Correction to "Bi-Iteration Recursive Instrumental Variable Subspace Tracking and Adaptive Filtering"

Peter Strobach

The quasicode listed in Table III on page 2714 of the above paper was reproduced incorrectly. The true quasicode of subspace tracker RIVST 3 is given as follows.

| Q₄(t = 0) = Q₄(t = 0) = |Ir | ; R₄(t = 0) = R₄(t = 0) = 0 |

for 4 = 1, 2, 3, ... for each time step do

Input: x(t), w(t)

ι(t) = Ql(t-1)x(t)
M(t) = Qi(t-1)w(t)

x₄(t) = ||x₄(t) ||^2
φ(t) = 11 w₄(t)

ι(t) = Ql(t-1)x₄(t)

R₄(t) = Γ(t)Γ(t)

Q₄(t) = Q₄(t-1)w₄(t)

Q₄(t) = Q₄(t-1)w₄(t)

\[ \begin{bmatrix} Q₄(t) \\ Q₄(t) \end{bmatrix} = G₄(t) \begin{bmatrix} \phi(t) \\ \phi(t) \end{bmatrix} \]

\[ \begin{bmatrix} Q₄(t) \\ Q₄(t) \end{bmatrix} = G₄(t) \begin{bmatrix} \phi(t) \\ \phi(t) \end{bmatrix} \]

The author is with Fachhochschule Furtwangen, Furtwangen, Germany.

Abstract— This correspondence proposes an alternative scheme for characterizing the wide sense cyclostationary (WSCS) processes and applies it for the analysis of the filter banks whose output is WSCS for wide sense stationary and WSCS inputs. The proposed representation clearly illustrates the kind of transformation taking place in multiscale stochastic systems. These relationships are extended to the filter bank to make clear the effect of the various stages of the filter bank on WSCS processes.

Index Terms—Cyclostationarity, filter banks, multirate.

I. INTRODUCTION

Multirate analysis has aroused a lot of interest in recent times [1]-[3]. The subband coder, which is used as basis for multiresolution analysis, is widely used in signal compression, signal representation, and other such applications [6]-[8]. Filter banks, which play a pivotal role in multirate signal processing, have been widely used and applied. The design of filter banks, for deterministic as well as stochastic signals, has been an area of frenzied activity.

It is well known that the output of a filter bank is wide sense cyclostationary (WSCS) with period M (where M is the rate of decimation and interpolation) [4]. The cyclic spectral densities and the cyclic correlation sequences are normally used for characterizing cyclostationary processes. Ohno and Sakai [5] have derived the cyclic spectral density of the output of a filter bank for WSCS input.

The representation of the cyclostationary processes, in terms of cyclic spectral density, which, although a means of characterization, is not very illustrative, particularly in the context of filter bank analysis. The actual transformations that take place in the various stages (as we move from the input to the output) are not explicitly brought out in this kind of a representation. In this correspondence, an alternative scheme for characterization of cyclostationary processes is presented. The time frequency representation (TFR) is proposed, developed, and later used for representation of the cyclostationary processes. This representation very clearly demonstrates the various transformations that take place as a wide sense cyclostationary process propagates through a filter bank.

The representation scheme proposed in this correspondence, it is conjectured, has the potential of being extended for the representation of nonstationary processes. As the period of a WSCS process tends to infinity, the process becomes nonstationary. This can be utilized to represent the nonstationary processes by letting the period tend to infinity in the proposed representation scheme for the WSCS processes.

This correspondence is organized as follows. Section II introduces the aforementioned alternative representation scheme. Section III contains the input-output relationships in this setting for some basic filter types and configurations. These relationships are shown to clearly illustrate the transformations taking place as a WSCS process.

The authors are with the Department of Electrical Engineering, Indian Institute of Technology, Delhi, India.

Manuscript received March 18, 1997; revised September 29, 1998. The associate editor coordinating the review of this paper and approving it for publication was Dr. Truong Q. Nguyen.

The correlations $B^u(u)$ are a function of both $m$ and $u$. We can, therefore, define a 2-D spectrum of $B^u(u)$, which would draw out the frequency information of $B^u(u)$ both as a function of $m$ and that of $u$. The correlations are functions defined on a group that is the external direct product of the group $(\mathbb{Z}M, +)$ (integers with modulo $M$ addition) and the group $(\mathbb{Z}, +)$ (integers under addition), i.e., $G = (\mathbb{Z}M \times \mathbb{Z})$. A natural definition of the Fourier transform for functions defined on this group is the one used in (10), given below, to calculate the 2-D transform of the correlations [12]. The 2-D spectrum of $B^u(u)$ can be defined as

$$S^\omega_{\text{2-D}}(\omega) = \frac{1}{M} \sum_{m=0}^{M-1} \sum_{u} B^m(u) e^{-j2\pi u \omega} e^{-j2\pi km/M}.$$  

Rearranging the summation in (10) and using (2) and then (3), we get

$$S^\omega_{\text{2-D}}(\omega) = E \left[ \frac{M}{M} \left( \sum_{u} S^m(u) e^{-j2\pi u \omega} \right)^2 \right].$$

Using (5) in (13), we obtain

$$S^\omega_{\text{2-D}}(\omega) = \sum_{m} \sum_{u} B^m(u) e^{-j2\pi u \omega} e^{-j2\pi km/M}.$$  

Equation (14) gives a characterization, in the spectral domain, for nonstationary processes. The correlation sequences of a nonstationary process are a function of the time index at which they are obtained. This two-dimensional (2-D) correlation data, therefore, expects a 2-D spectral domain representation, which is given by (14).

### III. EFFECT OF MULTIRATE FILTERS ON WSCS AND WSS PROCESSES

With the basic framework of TFR for WSS processes, we now, in this section, focus our attention on deriving correlation and spectrum based input-output relationships of certain filters, which are useful in the analysis of filter banks. These relationships, as will be seen later, clearly illustrate the transformations taking place in each of the filters, and are now derived.

1) *LTI Filter:* The input-output relationship of an LTI filter [Fig. 1(a)] is given by

$$y(n) = \sum_{k} h(k)x(n - k).$$

Here, $x(n)$ is WSS with period $M$. The correlations of the output sequence $y(n)$ are then given by

$$B_x(m, n) = \mathbb{E} [x(n - k) y(m - k) h(l) x(n - l - k)].$$

or

$$B^m_x(u) = \sum_{l} h(l) h(k) B^m_{x-l}(u + l - k).$$

However, since $x(n)$ is WSS with period $M$

$$B^m_{x-M-1}(u + l - k) = B^m_{x}(u + l - k).$$
Fig. 1. (a) LTI filter with \( h(n) \) as the impulse response, \( x(n) \) as the input, and \( y(n) \) as the output, (b) Downsampler that downsamples by a factor \( D \) with \( x(n) \) as the input and \( y(n) \) as the output, (c) Upsampler that inserts \( L-1 \) zeros between adjacent samples of the input \( x(n) \) to give the output \( y(n) \).

Using (18) and replacing \( m \) by \( m + M \) in (17), we obtain

\[
B_y^{m+M}(u) = B_x^{m}(u). \tag{19}
\]

Equation (19) shows that \( y(n) \) is WSCS with period \( M \). To obtain the input-output relationship of the spectrums, use (5) in (17)

\[
S_y^{m}(\omega) = H(\omega) \sum h(l)S_x^{m-l}(\omega)e^{j\omega l}. \tag{20}
\]

2) Downsampler: In this discussion, \( M \) is assumed to be a multiple of \( D \). The input-output relationship of a downsampler [Fig. 1(b)] is given by

\[
y(n) = x(Dn). \tag{21}
\]

The correlations of the output are given by

\[
B_y(m,n) = E[x(Dm)x(Dn)]. \tag{22}
\]

Therefore

\[
B_y(a) = B_x^{M}(Du). \tag{23}
\]

Using (23) and the fact that \( x(n) \) is WSCS with period \( M \), we obtain

\[
B_y^{m+M(D)}(u) = B_x^{m}(Du). \tag{24}
\]

Equation (24) shows that \( y(n) \) is WSCS with period \( M/D \).

To obtain the spectrums, we calculate the Fourier transform of respective correlation sequences. It is straightforward to see that the spectrums are given as [10]

\[
S_y \left( \frac{\omega}{M} \right) = \frac{1}{M} \sum_{l=0}^{M-1} S_x(l) e^{-j\omega l}. \tag{25}
\]

3) Upsampler: The input-output relationship of an upsampler [Fig. 1(c)] is given by

\[
y(n) = x(nL). \tag{26}
\]

The output correlations are given by

\[
B_y(m,n) = E[x(mL)x(nL)] \tag{27}
\]

or

\[
h_u(n) = \begin{cases} B_x^{m}(L^{-1}n) ; & mn = Lk, k \text{ int} \\ 0; & \text{otherwise.} \end{cases} \tag{28}
\]

Replacing \( m \) by \( m + ML \) in (28) and using the fact that \( x(n) \) is WSCS with period \( M \), we can show that \( y(n) \) is WSCS with period \( ML \). Again, to obtain the spectrums, we calculate the Fourier transforms of respective correlation sequences of the output. The spectrums are [10]

\[
S_y \left( \frac{\omega}{ML} \right) = \frac{1}{ML} \sum_{l=0}^{M-1} g(l)S_x(l)e^{-j\omega l}. \tag{29}
\]

Using relationships derived above, we now obtain the correlations and spectrums of the output, in terms of the input, of two useful filter configurations (useful in a subband coder and in filter banks). Consider the setup in Fig. 2(a), where \( x(n) \) is WSCS with period \( M \). This implies that \( v(n) \) is also WSCS with period \( M \) (as previously derived) since \( h(n) \) is an LTI. After passing through the downsample (by \( M \)), the process becomes wide sense stationary. Thus, \( y(n) \) is stationary, and its correlation sequence, in terms of correlations of \( x(n) \) and the filter coefficients, is given by [using (17) and (23)]

\[
B_y^{m}(u) = \sum_k b(l)h(k)R_x^{m}(Mu + l - k). \tag{30}
\]

Equation (30) is the desired expression between input and output correlation sequences, and in addition, we can clearly see that if \( x(n) \) is WSCS with period \( M \), then \( y(n) \) is stationary. The spectrum of \( y(n) \) is [using (20) and (25)]

\[
S_y(\omega) = \frac{1}{M} \sum_{k=0}^{M-1} H(\omega - \frac{2\pi k}{M}) \sum_l h(l)S_x^{l-1} e^{-j(\omega - \frac{2\pi k}{M})l}. \tag{31}
\]

Looking at the setup in Fig. 2(b), we see that \( x(n) \) is a wide sense stationary process. Therefore, \( v(n) \) is WSCS with period \( M \). Since \( v(n) \) is WSCS with period \( M \), \( y(n) \) is also WSCS with period \( M \) as \( g(n) \) is an LTI. The correlations of \( y(n) \) in terms of the correlation sequence of \( x(n) \) and the filter coefficients are [using (17) and (28)]

\[
B_y^{m}(u) = \sum_k g(l)h(k)R_x^{m}(\frac{u + l - k}{M}). \tag{32}
\]

where \( l = m - Mp \) and \( k = a + m - Mp \) with \( p \) integer.

Equation (32) shows that if \( x(n) \) is wide sense stationary, then \( y(n) \) is WSCS with period \( M \). The spectrums of \( y(n) \) [in terms of the spectrum of \( x(n) \) and the filter \( g(n) \)] are [using (20) and (29)]

\[
S_y(\omega) = G(\omega) \sum_l g(l)S_x(M\omega)e^{j\omega l}, \tag{33}
\]

where \( l = m - Mk \), \( k \) int.
IV. ANALYSIS OF FILTER BANKS

A filter bank is shown in Fig. 3. The input \( x(n) \) is WSCS with period \( M \). This implies that all \( c_i \)'s are wide sense stationary (as shown in the previous section). The \( O_j \)'s are also jointly stationary [5], as illustrated (by the following discussion) using the alternative setting introduced in this correspondence.

\[
B_{V_iV_j}(m,n) = E \left[ \sum_i h_i(k)x(Mm - k) \sum_l h_j(l)x(Mn - l) \right].
\]

This can be put in the form (using the results obtained in the previous section)

\[
B_{V_iV_j}^{(M)}(u) = \sum_k \sum_l h_i(k) h_j(l) \delta^2(2 \pi k/M) \delta^2(2 \pi l/M) B_{x}(u - k - l),
\]

which shows that the correlations are not a function of \( m \). This implies that the correlation sequence is the same for all time instances, which means that \( V_i \)'s are jointly stationary. The joint spectrum can be put in the form

\[
S_{V_iV_j}(\omega) = \frac{1}{M} \sum_{l=0}^{M-1} H_i(\omega - l) H_j(\omega + l) S_{x}(\omega - 2\pi l/M).
\]

If we fix \( U \) and find the DFT of resulting sequence in \( m \), we obtain the joint cyclic spectrum density derived in [5]. Substituting (36) in (38) to obtain the relationship between the input and the output time-frequency representation

\[
S_{x}(\omega - \omega_k) = \sum_{l=0}^{M-1} \sum_{l=0}^{M-1} g_j(l) e^{j\omega l}.
\]

where \( w = (2\pi - \omega k/M) \), and \( l = m - M \) with \( s \) integer. In (39), \( y(n) \) is WSCS with period \( M \) [even if \( x(n) \) is stationary] because the values of \( l \) depend on the value of \( m \). The results obtained in this section can formally be stated as follows.

Given a filter bank with \( M \) as the decimator and interpolator rate, the output is WSCS with period \( M \) if the input is WSCS with period \( M \) or if the input is stationary. The spectrum-based input-output relationship of the filter bank is given by (39). The correlations- and spectrums-based input-output relationships of the analysis stage are, respectively, given by (35) and (36). In addition, the correlations- and spectrums-based input-output relationships of the synthesis stage are, respectively, given by (37) and (38). We conclude this section with a few remarks on the results obtained in this correspondence.

1) The filter bank consists of two stages: the analysis stage and the synthesis stage. The output of the analysis stage is the \( V_i \)'s obtained by extracting the appropriate frequency content and downsampling. The \( V_i \)'s are jointly stationary if \( x(n) \) is WSCS with period \( M \). They are also jointly stationary if \( x(n) \) is stationary. This is easy to see from (35) by removing the superscript of \( V_i \) (since the correlation sequences of stationary processes are independent of time). Equation (35) also shows how the correlation sequences of \( x(n) \) are weighted by the filter parameters, scaled, translated, and then added to obtain the cross correlation sequence of \( O_j \)'s. Thus, not only is there a convolution sum defined within the sequences, but there is a convolution sum defined across sequences as well. Equation (36) shows how the cross spectrum of \( V_i \)'s is obtained from the spectra of \( x(n) \). Spectra of \( x(n) \) are scaled, weighted by the filter parameters as well as the filter transfer

where \( l = m - M \) with \( k = u + m - M \) with \( p \) and \( q \) integers.

The time-frequency representation of \( y(n) \) can be derived by finding the Fourier transform of (37) with \( u \) as the time variable, and it has the form

\[
S_{y}^{(M)}(u) = \sum_{l=0}^{M-1} \sum_{l=0}^{M-1} g_j(l) e^{j\omega l}.
\]
functions, and then added to obtain the cross spectrum. Thus, there is a convolution of the spectrums of $x(n)$ to obtain the cross spectrum of $v_i$.

2) The synthesis stage regenerates the process from $c_j$’s, which is the output of the analysis stage. If $t_j$’s are jointly stationary, then the output of the filter bank $y(n)$ is WSCS with period $M$.

This is clear from the discussion that precedes the derivation of the input-output relationships of the synthesis stage. This implies that if $x(n)$ is WSCS with period $M$ or if it is stationary, then the output of the filter bank is WSCS with period $M$. From (37) $i^M$’s appear to be independent of $m$, but that is not the case because one of the summations on the RHS of (37) is over $1$, and $I$ depends on $m$. There are $M$ distinct sets of $1$ over which the summation occurs for different values of $m$. This is obvious from the constraint on $1$ that follows (37). The same holds for (38).

3) Equation (39) shows the input-output relationship of the spectrums of the filter bank. It is easy to see that the output is WSCS with period $M$. A convolution of the spectrums of the input leads to the spectrums of the output after they are appropriately weighted by the filter coefficients. If $x(n)$ were wide sense stationary, then $i^M$ would be independent of $m$, and therefore, the superscript of $B_i$ would not appear in (39). Even in this case, the output would be WSCS with period $M$ because of the constraint on $1$, which leads to different sets of $I$ for different values of $m$.

V. CONCLUSIONS

In this correspondence, a time-frequency representation for wide sense cyclostationary processes has been presented. Its relationship with the existing representation scheme, namely, the cyclic spectral density, has been given. A scheme for representing nonstationary processes by extending the representation scheme for WSCS processes is also presented. The proposed representation scheme has been used to illustrate the transformations that take place in certain filters (which are useful in the implementation of filter banks). The output spectrums, in this alternative representation scheme, have been derived in terms of the input spectrums. The input-output relationships of the spectrums and the correlations of a filter bank have also been derived in this new setting.

REFERENCES


Statistical Methods for the Estimation of Quantization Effects in FIR-Based Multirate Systems

Arda Yurdakul and Günhan Dündar

Abstract—In this correspondence, statistical methods are used to estimate the error on the output signal due to quantization of filter coefficients and the fractional part of the data bus in FIR-based multirate systems. In addition, a convex programming model is formed to estimate the filter coefficient wordlengths given the output signal error. The model is capable of handling folded architectures as well as regular ones.

Index Terms—Digital filter wordlength effects, error analysis, finite wordlength effects, mean square error methods, multirate systems, nonlinear programming, quantization, wordlength estimation.

I. INTRODUCTION

Digital signal processing (DSP), which is the most studied area in design automation of electronic systems, has been stimulated by the progression of multirate techniques. Multirate systems are formed by combining digital filter banks with interpolation and decimation units. Among the digital filter banks, FIR ones are usually more preferable in hardware realizations because of their linear phase, stability against quantization, and good performance in cascaded systems. On hardware implementation of multirate systems, filter coefficients have to be of infinite or very high precision because of the finite silicon area. Even though there has been a lot of study on this subject in the literature, none presents a systematic approach for multirate systems [1]-[6].

In this correspondence, a method is developed to estimate the wordlength of a fixed-point FIR-based multirate filter for a given output error. However, the type of quantization step size is important to decrease the error. In this analysis, limiting coefficients of a filter to a wordlength enables the usage of folding, which shrinks the effective chip area. These are explained in the following section. In the design of cascaded systems, data bus limitations have to be taken into account because a common trend is the usage of a fixed-size data bus for both intra- and inter-chip operations (Section III). Then, a model can be easily formed to estimate the optimal chip area for a given output error, as described in Section IV. Finally, Section V

Manuscript received September 23, 1997; revised November 23, 1998. The associate editor coordinating the review of this paper and approving it for publication was Dr. Xiang-Gen Xia.

The authors are with the Department of Electrical Engineering, Bogazıçi University, Instanbul, Turkey.

Publisher Item Identifier S 1053-587X/99/03690-9.