Abstract — Using a series approach, expressions for the average symbol error probability (SEP) of coherent binary signals over a correlated Rayleigh fading channel with dual predetection equal gain combining (EGC) and selection diversity (SD) are derived. For both EGC and SD cases, the SEP is in terms of the correlation coefficient of the branch amplitudes, which is easy to compute and depicts clearly the effect of correlated fading on the error performance.

I. INTRODUCTION

Previously published studies of the performance analysis of diversity combining in Rayleigh fading with both independent [1] [2] [3] and correlated diversity branches [4] [5] [6] have focused mainly on maximal ratio combining (MRC) [1] [3] [4] [5]. Equal gain combining (EGC) with coherent binary keying was studied with two and three independent branches in [6]. Analysis for selection diversity (SD) with two correlated branches was presented in [7], where the results were expressed in terms of finite integrals. In this paper we use a series approach instead of an integration approach to derive computationally simple expressions for the average symbol error probability (SEP) for coherent detection of binary signals with dual-diversity predetection EGC and SD. Both the cases of equal branch signal-to-noise-ratios (SNR)’s and unequal branch SNR’s are included in our framework.

11. PRELIMINARIES

Let \( \alpha_1 \) and \( \alpha_2 \) be two correlated random variables which are marginally Rayleigh with second moments

\[
E(a_i) = R_i, \quad i = 1, 2. \tag{1a}
\]

and correlation coefficient

\[
\text{cov}(\alpha_i, \alpha_j) = \rho \neq 0, \quad 5\rho < 1. \tag{1b}
\]

The joint probability density function (p.d.f.) of \( \alpha_1, \alpha_2 \) is given by

\[
f_{\alpha_1, \alpha_2}(r_1, r_2) = \frac{e^{-\frac{1}{2} \left( \frac{r_1^2}{R_1} + \frac{r_2^2}{R_2} \right)}}{R_1 R_2 \left( 1 - \rho \right) \left( 1 - 2 \rho \right)^{\frac{1}{2}}} 
\times \frac{r_1^{\frac{1}{2} - 1} r_2^{\frac{1}{2} - 1}}{\left( 1 - \rho \right)^{\frac{1}{2}}}, \quad r_1, r_2 \geq 0. \tag{2}
\]

The joint cumulative distribution function (c.d.f.) of \( \alpha_1, \alpha_2 \) can be expressed in terms of the infinite series [8]

\[
F_{\alpha_1, \alpha_2}(r_1, r_2) = \left( 1 - \rho \right) \sum_{n = 0}^{\infty} \frac{\left( \frac{1}{2} \right)^n}{n!} \Gamma \left( k + 1, \frac{r_1^2}{R_1 (1 - \rho)} \right) \times \Gamma \left( k + 1, \frac{r_2^2}{R_2 (1 - \rho)} \right), \quad n, r_2 > 0, \tag{3}
\]

where the incomplete gamma function \( \Gamma(k + 1, s) \) has the representation

\[
\Gamma(k + 1, x) = \int_0^x u^k e^{-u} du = k! \left[ 1 - e^{-x} \sum_{n=0}^{k} \frac{x^n}{n!} \right]. \tag{4}
\]

In the performance analysis with EGC, we will use the joint characteristic function (c.f.) of \( \alpha_1, \alpha_2 \), which is given by [1] (p. 409)

\[
\Psi_{\alpha_1, \alpha_2}(j\omega_1, j\omega_2) = E \left\{ e^{j(\omega_1 \alpha_1 + \omega_2 \alpha_2)} \right\} = G_0(j\omega_1; \Omega_1, \rho) \times G_0(j\omega_2; \Omega_2, \rho), \tag{5}
\]

where, for \( i = 1, 2, \)

\[
G_0(j\omega; \Omega_i, \rho) = \Gamma(k + 1, \frac{1}{2}) \Gamma \left( k + \frac{1}{2}, -\frac{\Omega_i (1 - \rho)}{2} \right) \times \Gamma \left( k + \frac{1}{2}, -\frac{\Omega_i (1 - \rho)}{2} \right). \tag{6}
\]

Note that \( \Gamma(\cdot) \) denotes the gamma function and \( l^{-l}(\cdot; \cdot; \cdot) \) denotes the confluent hypergeometric function.

For SD, we will use the c.f. of

\[
7_{\alpha_i} = \max \left\{ \frac{1}{c_i} \alpha_i \right\} \tag{7}
\]

where \( c_1 \) and \( c_2 \) are positive scale factors. Now the c.d.f. of \( \gamma_{\alpha_i} \) can be expressed as

\[
F_{\gamma_{\alpha_i}}(v) = F_{\alpha_1, \alpha_2} \left( c_1 \left( v^\frac{1}{2}, c_2 v^\frac{1}{2} \right) \right), \tag{8}
\]

and, using (3) and (4), its p.d.f. is given by

\[
f_{\gamma_{\alpha_i}}(v) = \frac{d}{dv} F_{\alpha_1, \alpha_2} \left( c_1 v^\frac{1}{2}, c_2 v^\frac{1}{2} \right). \tag{9}
\]
\[ b_k = v_0, \]  
\[ \text{if } k = 1,2. \]  
\[ \text{(9)} \]

To obtain the c.f. of \( G_{\text{D}} \), we use the result \[ f_{G_{\text{D}}}(u) = J_0(u) \]  
\[ \text{in } (9). \]  
This gives
\[ f_{G_{\text{D}}}(u) = (1 - \rho) \left\{ \frac{\theta_k}{(1 - \rho) - j u} + \frac{\theta_k}{(1 - \rho) - j u} \right\}^{-1} \]  
\[ - \sum_{k=0}^{\infty} \rho^k \sum_{i=0}^{k} \left( \frac{k^2 + 1}{k!} \right) \left( \frac{j u}{1 - \rho - j u} \right)^i. \]  
\[ \text{(11)} \]

### 111. EQUAL GAIN COMBINING

We consider a coherent dual-diversity reception system with a correlated flat Rayleigh fading channel, in which the receiver employs matched filter detection. With EGC, the received signals of each diversity branch are co-phased, combined, and coherently demodulated. The complex baseband signal received over the kth diversity branch in a symbol interval \( 0 \leq t < T \), can be represented as
\[ Q(t) = a_k e^{j e_k S(t)} + n_k(t), \quad k = 1,2, \]  
\[ \text{(12)} \]
where \( s(t) \) is the information-bearing signal, \( Y_k \) and \( e_k \) are the fading magnitude and phase of the kth diversity branch, and \( n_k(t) \) represents the additive noise. The noises \( n(t), n(t) \) are assumed to be independent zero-mean complex white Gaussian random processes with two-sided power spectral densities \( 2N_0 \) and \( 2N_0 \) respectively. We also assume independence among the random sequences \( \{Y_k\}, \{e_k\} \) and \( \{n_k(t)\}. \)

The fading magnitudes \( 0_1, 0_2 \) are assumed to be correlated Rayleigh random variables satisfying (1) with joint p.d.f. \( f_{u_1, u_2} \) given by (2).

We focus on the coherent detection of binary signals in which, over a symbol interval, \( s(t) = s_i(t) \) if symbol \( i \) is transmitted, where \( i = 0,1 \). The complex waveforms \( s(t) \) and \( s(t) \) have support \( [0, T) \) and satisfy
\[ \int_0^T |s_i(t)|^2 dt = 2E_s, \quad i = 0,1, \]  
\[ \mathcal{R}\left\{ \int_0^T s_i(t) s_i(t) dt \right\} = 2E_s, \quad -1 \leq \epsilon < 1. \]  
\[ \text{(13)} \]
\[ ^{1}\text{The notation } \mathcal{R}\{ \} \text{ stands for the real-part operator.} \]

Note that \( E \) is the signal correlation coefficient (correlation coefficient of \( s(t) \) and \( s(t) \)). The decision rule of the receiver is given by
\[ \mathcal{R}\left\{ \sum_{k=0}^{2} \int_0^T r_k(t) [S_i(t) - s_i(t)] dt \right\}, \quad 0 \leq k < 1. \]  
\[ \text{(14)} \]

This can be simplified using (13) to yield (6)
\[ \begin{align*} 
D_1 & = + (0_1 + 0_2) + (W_1 + W_2) > 0, \\
& \text{if symbol } 1 \text{ is transmitted,} \\
& 0, \\
D_2 & = - (0_1 + 0_2) + (W_1 + W_2) < 0, \\
& \text{if symbol } 0 \text{ is transmitted,} \\
& 0.
\end{align*} \]  
\[ \text{(15a)} \]
\[ W_k = \frac{1}{2} \int_{-\infty}^{\infty} \mathcal{R}\{ e^{j \omega} \int_0^T s_k(t) s_k(t) dt \} e^{-j \omega} d\omega. \]  
\[ \text{(15b)} \]
are independent zero-mean real Gaussian random variables with variances
\[ \mathcal{E}\{ W_k^2 \} = \frac{N_0}{E_s(1 - \epsilon)}, \quad k = 1,2. \]  
\[ \text{(15c)} \]
By symmetry of \( W_1 + W_2 \), the average SEP is given by
\[ P_{\text{E,EGC}} = P_{\text{D}_1} < 0 = P_{\text{D}_2} > 0. \]  
\[ \text{(16)} \]
If \( F(-\cdot) \) denotes the c.d.f. of the decision variable \( D_1 \) and \( \Psi(-\cdot) \) denotes its c.f., then, invoking the inversion theorem \[ [11], \] we get from (16) \[ ^{2}\text{The notation } \mathcal{R}\{ (-\cdot) \} \text{ stands for the real-part operator.} \]
\[ \begin{align*} 
P_{\text{E,EGC}} &= F_{D_1}(0) = \frac{1}{2} - \frac{1}{\pi} \int_{-\infty}^{\infty} \mathcal{R}\{ \Psi_{-D_1}(j \omega) \} e^{-j \omega} d\omega. \end{align*} \]  
\[ \text{(17)} \]

The c.f. of \( D_1 = (a_1 + c_2) + (W_1 + W_2) \) is given by
\[ \Psi_{D_1}(j \omega) = \Psi_{W_1}(j \omega) \Psi_{W_2}(j \omega) \Psi_{u_1 + u_2}(j \omega) \]  
\[ = e^{-\frac{1}{2} \epsilon^2} \int_{-\infty}^{\infty} \mathcal{R}\{ \psi_{W_1}(j \omega) \psi_{W_2}(j \omega) \} \psi_{u_1 + u_2}(j \omega) d\omega, \]  
\[ \text{(18)} \]
where \( \psi_{W_1}(j \omega) \) can be obtained from (5). Noting that
\[ \Gamma(k + 1) = k!, \quad \Gamma \left( k + \frac{3}{2} \right) = \frac{(2k + 1)!!}{k!2^{k+1}} \]  
\[ \text{(19)} \]
\[ ^{3}\text{The notation } \psi_{W_1}(j \omega) \text{ stands for the imaginary-part operator.} \]
we get from (18) and (5) the relation
\[
\mathcal{G}(\mathbf{H}, \mathbf{K}) = e^{-\frac{\omega^2}{4}} \sum_{k=0}^{\infty} \frac{(2k+1)!}{(2k+1)m!} \left( \frac{\omega}{4} \right)^{2k+1} \times \left[ \begin{array}{c} \Omega_1^2 g_k(\omega; \Omega_1[1 - \rho], \Omega_2[1 - \rho]) \\
+ \Omega_2^2 g_k(\omega; \Omega_2[1 - \rho], \Omega_1[1 - \rho]) \end{array} \right].
\]

where
\[
g_k(\omega; a_1, a_2) = \sum_{m=0}^{k} \frac{(-1)^{m} \left( \omega^{2} \right)^{m}}{m!(2k+1-2m)!},
\]

Now [9] (p. 1074)
\[
\left. \frac{d}{d\omega} \right|_{\omega=0} \left[ \begin{array}{c} \Omega_1^2 \\
+ \Omega_2^2 \end{array} \right] = \frac{1}{2} \sum_{k=0}^{\infty} (-1)^{k} \left( \frac{\omega^2}{4} \right)^{2k} \left( \frac{1}{2} \right)^{k+1} \times \left( \begin{array}{c} \Omega_1^2 \\
+ \Omega_2^2 \end{array} \right)
\]

where \( H_{2k+1}(\omega) \) is the Hermite polynomial of order \( 2k + 1 \).

Using the result [12] (7.621(4))
\[
\int_{-\infty}^{\infty} e^{-\omega^2} f(\omega) d\omega = \sqrt{\pi} \text{erf} \left( \frac{a}{\sqrt{2}} \right)
\]

in (23), we get
\[
h_k(a_0, a_1, a_2) = (-1)^{k} \frac{\omega^2}{4} \left( \begin{array}{c} \Omega_1^2 \\
+ \Omega_2^2 \end{array} \right) \left( \begin{array}{c} \Omega_1[1 - \rho], \Omega_2[1 - \rho] \\
\end{array} \right).
\]

Now (23), (22) and (24) yield
\[
\text{SNR}_3 = \left( \frac{\Omega_2 E_s}{\text{SNR}_3} \right)^{\frac{1}{2}}
\]

and let
\[
g = \frac{1 - e^{-iu}}{2}.
\]

Using the fact that
\[
\Gamma \left( k - m + \frac{1}{2} \right) = \frac{(2k - 2m)!}{(k - m)!} \left( \frac{1}{2} \right)^{k-m}
\]

and changing the summation index \( m \) to \( k - m \) in (24), we combine (25), (24), and (17) to obtain the final expression for the SkP, which is
\[
P_{e,E,Gc} = \frac{1}{2} \sum_{k=0}^{\infty} (2k)! \left( \begin{array}{c} \text{SNR}_3 \\
+ \text{SNR}_2 \end{array} \right)^{2k} \times \left( \begin{array}{c} \text{SNR}_1 \\
+ \text{SNR}_2 \end{array} \right)^{k-m+\frac{1}{2}}
\]

\[
\sum_{m=0}^{k} \frac{(-1)^{m} \left( \text{SNR}_3 \right)^{m}}{m!(2k+1-2m)!} \left( \begin{array}{c} \text{SNR}_1 \\
+ \text{SNR}_2 \end{array} \right)^{m+\frac{1}{2}}
\]

\[
\times \left( \begin{array}{c} \text{SNR}_1 \\
+ \text{SNR}_2 \end{array} \right)^{k-m+\frac{1}{2}}
\]

\[
\times \left( \begin{array}{c} \text{SNR}_1 \\
+ \text{SNR}_2 \end{array} \right)^{k-m+\frac{1}{2}}
\]

\[
(27)
\]

934
The expression (27) is in terms of a series of powers of \( p \), enabling easy computation of the SEP, and quantifying the effect of \( p \) on the SEP.

In the case of independent branches (\( p = 0 \)), only the \( f_c = 0 \) term of the summation over \( k \) in (27) is non-zero. Using the fact that [13] (pp. 556-561)

\[
2F_1\left( -\frac{1}{2}, \frac{1}{2}; 1; z \right) = (1 - z)^{\frac{1}{2}},
\]

(27) simplifies to

\[
P_{e,\text{EGC}} = 4 \sqrt{SNR_1 (SNR_1 + 1)} + \sqrt{SNR_2 (SNR_2 + 1)}
\]

which is the same as equation (23) of [6].

IV. SELECTION DIVERSITY

In this case, the SEP, conditioned on the instantaneous output SNR \(-y_sD\), for coherent binary keying can be expressed as

\[
P_{e,SD}(\gamma_{SD}) = Q \left( V(1 - e^{-\gamma_{SD}}) \right) = Q \left( \sqrt{2\gamma_{SD}} \right),
\]

where \( \gamma_{SD} = \max \left\{ \frac{E_0 \alpha_1^2}{N_0}, \frac{E_2 \alpha_2^2}{N_0} \right\} \).

The c.f. of \( \gamma_{SD} \) is given by (11) with

\[
\gamma = \frac{N_0 \gamma}{10^8(1 - e^{-\gamma})}, \quad k = 1, 2.
\]

The average SEP is given by

\[
P_{e,SD} = E[\gamma_{SD}(\cdot)]
\]

\[
= \int_0^\infty P_{e,SD}(\gamma) f_{SD}(\gamma) d\gamma.
\]

Using the technique of [14], the average SEP can be written in terms of the c.f. of \( \gamma_{SD} \) as

\[
P_{e,SD} \approx \frac{1}{\pi} \int_0^\infty \sin^{2k} \phi \left( 1 - \frac{1}{\sin^2 \phi} \right) d\phi,
\]

where \( \gamma_{SD}(\cdot) \) is given by (11). We can therefore rewrite (32) as

\[
P_{e,SD} = \frac{1}{\pi} \int_0^\infty \sin^{2k} \phi \left( 1 - \frac{1}{\sin^2 \phi} \right) d\phi.
\]

The definite integral

\[
\int_0^{\pi} \sin^{2k+1} \phi \sin^2 \phi d\phi = \alpha_k, \quad k = 0, 1, 2, \ldots
\]

can be evaluated to yield

\[
J_k(a) = 1 + \sum_{l=1}^k (k+l) \left( \frac{a}{a+1} \right)^{l-1} \times \sum_{m=0}^{l-1} \left( 1 - 1 \right) \left( \frac{2m}{m} \right) \frac{1}{(4a)^m}.
\]

Applying (34) in (33), we get

\[
P_{e,SD} = \frac{1}{2} \left[ J_1 \left( \frac{g}{b_1(1 - \rho)} \right) + J_1 \left( \frac{g}{b_2(1 - \rho)} \right) \right.
\]

\[
- \frac{1 - \rho}{2} \sum_{k=0}^\infty \sum_{i=0}^k \left( k+i \right) \times \left( \frac{b_1+b_2}{b_1+b_2} \right)^{k+i+1} \cdot J_{k+i+1} \left( \frac{g}{b_1+b_2} \right).
\]

Let the branch SNR's be defined in this case by

\[
SNR_1 = \frac{\Omega_1 E_o}{\Omega_0}, \quad SNR_2 = \frac{\Omega_2 E_o}{\Omega_0},
\]

On further simplification of (36) using (30) and (37), we get the final expression for the SEP, which is

\[
P_{e,SD} = \frac{1}{2} \left[ 2 - \sqrt{\frac{SNR_1}{2 \Omega_0 SNR_1 + 1}} - \sqrt{\frac{SNR_2}{2 \Omega_0 SNR_2 + 1}} \right.
\]

\[
- \frac{1 - \rho}{2} \sum_{k=0}^\infty \sum_{i=0}^k \left( k+i \right) \times \frac{(SNR_1 SNR_2)^{k+i+1}}{(SNR_1 SNR_2)^{k+i+1}} \times J_{k+i+1} \left( \frac{g}{SNR_1 SNR_2} \right),
\]

where \( J_{k+i+1}(\cdot) \) is given by (35). Like the EGC expression (27), (38) is also in terms of a series of powers of \( P \).

For independent branches (\( p = 0 \)), only the \( k = 0 \) term of the summation over \( k \) in (38) is non-zero, and we get

\[
P_{e,SD} \approx \frac{1}{2} \left[ 1 - \sqrt{\frac{SNR_1}{2 \Omega_0 SNR_1 + 1}} - \sqrt{\frac{SNR_2}{2 \Omega_0 SNR_2 + 1}} \right.
\]

\[
+ \frac{1 - \rho}{2} \sum_{k=0}^\infty \sum_{i=0}^k \left( k+i \right) \times \frac{(SNR_1 SNR_2)^{k+i+1}}{(SNR_1 SNR_2)^{k+i+1}} \times J_{k+i+1} \left( \frac{g}{SNR_1 SNR_2} \right),
\]

which is a known result [16].

V. NUMERICAL RESULTS

The SEP of BPSK, which corresponds to \( g = 1 \), is plotted against the branch SNR \( SNR_1 \) for EGC in Fig. 1 and for SD in Fig. 2, with different values of the correlation coefficient \( p \) (\( p = 0, 0.5, 0.7, 0.9 \)). We have considered both
the situations of equal branch SNR’s (SNRI = SNR2) and unequal branch SNR’s (SNR1 > SNR2).

In the case of EGC, the Gaussian hypergeometric functions in (27) have been calculated using a truncated series formula with a relative error tolerance of 0.001. In computing P_{EGC}, the number of terms taken in the summation over k in (27) are: 15 for p = 0.5, 25 for p = 0.7, and 50 for p = 0.9. The maximum relative error obtained over all P_{EGC} computations is 2.39 %. In the case of SD, the number of terms considered in the summation over k in (27) for computing P_{sd} are: 15 for p = 0.5 and 25 for p = 0.7, 0.9, and the maximum relative error obtained over all P_{sd} computations is 1.28 %.

The plots reveal that the SEP decreases with increase of branch SNR for a given p, and for given branch SNR’s, the SEP increases with increase of p. In addition, the plots indicate that the performance with equal branch SNR’s is better than that with unequal branch SNR’s.

REFERENCES


