COUPLED RADIATIVE AND CONDUCTIVE THERMAL TRANSFERS ACROSS TRANSPARENT HONEYCOMB INSULATION MATERIALS

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Abstract—Combined radiative and conductive heat transfer across transparent, square-celled, honeycomb insulation is investigated. The governing equations for one-dimensional heat transfer are formulated using an exponential kernel approximation. The resulting non-linear equation is linearised by least-squares approximation and solved analytically. Computational results for conductive, radiative and total heat transfers through the transparent insulation material (TIM) of three bounding plate conditions: black-black, selective-black and selective-selective are presented. For the case of selective bounding plates, there is a strong coupling of conductive and radiative heat transfer because of the intermediate wall emissivity of the TIM. The effect of cell aspect ratio, wall emissivity and the absorber temperature on the total heat transfer is also studied and it is pointed out that the following configurations of TIM have the lowest heat loss coefficients: (1) TIM with black end plates and cellular walls of high emissivity; (2) TIM with selective end plates and cellular walls fully transparent to IR radiation.

Keywords—Coupled radiative conduction, transparent honeycomb device, thermal insulation, solar thermal system.

1. INTRODUCTION

Transparent insulation materials (TIM) are characterized [1] by good thermal insulation and high transmission for radiation in the range of the solar spectrum. These properties suggest a large range of their applications for efficient solar heat recovery [2] using high temperature flat-plate collectors, integrated collector cum storage systems, building wall/roof heating and solar desalination, as well as outdoor systems such as industrial storage tanks and transport pipelines of hot fluids. TIM consists of a transparent cellular structure immersed in an air layer. It can be classified based on various parameters but the following four types, based on geometry, describe them in detail [2]:

(1) absorber-parallel;
(2) absorber-vertical;
(3) cavity structures;
(4) homogeneous.

The most documented version of the TIM is the absorber-vertical type; it includes honeycombs and parallel slat arrays.

Solar transmittance of TIM has been studied in detail both experimentally and theoretically using Monte Carlo ray tracing algorithms [3, 4] in earlier days and by explicit models [5–7] in recent years. Heat is transferred through TIM by convection through the cell and the conduction and radiation through the cell and wall. By properly choosing the dimensions of the cellular structure, the convection through the cell can be suppressed; it has been demonstrated both theoretically and experimentally [8–10]. The total heat loss across such a TIM has often been calculated by an analysis [11–13] which assumes that the total heat loss is the sum of pure conductive (calculated by ignoring radiation) and pure radiative (calculated by ignoring conduction) losses and this method gives results in agreement with experiments for the black end plates [13]. However, when either the absorber or the top cover is of the selective type, the above method gives erroneous results because of strong coupling of conductive and radiative heat transfer [14, 15]. In the present paper, the coupled mode heat transfer across the TIM is investigated in detail. The problem is treated in similitude with the coupled conductive and radiative heat transfer through a plane layer of
absorbing and emitting gas [16]. The aim of the investigation is to evolve the materials and design characteristics of cellular structures embedded in TIM.

2. MODEL AND ANALYSIS

The propagation of IR radiation through a transparent cellular structure immersed in an air layer is portrayed in Fig. 1(a). The IR radiation which is striking the side walls is partly reflected, partly transmitted and the remainder absorbed. For every ray which is leaving the cell wall by transmission, there is another ray entering from the adjacent cell in such a way that it is a perfect mirror reflection of the original incident ray. So for modelling the radiative heat transfer a single cell can be considered with half the original thickness, total reflectance equal to the sum of the reflectivity and transmissivity and emittance equal to emissivity of the wall itself [13]. The side walls are assumed to be of specular reflectors and diffuse emitters and the two bounding plates as diffuse emitters and reflectors. Following Edwards and Tobin [17], the net radiative heat flux at a distance $x$ from the absorber plate is given by

$$q_r(x) = \sigma T_4^4 \tau_p(x) - \sigma T_4^4 \tau_p(L - x) - \int_0^L \sigma T_4^4 \frac{dx'}{(x' - x)} \frac{d\tau_p(x' - x)}{d(x' - x)} + \int_{x}^{L} \sigma T_4^4 \frac{dx'}{(x' - x)} \frac{d\tau_p(x' - x)}{d(x' - x)} \frac{d\tau_p(x' - x)}{d(x' - x)} \frac{dx'}{dx},$$  \hspace{1cm} (1)

Fig. 1. (a) Schematic of IR radiation propagation through TIM; (b) plane layer of absorbing and emitting medium.
$T_1$ and $T_2$ are the bottom (hot absorber) and top (cold cover) plate temperatures and $\tau_p(x)$ is the diffuse radiation transmittance which is emitted by the absorber for a height $x$ and given by

$$
\tau_p(x) = \frac{\int_{-\pi/2}^{\pi/2} \int_{0}^{\pi/2} T_{br}(\theta, r) \cos \theta \sin \theta \, d\theta \, dr}{\int_{-\pi/2}^{\pi/2} \int_{0}^{\pi/2} \cos \theta \sin \theta \, d\theta \, dr}, \quad (2)
$$

$T_{br}(\theta, r)$ is the direct radiation transmittance of the TIM for the distance $x$ with angle of incidence $\theta$ and azimuth angle $r$ calculated using the procedures outlined by Hollands et al. [5] and Kaushika et al. [6].

The total conductive heat flux through the TIM is given by

$$
q_c = -ke \frac{dT}{dx}. \quad (3)
$$

The equivalent area weighted thermal conductivity ($ke$) of the TIM in the above equation is

$$
ke = \frac{(Ac \ast ka + Aw \ast kw)}{(Ac + Aw)}, \quad (4)
$$

where $Ac$, $Aw$ and $ka$, $kw$ represent the area and thermal conductivities of the cell and wall, respectively.

The energy equation for the TIM with combined conduction and radiation under thermal equilibrium is given by

$$
-ke \frac{d^2T}{dx^2} + \frac{dq_r}{dx} = 0. \quad (5)
$$

The above equation upon integration yields the total heat flux ($q_{tot}$)

$$
q_{tot} = -ke \frac{dT}{dx} + q_r = \text{constant.} \quad (6)
$$

The corresponding boundary conditions are $T(0) = T_1$ and $T(L) = T_2$.

The above problem may be treated in similitude with the combined conductive and radiative heat transfer through a participating medium of length $L$, volumetric absorption coefficient $\beta$ and thermal conductivity $ke$ [Fig. 1(b)]. The energy equation for this plane layer is also given by equations (5) and (6). The equivalent absorption coefficient $\beta$ for the TIM is given by [15]

$$
F12 = \frac{q_r}{(\sigma T_1^4 - \sigma T_2^4)} = \frac{1}{1 + \frac{3}{4}(\beta L)}, \quad (7)
$$

$$
\beta = \left( \frac{1}{F12} - 1 \right)^{1/3}, \quad (8)
$$

$F12$ is the dimensionless radiative heat flux (shape factor) for the TIM with wall emissivity $\varepsilon_w$ and aspect ratio ($A$) with black end plates which was calculated [13] from the net radiative heat flux equation (1) by the method of undetermined coefficients in a manner similar to the Karman–Pohlhausen integral technique; it involves the computation of elemental and integral IR transmittance of the TIM.

The total dimensionless heat flux $\psi$ for the TIM with equivalent absorption coefficient $\beta$ may be expressed in terms of the total heat flux for the absorbing emitting medium of length $L$ [15] as follows:

$$
\psi = \frac{q_{tot}}{\sigma T_1^4} = -4N \frac{d\theta}{dr} + 2\chi_1 E_3(\tau) - 2\chi_2 E_3(\tau_0 - \tau) + 2 \int_{\tau_0}^{\tau} \theta^4(t)E_2(\tau - t) \, dt \\
-2 \int_{\tau}^{\tau_0} \theta^4(t)E_2(t - \tau) \, dt, \quad (9)
$$
\( \theta(\tau) = T(\tau)/T_1, N = k\beta/\alpha T_1^4, \chi = B/\sigma T_1^4, \tau = \beta x \) are the dimensionless variables with \( \theta(0) = 1.0 \) and \( \theta(\tau_0) = T_2/T_1 = \theta_2 \). Employing the following exponential kernel approximations of

\[
E_2(t) = \frac{1}{2} e^{-3t/2} \\
E_3(t) = \frac{1}{2} e^{-3t/2}
\]

and differentiating twice for elimination of exponential terms and integrating again the above equation reduces to

\[
4N \frac{d^2 \theta}{d\tau^2} - 9N \theta - 3\theta^4 = \frac{9}{4}\psi \tau - \alpha.
\]

The dimensionless total heat flux \( \psi \) is given by

\[
\psi = \frac{4}{(\tau_0 + \frac{3}{4})} \{ N(1 - \theta_2 - \frac{3}{2}(S_1 + S_2) + \frac{1}{2}(\chi_1 - \chi_2)) \}
\]

and the integration constant

\[
\alpha = 9N + 3\chi_1 - 6NS_1 - \frac{3}{2}\psi
\]

with dimensionless radiosities

\[
\chi_1 = 1 - \left(1 - \frac{\epsilon_b}{\epsilon_c}\right)(\psi + 4NS_1),
\]

\[
\chi_2 = \theta_2^4 + \left(\frac{1 - \epsilon_c}{\epsilon_c}\right)(\psi + 4NS_2),
\]

\[
S_1 = \left. \frac{d\theta}{d\tau} \right|_{\tau = 0}
\]

and

\[
S_2 = \left. \frac{d\theta}{d\tau} \right|_{\tau = \tau_0}
\]

specify the boundary conditions.

Equation (12) is a non-linear differential equation. The piecewise linearisation is made to linearise the non-linear term \( \theta^4 \):

\[
\theta^4 = b_p + c_p \theta,
\]

\[
b_p = \frac{4x_3x_5 - 5x_2x_6}{20x_1x_3 - 15x_2^2}, \quad c_p = \frac{10x_1x_5 - 6x_2x_5}{20x_1x_3 - 15x_2^2}, \quad x_n = 1 - \theta_2^n.
\]

Equation (12) reduces to

\[
\frac{d^2 \theta}{d\tau^2} - \left(\frac{9}{4} + \frac{3c_p}{4N}\right) \theta = \frac{3b_p}{4N} + \frac{9}{4}\psi \tau - \frac{\alpha}{4N}.
\]

Introducing the following constants for simplification:

\[
e_1 = \frac{1}{2} \left(\frac{2 - \epsilon_b}{\epsilon_c}\right), \quad e_2 = \frac{1}{2} \left(\frac{2 - \epsilon_c}{\epsilon_c}\right),
\]

\[
c_1^2 = \left(\frac{9}{4} + \frac{3c_p}{4N}\right), \quad p_1 = \frac{y_0}{4} + \left(\frac{e_1 + e_2}{9}\right),
\]

\[
p_2 = \frac{N(1 - \theta_2) + \frac{1}{2}(1 - \theta_2^2)}{p_1},
\]

\[
a_0 = \frac{9p_2}{16Nc_i^2}, \quad a_1 = -\frac{e_1}{4p_1c_i^2}, \quad a_2 = -\frac{e_2}{4p_1c_i^2},
\]

\[
b_0 = \frac{e_1p_2 - 9N + 3(b_p - 1)}{4NC_i}, \quad b_1 = \frac{e_1}{c_i} \left(1 - \frac{e_1}{9p_1}\right), \quad b_2 = -\frac{e_1e_2}{9p_1c_i}.
\]
So we have

$$\psi = \frac{q_r + q_c}{\sigma T_i^4} = \frac{16Nc_i^2}{9} (a_0 + a_1s_1 + a_2s_2).$$

(20)

The temperature distribution inside the TIM upon solving the equation (19) is given by

$$\theta(z) = y_1e^{n_1z} + y_2e^{-n_1z} - \frac{9c_i^2\psi z}{16N} \frac{(b_0 + b_1s_1 + b_2s_2)}{c_i}.$$  

(21)

The constants $y_1$ and $y_2$ are given by

$$y_1 = \frac{1}{2} \left( 1 + \frac{S_1 + (a_0 + b_0 + S_1(a_1 + b_1) + S_2(a_2 + b_2))}{c_i} \right),$$

$$y_2 = \frac{1}{2} \left( 1 - \frac{S_1 + (b_0 - a_0) + S_1(b_1 - a_1) + S_2(b_2 - a_2)}{c_i} \right).$$

$S_1$ and $S_2$ can be calculated from the following simultaneous linear equation:

$$U_1S_1 + U_2S_2 = Z_1$$

$$U_3S_1 + U_4S_2 = Z_2,$$
Fig. 4. Conductive, radiative and total heat loss coefficients for selective-black bounding plates.

Fig. 5. Conductive, radiative and total heat loss coefficients for selective-selective bounding plates.

Fig. 6. Effect of wall emissivity on total heat loss coefficient for different bounding plate conditions.
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where

\[
U_1 = \frac{1}{2c_1} \{ (1 + a_1 + b_1) e^{c_1 t_0} - (1 + a_1 - b_1) e^{-c_1 t_0} \} - \frac{b_1}{c_1},
\]

\[
U_2 = \frac{1}{2c_1} \{ (a_2 + b_2) e^{c_1 t_0} - (a_2 - b_2) e^{-c_1 t_0} \} - \frac{b_2}{c_1},
\]

\[
Z_1 = \frac{1}{2c_1} \{ (a_0 + b_0 + c_1) e^{c_1 t_0} - (a_0 - b_0 - c_1) e^{-c_1 t_0} \} - \frac{b_0}{c_1} - \theta_2,
\]

\[
U_3 = \frac{1}{2} \{ (1 + a_1 + b_1) e^{c_1 t_0} + (1 + a_1 - b_1) e^{-c_1 t_0} \} - a_1,
\]

\[
U_4 = \frac{1}{2} \{ (a_2 + b_2) e^{c_1 t_0} + (a_2 - b_2) e^{-c_1 t_0} \} - a_2 - 1,
\]

\[
Z_2 = \frac{1}{2} \{ (1 + a_0 + b_0) e^{c_1 t_0} - (1 - a_0 + b_0) e^{c_1 t_0} \} - a_0.
\]

Fig. 7. Effect of cell aspect ratio on total heat loss coefficient.

Fig. 8. Effect of absorber temperature on total heat loss coefficient.
3. COMPUTATIONAL RESULTS AND DISCUSSION

The radiation shape factor $F_{12}$ is calculated [13] from the transmittance and integral transmittance of the TIM; it is a function of cell and wall dimensions, as well as the thermophysical properties of the wall material. The equivalent absorption coefficient $\beta$ is then calculated from equation (8) and is used in finding the temperature distribution inside the TIM from equation (21); equation (20) is used to calculate the total heat flux ($\psi$).

The temperature distribution inside the TIM for the three cases of bounding plates: black–black, selective–black, selective–selective, is shown in Fig. 2. The temperature gradient near the bounding plates is relatively higher. The temperature at any point inside the TIM for the selective–black case is always lower than the black–black one, whereas for the selective–selective bounding plates the temperature inside the TIM is lower than the black–black case, up to the mid-plane and vice versa after that. From this, it can be inferred that with the selective case there is a strong coupling of conductive and radiative heat transfers. The calculated conductive, radiative and total heat loss coefficients inside the TIM are depicted in Figs 3–5; three bounding cases of black–black, selective–black and selective–selective are considered. The total heat loss is maximum for the black–black bounding plates and minimum for the selective–selective ones. In general, in the interior of the TIM, the conductive heat loss is lower than the radiative heat loss. However, the conductive heat loss exhibits escalation in magnitude near the bounding plates; the effect is more pronounced in the case of selective boundaries, where the conductive heat loss even exceeds radiative heat loss. These aspects may be explained in terms of the coupling of modes, wherein the heat is first transferred by conduction near the end plate to the cellular walls and, because of the intermediate wall emissivity, it is radiantly emitted and transported to the cold plate.

The effect of wall emittance of the TIM on the total heat loss coefficient for various bounding plate conditions is shown in Fig. 6. For the black end plates the total heat loss coefficient decreases with an increase in wall emissivity; the minimum value corresponds to re-radiating walls (one which absorbs and emits completely). For the selective end plates the heat loss is minimum for a wall emissivity of zero; this represents 100% IR transmittance and increases in the intermediate range of 0.1–0.4 and decreases again. The variation of total heat loss coefficient as a function of cellular aspect ratio and absorber temperature is investigated in Figs 7 and 8. The total heat loss coefficient is higher at high temperatures and decreases with an increase in aspect ratio which seems to level around an aspect ratio of 25.

4. CONCLUSION

Combined conduction and radiation heat transfers are investigated in transparent honeycomb insulation material comprising an air-layer rendered non-convective by the vertical cellular structure and horizontal rigid boundaries at the hot and cold ends. Selective, as well as black, boundaries are considered. The approach involving the similitude of the above configuration with the participating medium is used. It is pointed out that when either the absorber or the top cover is of the selective type, there is a strong coupling of conductive and radiative heat transfer, therefore a coupled mode analysis must be used to find the net heat transfer. The following configurations of TIM are found to have the lowest heat loss coefficients:

1. TIM with black end plates and cellular walls of high emissivity;
2. TIM with selective end plates and cellular walls fully transparent to IR radiation.

High aspect ratios, up to 25, are favoured for lowering the total heat loss coefficient at high temperatures and the selective absorber is desirable for further reduction of heat loss; however, the degradation of selective materials at high temperatures places a disadvantage on the use of selective coatings.

REFERENCES

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