An Approximate Steady-state Characteristic for HVDC Converters Connected to Alternators

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Abstract—This paper attempts a novel formulation for the steady-state output dc voltage of a three-phase alternator fed HVDC converter, expressed as a function of the output dc current of the configuration. The formulation includes a detailed representation of alternator flux-linkages, and involves relevant self and mutual inductance parameters as well as the internal load angle of the machine. It is assumed that under steady-state conditions, all emf contributions by alternator damper current and the transformer emf contribution due to the alternator field current are negligible, so that only the speed emf contribution due to field current needs to be considered.

For different example cases, converter characteristics generated by the new formulation are compared with results of corresponding dynamic simulation studies. The concurrence between the two is found to be far superior as compared to what exists between the conventional unit-connected converter characteristics and dynamic simulation results.

Keywords: HVDC converters, unit connection.

I. INTRODUCTION

The unit-connected HVDC converter has generally been a popular configuration principally due to the economic advantages associated with it [1, 2]. However, to date the steady state performance analysis of such a configuration continues to offer certain interesting challenges [3]. While the nature of this problem is understood to a certain degree of clarity, most solutions that have been suggested are either too elementary to provide accurate output dc characteristics, or achieve good accuracy at the cost of very high computational burden [4, 5].

Through studies and experience, it is now generally accepted that the conventional HVDC converter characteristic [6] does not yield acceptable results when applied to unit-connected converters [3,4]. Fig.1 shows a general HVDC converter configuration comprising of a three-phase ac voltage source, a converter transformer, and a six-pulse Graetz bridge. In the conventional formulation [6], the three-phase source is assumed to have a "stiff voltage magnitude and frequency. If the three phase voltages, referred to the transformer secondary, are specified as

\[ e_a = \frac{E_n}{\sqrt{3}} \cos(\alpha + n/3) \]
\[ e_b = \frac{E_n}{\sqrt{3}} \cos(\cot - n/3) \]
\[ e_c = -\frac{E_n}{\sqrt{3}} \cos(cof) \]

where \( E_n \) being the rms line voltage on transformer secondary, with (co2\pi) as the frequency, then the converter output characteristic is obtained as

\[ V_d = \left(3\sqrt{2} \frac{E_n}{\text{n}}\right) \cos a - \left(3\cot \frac{L}{n}\right) I_d \]

where \( V_d \) is the output dc voltage, expressed as a function of the output dc current \( I_d \), the converter firing angle \( a \), the transformer per-phase leakage inductance \( L \), and the line voltage [6].

The representation (1) for ac phase voltages is invalid in the case of unit-connected converters because, for such configurations, current harmonic filters are generally not included at the ac bus. The "stiff three-phase ac voltage source is replaced by a three phase alternator; and the current harmonics generated by the converter flow unchecked into the alternator windings. As a consequence the air-gap flux of the alternator includes the corresponding harmonic components, which in turn introduce harmonic frequency components in the open circuit emf (and hence, the terminal voltage) of the machine.

Several important inferences follow from the above:

![Fig.1 HVDC converter connected to voltage source through transformer](image)

(i) The output dc characteristic of the unit-connected converter can not be described by (2), since the terminal voltage of the alternator may not be a pure sinusoid of line-to-line rms value $E_n$ and frequency $(col/2)\omega$.

(ii) Any attempt to "modify" (2) by using the open circuit alternator emf as a "stiff voltage may not be very accurate [3, 4], since this emf is generated by the air-gap flux rich in harmonics and can not be represented in a form similar to (1).

(Hi) The so called commutation inductance that is responsible for converter overlap [6], can not be represented adequately as the transformer leakage inductance $L_L$. It must in some way include a detailed representation of alternator inductance parameters, inclusive of relevant space variation functions.

The objective of this paper is to derive and discuss a novel formulation for steady-state output dc characteristic of an unit connected HVDC converter. The principal expressions that define the characteristic are described in section II. Details of the derivation are included as Appendix 1.

In section III, three cases of dynamic simulation studies are presented in order to verify the new formulation; and to compare its effectiveness with that of conventional HVDC converter characteristics. Some details of the simulated systems are included as Appendix 2.

II. THE ALTERNATOR / CONVERTER CHARACTERISTIC

Consider the commutation for which $V_A$ (the incoming valve) takes over the dc current from $V_s$ (the outgoing valve) in the positive valve-group (Fig.1). $V_s$, being the only member of the negative valve-group that is in conduction, carries the entire dc current $I_s$. For this overlap-period (three-valve) and the subsequent dc-period (two-valve), the conventional model of the converter transformer [6] yields an expression for the instantaneous dc-side voltage that is given by

$$V_d = (e_b - dX_{bl}dt) - (e_c - dX_{cl}dt) = (et - ec) - (dl_{bl}dt - dl_{cl}dt) \quad (3)$$

where $e_b$ and $e_c$ are the $b$ and $c$ phase voltages of an ideal three-phase ac voltage source, while $v_b$ and $v_c$ are the total flux linkage of phases $b$ and $c$, respectively. In the conventional model, the flux linkage in each phase is entirely due to leakage in the respective transformer windings.

Equation (3) would apply equally well to a converter fed by a generating unit comprising of a three phase alternator and a transformer, provided the three-phase voltage source (the $e$ variables) and the flux linkages (the $X$ variables) are interpreted correctly for this context. The total flux linkage of a particular phase must now include (i) the magnetising flux due to armature, field, and damper currents, (if) the leakage flux in armature winding, (Hi) the leakage flux in field winding, (iv) the leakage flux in damper circuit, and (v) the leakage flux in the transformer windings. This covers all flux linkages contributing to transformer- and speed-emf s in the ac phase in question, so that the three-phase voltage source (e-variables) must be set to zero. (3) would then reduce to

$$v_d = -d(e_b - X_c)/dt \quad (4)$$

The dc voltage at the converter terminals can be obtained by averaging $v_d$ over the particular dc voltage pulse (spanning over $\pi/3$ radians) as

$$V_d = \left[ \frac{1}{\pi} \int_a^b (d(e_b - X_c)/dt).dcot \right]_{a+\pi/3}^{a+\pi/3} \quad (5)$$

Under the assumptions that, at steady-state, (a) the alternator field current may be assumed to be approximately constant, (b) the alternator damper current may he assumed to be approximately zero, and (c) the alternator speed is approximately constant, (5) is analysed further in Appendix 1 using standard expressions for various alternator inductance parameters [7], In section III, the validity of these assumptions is examined by dynamic simulation of typical cases.

The following expression for $V_d$ is obtained by the analysis described in Appendix 1:

$$V_d = (6\sqrt{3} \, CO/TI)MF/F \cos(a + \frac{1}{2}S) - (3\cos(n/3))[L_t + L_s + M_s + 6L_m'] \quad (6)$$

where,

$$L'_m = L_m \sin(2a + 2d_ - + n/6) \quad (7)$$

$I_l$ and $I_2$ being the steady-state values of dc link current and alternator field current respectively, all referred to the transformer secondary. Of the different inductance parameters that appear in (6-7), $L_s$ is the per-phase leakage inductance of the converter transformer. $M_k$ is the maximum value of the mutual inductance between the field winding and each armature phase winding of the alternator. $L_2$, $L_3$, and $M_k$ are other inductance parameters of the alternator, and are defined as follows [7]:

$$L_s = (L_d + L_q + L_o)/3$$

$$L_m = (L_d - L_q)/3$$

$$M_k = (L_d + L_q - 2L_o)/6$$

$$L_s$$
where $L_d$, $L_q$, and $L_o$ are respectively the direct axis, quadrature axis, and zero sequence inductance parameters of the alternator. Of the different angle variables that appear in (6-7), $a$ is the firing angle of the converter with respect to the fundamental frequency component of alternator terminal voltage. $A_5$ and $A_8$ are functions of the internal power-angle of the alternator, and are defined as

$$ Ad = \frac{(S_{a}+K/3 - S_{b})}{2} $$

$$ \bar{A}_{-} = \frac{(S_{a}+n/3 - S_{b})}{2} $$

where $S_a$, $S_{a+n/3}$ and $S_{a+K/3}$ are the values assumed by the internal power-angle of the alternator at the firing-instant of a converter valve, and at one-sixth of a cycle (i.e., one dc pulse) after the firing-instant, respectively.

As the direct- and quadrature-axis components of alternator flux linkage change due to harmonic effects mentioned above, a corresponding variation of alternator terminal voltage can be expected. As a consequence, the internal power angle $\bar{A}_d$ must differ from $A_8$ and $\bar{A}_8$ must differ from $A_7$, and $A_8$ and $\bar{A}_8$ can change from valve to valve, and $A_8$ and $\bar{A}_8$ can change from $dc$ pulse to $dc$ pulse. Some form of “averaging” of alternate internal power angle may therefore be essential in order to extend the applicability of (5) to sustained steady-state condition. This is taken up further in the next section.

III. CASE STUDIES

In this section three case-studies are presented to judge the correctness and effectiveness of the formulation (6-9):

I. Twelve pulse converter fed by a 100 MVA, 13.8kV alternator; each six pulse bridge fed through a 50MVA, 13.8kV/30.36kV transformer.

II. Twelve pulse converter fed by twelve 100 MVA, 13.8kV, parallel connected alternators; each six pulse bridge fed through a 368.92MVA, 13.8kV/112 kV transformer. The ac side harmonic currents affect each alternator-excitation unit individually. The alternators are however simulated sequentially [8], and this introduces some approximation.

III. Same as II, except that the parallel connected alternators are simulated as a single equivalent alternator. The alternators are collectively affected by the ac side harmonics. This simulation, however, does not involve any error due to sequential simulation of machines.

Cases II and III together provide a complete picture for the 1200MVA generation system. Details of these simulations are provided in Appendix 2.

The simulations use EMTDC version 2.0 on a DOS platform [8]. The converters are provided with controls similar to those used at the rectifier end of typical long distance, bipolar links [8] with the exception that no voltage dependent current order limiter is incorporated in any of the cases. The alternators are provided with suitable static excitation systems.

For each case, the value of dc terminal voltage of the twelve pulse unit is predicted by four methods:

1. Conventional steady-state characteristic, given by (2).
2. Modified conventional characteristic using the fundamental frequency component of the open circuit emf of alternator as the commutation voltage, and a series combination of transformer leakage and alternator subtransient inductance parameters as the commutation inductance [3, 4].
3. Alternator / converter characteristic (6-9), for which the values for current and internal power angle of the alternator are obtained as described below.
4. Dynamic simulation at steady-state, achieved over a 3.0s period.

The alternator field current required for the alternator / converter characteristic is to be equal to the instantaneous value obtained at the end of the 3.0s dynamic simulation (at which point the system is found to have attained steady-state). The values for $S_a$ and $S_{a+n/3}$ and $S_{a+K/3}$ are obtained separately for the firing instants corresponding to $V_6$, $V_5$, and $V_5$ using the last cycle of the 3.0s simulation. The values of $AS$ and $A_5$ are computed for each of these three valves according to (9), and the average $AS$ and $A_5$ are used to compute the characteristics by (6-7).

Before a discussion on the results can be undertaken, the validity of assumptions (a), (b) and (c) described in section II should be examined for each of the three cases. In order to do so, the values of alternator field and damper current are recorded at the end of each 3.0s steady-state simulation mentioned above. In all cases, no perceptible variation is observed in the alternator field current so recorded, indicating that assumption (a) is justified. As reported in [3], the damper current on $d$- and $\alpha$-axis ripple around zero; and for the entire range of dc current in each case, the damper ripple current attains the following levels:

Case I:

$$ i_{dc} \lessgtr 0.022pu \text{ on alternator base (100MVA).} $$

$$ i_{dc} \lessgtr 0.017pu \text{ on alternator base.} $$

Case II:

$$ i_{dc} \lessgtr 0.0024pu \text{ on alternator base (100MVA).} $$

$$ i_{dc} \lessgtr 0.003pu \text{ on alternator base.} $$

Case III:

$$ i_{dc} \lessgtr 0.083pu \text{ on alternator base (1200MW).} $$

$$ i_{dc} \lessgtr 0.003pu \text{ on alternator base.} $$

It follows that for all cases, assumption (b) can be assumed to hold approximately; and since the mutual inductance between
the alternator damper and armature is very small, the transformer and speed emf's induced by the damper currents in the armature may be assumed to be negligible. Assumption (c) is valid for alternators that have high mechanical inertia. This is true for most unit connected HVDC systems, which typically employ hydro generating units [1-2].

The converter dc output voltage $V_d$ as predicted by each of the methods 1 through 4, is plotted for different values of dc current $I_d$. Fig.2 shows the plot obtained for case I, the converter being operated at different values of firing angle (a). Fig.3 and 4 show the plots obtained for cases II and III respectively, in each of which the converter is allowed to operate at the minimum value of a. In each figure, the operating points as predicted by the alternator/converter-characteristic is indicated by solid line.

It is evident that steady-state conditions predicted by the alternator/converter formulation (as compared to predictions by conventional steady-state, and modified characteristics) are in very close agreement with the corresponding dynamic simulation results. For large systems (cases II and III), small discrepancies between the two are observed under certain heavy load conditions (Figs. 3, 4). At these operating points the unit-connected configuration is found to have excessive ripple both in dc voltage and ac current; and as a consequence the values $\delta_a$ and $\delta_{a+\%}$ of the internal load angle of the alternator, also differ significantly from valve to valve. The excessive ripple in instantaneous values of variables at the end of a 3.0s steady-state simulation (as employed in the cases reported here), may therefore introduce some error in the alternator/converter characteristics. Further averaging of variables over several dc pulses can be resorted to in order to obtain better agreement with dynamic simulation results.

IV. CONCLUSIONS AND FUTURE WORK

A fundamental analysis of steady-state dc conditions in unit-connected HVDC converters has been presented in this paper; aiming to relate space variation of alternator inductance parameters with overlap conditions of the converter. Involved harmonic analysis [5, 6] has been carefully avoided.

An important application of steady-state HVDC converter characteristics is in the formulation of ac/dc load flows [4]. There is therefore, a considerable motivation to investigate the applicability of (6-9) in load flow algorithms. This avenue is currently under investigation, and is likely to be reported in future publications.

V. REFERENCES

APPENDIX 1

The following inductance parameters for an alternator have been defined in [7]:

1. Stator self inductance parameters:
   \[ \begin{align*}
   L_{aa} &= L_a + \text{Im} \cos 26 \\
   L_{bb} &= L_s + L_m \cos 2(d - 2n/3) \\
   L_{cc} &= L_s + L_m \cos 2(0 + 2n/3)
   \end{align*} \] (A1.1)

2. Stator mutual inductance parameters:
   \[ \begin{align*}
   L_{ab} &= -M_{s} - M_{m} \cos 2(6 + n/3) \\
   L_{bc} &= L_{cb} = -M_{s} - M_{m} \cos 2(6 - n/3) \\
   L_{ca} &= L_{ac} = -M_{s} - M_{m} \cos 2(6 + n/3)
   \end{align*} \] (A1.2)

3. Stator to field mutual inductance parameters:
   \[ \begin{align*}
   L_{af} &= L_{fa} = M_f \cos 6 \\
   L_{bf} &= L_{fb} = M_f \cos (d - 2n/3) \\
   L_{cf} &= L_{fc} = M_f \cos (6 + 2n/3)
   \end{align*} \] (A1.3)

4. Stator to q-axis damper mutual inductance parameters:
   \[ \begin{align*}
   L_{q}d &= L_{a} + M_d \cos 6 \\
   L_{bd} &= L_{db} = M_d \cos (6 - 2n/3) \\
   L_{cd} &= L_{dc} = M_d \cos (6 + 2n/3)
   \end{align*} \] (A1.4)

Using these definitions, the total flux linkage in phases \( b \) and \( c \) of the alternator (as referred to in section II) are given by [7]

\[ \begin{align*}
\lambda_b &= L_{bb}i_b + L_{bc}i_c + L_{bf}i_a + L_{bd}i_d + L_{bg}i_q \\
\lambda_c &= L_{cc}i_a + L_{ca}i_b + L_{cf}i_d + L_{cd}i_d + L_{cq}i_q
\end{align*} \] (A1.6)

where the different inductance parameters in (A1.6) can be substituted for, using the expressions (A1.1-A1.5). Subtraction of \( X_c \) from \( X_b \) together with some trigonometric simplifications, yields

\[ (X_b - X_c) = \]

\[ \begin{align*}
&= \left[ \sqrt{T L_s \sin (26)} \right] i_a + [L_s + M_s - j M_m \cos (26 + 2n/3)] i_c \\
&\quad + [L - j M_f \sin 6] i_d
\end{align*} \] (A1.7)

If it can be assumed that over the commutation period, \( i_p \ll 0, i_q \ll 0, i_c = I_f \) which is a constant, then (A1.7) reduces approximately to

\[ (\lambda_b - \lambda_c) \approx \]

\[ \begin{align*}
&= \left[ \sqrt{T L_s \sin (26)} \right] i_a + [L_s + M_s - j M_m \cos (26 - 2n/3)] i_b \\
&\quad - [L_s + M_s - j M_f \cos (26 + 2n/3)] i_c
\end{align*} \] (A1.8)

At the beginning of the \( V_i \rightarrow V_j \) commutation period, \( \alpha = a, i_a = I_d, i_b = 0, \) and \( i_c = -I_d \). Designating the value of \( \theta \) at this point of time as \( \theta_a \), (A1.8) simplifies to

\[ (X_b - X_c)_{\theta_a} = \]

\[ \begin{align*}
&= [L_s + M_s - 3L_m \sin (29a - 6n/3)] I_d \\
&\quad + [J M_f \sin 6] I_f
\end{align*} \] (A1.9)

Similarly at the end of the \( V_j \rightarrow V_3 \) commutation period, \( \alpha = a + u \) with \( u \) as the overlap angle, \( i_a = 0, i_b = I_d, \) and \( i_c = -I_d \). Designating this value of \( \theta \) at this point of time as \( \theta_{a+u} \), (A1.8) simplifies to

\[ (\lambda_b - \lambda_c)_{\theta_{a+u}} = \]

\[ \begin{align*}
&= [2L_s + 2M_s - 3L_m \cos (26a + 6n/3)] I_d \\
&\quad + [\sqrt{T M_f \sin (9a + u)] I_f
\end{align*} \] (A1.10)

Subtraction of (A1.9) from (A1.10) gives the total change in flux linkage of the system during \( V_i \rightarrow V_3 \) commutation as
(A_{1.11})

\[
\begin{align*}
(\lambda_b - \lambda_c)_{d+n/3} \simeq & \left[ L_a + M_s - 3L_m \cos 2\theta_a + u \\
- & 3L_m \sin (29_a - n/6) \right] I_d \\
+ & \sqrt{3} M_F [\sin 9_a + u - \sin \theta_a] I_F 
\end{align*}
\]

(A1.12)

During the two-valve conduction period that follows the \( V_i \rightarrow V_{3} \) commutation, \( i_a = 0, i_b = I_b, \) and \( i_c = -I_b. \) The difference of \( b- \) and \( c- \) phase flux linkages can be obtained for any value of \( b \) within this period, by appropriate substitution of these current values in (A1.8), as

\[
(h - Ic) \simeq \left[ 2L_s + 2M_S - 3L_m \cos 29 \right] I_d \\
+ \left[ \sqrt{3} M_F \sin (\frac{2\theta_a + n/3}{2}) \right] I_F 
\]

(A1.13)

A summation of (A1.11) and (A1.13) gives the change in \( (k_a - X_c) \) over the entire dc pulse, that commences with \( V_3 \) receiving a firing pulse and spans over \( TC/3 \) radians.

\[
\frac{\Delta b}{c} \simeq \left[ \frac{\Delta a}{a} \right] \frac{n}{n/3} \simeq \left[ L_s + M_s - 3L_m \cos (2a + n/3) \\
- & 3L_m \sin (2a - n/6) \right] I_d \\
+ & \sqrt{3} M_F [\sin 9_a + n/3 - \sin 9_a] I_F 
\]

(A1.14)

In popular convention for analysis of alternator dynamics [7], the phase-a terminal voltage is assumed to pass through a positive peak at time \( t = 0. \) For such a representation, the angle between the phase-a winding axis and the \( a- \) axis is given by \( 9 = (at + 8 + \pi/2), \) where \( c_0 \) is the alternator frequency in electrical radians per second, and \( 8 \) is the internal load angle of the alternator. For ease of mathematical analysis, in this paper the alternator terminal phase voltages are represented using a convention similar to [6] (which for \( \sinusoidal \ terminal\ voltages, \) becomes identical to (1)). The phase-a voltage is thus \( \pi/3 \) phase advanced over the representation used in [7], so that the angle between the phase-a winding axis and the \( J- \) axis becomes

\[
9 = \cot + S + 5n/6 \quad \text{(A.15)}
\]

With this representation of phase voltages \( V_3 \) is fired at \( x > t = a, \) so that

\[
9_a = a + d_a + 5n/6 \quad \text{(A.16)}
\]

\[
9_a + n/3 = a + S + \pi/2 + 5n/6
\]

where \( 8_a \) and \( 8_{a+n/3} \) are the values of the internal load angle of the alternator at \( at = a, \) and \( at = a + \pi/3, \) respectively. Substitution of the \( 9- \)values from (A1.16) into (A1.14) gives

\[
\begin{align*}
\Delta b & \sim \left[ L_a + M_s - 3L_m \cos (2a + 2\theta_a + n/3 + 3n/3) \\
+ & 3L_m \cos (2a + 2\theta_a) \right] I_d \\
& + \sqrt{3} M_F [\sin (a + \theta_a + n/3 + 5\pi/6)] I_F 
\end{align*}
\]

(A1.17)

Application of the definitions (9) to (A1.17), followed by some simplification, leads to

\[
\Delta b \sim \left[ L_a + M_s - 3L_m \cos (2a + 2\theta_a + n/3) \\
+ & 3L_m \cos (2a + 5\pi/6) \right] I_d \\
& + \sqrt{3} M_F [\sin \left( a + \theta_a + n/3 + 5\pi/6 \right)] I_F 
\]

(A1.18)

Finally, substitution of (A1.18) into (5) results in (6-7), which is the approximate dc characteristic introduced in this paper.

APPENDIX 2

Ratings and selected parameters of components used in dynamic simulations:

**Alternators for cases I, II, and III:**

- 100MVA, 13.8kV, 50Hz three phase, \( x_d = 1.2pu, x = 0.8pu, x_0 = 0.0pu, x' = 0.2pu, x'' = 0.367pu; \) all parameters on alternator base.

**Transformers for case I:**

- 50MVA, 13.8kV / 30.86kV, 50Hz three phase, leakage reactance of 0.05pu per phase on the transformer base.

**Transformers for cases II and III:**

- 368.97MVA, 13.8kV / 112.0kV, 50Hz three phase, leakage reactance of 0.05pu per phase on the transformer base.

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