \(t\) and the DC current \(I_f\) are taken as specified quantities. Under this condition only the tap settings at the rectifier and inverter transformers change for every line outage. Table 1 gives the \(PI\) ranking of contingencies causing voltage limit violation. Rankings by both methods appear to be in good agreement. It can be seen from Table 1 that one low ranking contingency is not captured by the proposed method; the proposed method also captures two additional contingencies as violated ones. Table 2 gives \(PI\) ranking of contingencies causing line load limit violation.

It can be seen that rankings obtained by both methods are in complete agreement. Other DC variables remain unchanged.

Tables 4, 5 and 6 give results for a second control strategy where the cosine of the extinction angle \(cos\) \(f\), the direct current \(I_f\), the tap \(a\) and the DC voltage \(V_f\) at the inverter terminal are kept constant. Under this condition, the firing angle at the rectifier \(a\), the tap setting \(a_3\) at the inverter transformer and the rectifier power factor angle \(0\), change for every line outage. Table 4 gives \(PI\) ranking of line outages causing voltage limit violation. It can be seen from Table 4 that one lower ranking contingency is not captured by the proposed method; the proposed method also captures two additional contingencies as violated ones. Table 5 gives \(PI\) ranking of line outages causing line load limit violation. It can be seen that rankings with the two methods are in complete agreement.

Table 6 gives DC variables obtained for each line outage by the two methods which are in close agreement.

<table>
<thead>
<tr>
<th>Table 1: Ranking according to voltage performance index (first control strategy)</th>
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<tbody>
<tr>
<td>Rank</td>
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<tr>
<td>Proposed method line number</td>
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<th>Table 2: Ranking according to overload performance index (first control strategy)</th>
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<tr>
<td>Rank</td>
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<td>Proposed method line number</td>
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Table 4: Ranking according to voltage performance index (second control strategy)

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<tr>
<th>Rank</th>
<th>Proposed method line number</th>
<th>Load flow method line number</th>
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Table 5: Ranking according to overload performance index (second control strategy)

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<th>Rank</th>
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<th>Load flow method line number</th>
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Decoupled formulation of the proposed method has also been developed; however, the results do not appear to be satisfactory in comparison with those obtained with the latter method.

6 Conclusion

In this paper an efficient noniterative method has been developed for the contingency ranking of line outages causing overload and voltage problems in an AC–DC system. A coupled Jacobian matrix has been used for outage simulations, the line outage being simulated by providing real and reactive current compensations at the buses of the outage line. It has been shown how the line outage distribution factors, which depend only on network operating conditions, can be estimated in a simple and efficient manner which could then be used in applications such as security constrained optimisation to reduce the complexity of such problems. A mathematical model for handling an important feature of bus type switchings resulting from reactive power limit violations has also been developed. The results for two types of control strategies for a modified IEEE 14 bus test system indicate that both overload and voltage rankings obtained by the proposed method and rigorous load flow method are in good agreement. This clearly demonstrates the suitability of the proposed method for an AC system with a DC link.

7 References

Appendix: Compensation calculation using Thévenin’s equivalent

Using Thévenin’s equivalent seen from buses of outaged line \((k, m)\) compensations are calculated as follows:

Final voltage/angle across outaged line

\[ \text{deviation in voltage/angle across outaged line due to compensating injections} \]

In symbolic form, we have

\[ \begin{bmatrix} S'_{km} \\ V'_{km} \end{bmatrix} = \begin{bmatrix} S'_{km} \\ V'_{km} \end{bmatrix} - \begin{bmatrix} A_{THkm-in} & A_{THkm-in}^T \\ A_{THkm-in} & A_{THkm-in}^T \end{bmatrix} \begin{bmatrix} C_{km} \\ -d_{km} \end{bmatrix} \]

(13)

where the matrix on the right hand side of eqn. 13 represents Thévenin’s equivalent impedance in real variables, seen from the buses of the outaged line. The elements of this matrix are evaluated from the relevant submatrices of eqn. 2, e.g.

\[ A_{THkm-in} = A_{ikm} + A_{lmn} - A_{ilk} - A_{imk} \]

Using a model similar to the current mismatch load flow model, the final voltage across the buses of the outaged line in terms of real variables is given by

\[ \begin{bmatrix} V'_{km} \\ S'_{km} \end{bmatrix} = \begin{bmatrix} -X_{kkm} & R_{kkm} \\ -R_{kkm} - X_{kkm} \end{bmatrix} \begin{bmatrix} c_{km} \\ d_{km} \end{bmatrix} \]

(14)

Substituting for \([S'_{km} V'_{km}]^T\) from eqn. 14 in eqn. 13 and simplifying, we have

\[ \begin{bmatrix} c_{km} \\ d_{km} \end{bmatrix} = \begin{bmatrix} A_{TMkm-in} & A_{TMkm-in}^T \\ A_{TMkm-in} & A_{TMkm-in}^T \end{bmatrix} \begin{bmatrix} c_{km} \\ d_{km} \end{bmatrix} \]

(15)

Thus, the compensating currents to simulate line outage are given by

\[ \begin{bmatrix} V_{km} \\ S_{km} \end{bmatrix} = \begin{bmatrix} B_{TMkm} & G_{TMkm} \\ G_{TMkm} & B_{TMkm} \end{bmatrix} \begin{bmatrix} V_{km} \\ S_{km} \end{bmatrix} \]

where the matrix on the right-hand side of eqn. 16 is the inversion of the matrix in eqn. 15.