Channel Capacity in Evenly Correlated Rayleigh Fading with Different Adaptive Transmission Schemes and Maximal Ratio Combining

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Abstract — We present closed-form expressions for the single-user capacity over slow nonselective correlated Rayleigh fading channels having equal branch powers and the same correlation between any pair of branches. Maximal ratio combining (MRC) is used and three adaptive transmission schemes are analyzed: (1) optimal simultaneous power and rate adaptation, (2) optimal rate adaptation with constant transmit power, (3) channel inversion with Axed rate.

1. INTRODUCTION

Consider the coherent reception of some digitally modulated signal with $L$ diversity branches and predetection MRC. Let $\gamma_i$, $i = 1, \ldots, L$, denote the instantaneous signal-to-noise ratio (SNR) of the $i$th diversity branch. The random variables $\gamma_i, i = 1, \ldots, L$, are identically distributed, each having a marginal distribution which is Rayleigh with second moment $\gamma_i^2$, and therefore the marginal distribution which is Rayleigh with second moment $\gamma_i^2$.

The following expression for its probability density function:

$$ P_{\gamma_i}(\delta) = \frac{1}{\gamma_i^2} \left[ \left( \frac{r}{\sigma_i^2} \right)^{L-1} e^{-r} \sum_{k=1}^{L-1} \left( \frac{r}{\sigma_i^2} \right)^{L-k} \right], \forall \delta \geq 0. $$

The case of i.i.d. branches ($p = 0$) has been analyzed in [1].

11. CHANNEL CAPACITY

Under the condition of optimal simultaneous power and rate adaptation, the channel capacity $\text{Cora}$ (in bits/sec) is given by [2] [1] $\text{Cora} = \eta I \gamma (\delta + \eta I) f(\delta + \eta I) d\delta$, where $B$ (in Hz) is the channel bandwidth and 70 is the optimal cutoff SNR satisfying $I^*(\delta) = \eta I f(\delta) d\delta = 1$.

Denoting the event integral of order one by $E_{\gamma}(\sigma) = \gamma I e^{-\gamma I} (\delta + \eta I)^{-1} f(\delta + \eta I) d\delta$, and the Poisson distribution by $p_{I}(\sigma) = e^{-\sigma I} \sigma I^{\sigma I} / \sigma I!$ the following closed-form expression for the capacity per unit bandwidth (in bits/sec/Hz):

$$ \frac{C_{\text{Cor}}}{B} = \frac{1}{I \gamma} \left[ E_{\gamma}(r \gamma_0) \left( \frac{r}{\sigma_i^2} \right)^{L-1} - E_{\gamma}(r \gamma_0) \left( \frac{r}{\sigma_i^2} \right)^{L-1} \right] + \sum_{n=1}^{L-2} \frac{E_{\gamma}(r \gamma_n)}{n} \left( \frac{r}{\sigma_i^2} \right)^{L-n-1} - 1 \right]. $$

Since the transmission in suspended when $7 < 70$, there is an outage probability which is given by

$$ P_{out} = 1 - \left( \frac{r}{\sigma_i^2} \right)^{L-1} e^{-r \gamma_0} + \frac{L-1}{1} \sum_{k=1}^{L-1} \left( \frac{r}{\sigma_i^2} \right)^{L-k} P_{\gamma}(r \gamma_0). $$

In the case of optimal mte adaptation with constant transm-...r, the channel capacity is given by [3] [2] [1] $\text{Cora} = \eta I \gamma (1 + \gamma) f(\delta + \gamma) d\delta$, which yields the following expression for the capacity per unit bandwidth:

$$ \frac{C_{\text{Cor}}}{B} = \frac{1}{I \gamma} \left[ \left( \frac{r}{\sigma_i^2} \right)^{L-1} e^{-r} \sum_{k=1}^{L-1} \left( \frac{r}{\sigma_i^2} \right)^{L-k} \right] \times \left\{ P_{\gamma}(r \gamma_0) + \sum_{n=1}^{L-1} \frac{1}{n} P_{\gamma}(r \gamma_n) P_{\gamma}(r \gamma_n - 1) \right\}. $$

In the case of channel inversion with Axed rate, there are two schemes: truncated channel inversion with fixed rate, and channel inversion with fixed rate without truncation. With the truncation scheme, the channel capacity per unit bandwidth is expressed as [1]

$$ \frac{C_{\text{Cor}}}{B} = \frac{1}{I \gamma} \ln \left[ 1 + \sum_{n=1}^{L-1} \ln \left( \frac{r}{\sigma_i^2} \right) d\delta \right], $$

where $P_{out}$ is given by (1). The cutoff level 70 can be chosen either to achieve a specific outage probability $P_{out}$, or to maximize (2). A closed-form expression for the capacity can be obtained from (2). If we set $70 = 0$ in (2), we get $C_{\text{Cor}}$, the capacity for channel inversion with fixed rate and without truncation. In this case, $P_{out} = 0$.

111. NUMERICAL RESULTS

From plots of the channel capacity per unit bandwidth, we find that the capacity increases with increase of diversity order $L$ and increase of average received SNR per branch $E[\gamma_i] = 2 \gamma_i$, as expected. While the capacities $C_{\text{Cor}} / B$, $C_{\text{Cor}} / B$ and $C_{\text{Cor}} / B$ decrease with increase of $p$, the capacity $C_{\text{Cor}} / B$ increases sharply with $p$ for small positive values of $p$, reaches a maximum, and then decreases as $p$ increases further. It is also to be noted that the decrease in capacity with increase in $p$ is much sharper in the case of optimal power and rate adaptation as comparison to the other schemes. In the case of truncated channel inversion, the cutoff SNR 70 which maximizes the capacity decreases with increase of $p$. A comparison of the plots for the different schemes shows that for the same channel bandwidth $B$, $C_{\text{Cor}} > C_{\text{Cor}} > C_{\text{Cor}}$.

REFERENCES

