Application of Two Turbulence Closure Schemes in the Modelling of Tidal Currents and Salinity in the Hooghly Estuary


Centre for Atmospheric Sciences, Indian Institute of Technology, New Delhi 110016, India
National Water Research Institute, Environment Canada, Burlington, Ontario L7R 4A6, Canada
Hydraulic Study Department, Calcutta Port Trust, Calcutta 700043, India

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This paper has two purposes. The first is to study the circulation and salinity in Hooghly Estuary, along the east coast of India and the second is to compare the performance of two turbulence closure schemes by modelling it. A breadth averaged numerical model using a sigma co-ordinate system in the vertical is briefly described. Vertical diffusion of momentum and salt are parameterized by a simple first-order turbulent closure or by a one equation model for turbulent kinetic energy (TKE) which uses a specified mixing length. The results are compared with the available neap and spring tide observations along the estuary for both low and high discharge periods.

The computed elevations and currents are in reasonable agreement with the observations showing no major differences in vertical current profiles by both the turbulent schemes. However, there is a slight under-prediction of bottom currents. The salinity profiles predicted by TKE model show better matching with observations. Statistical tests are also conducted to study the comparative performance of the turbulent closure schemes. The maintenance of two layer structure in residual currents and salt variability are also studied by using the model.

Keywords: turbulence closure; estuary; numerical model; currents; salinity; India coast

Introduction

Tidal estuaries comprise the lowest reaches of the rivers where they enter the sea. Mixing in estuaries results from a combination of various effects such as tide, wind, the earth’s rotation and inflow from rivers. The interaction between dense saline waters entering from the sea and the fresh water derived from the upland discharge gives rise to a wide spectrum of circulation pattern and estuarine mixing.

The application of two dimensional models with different types of closure schemes to determine the tidal circulation in estuaries has been done in many part of the world. The multi-level breadth averaged models developed for this purpose involve the parameterization of Reynold’s stresses. Parameterization of eddy diffusivity has been carried out either by; using simple first order closure schemes (Boerrie & Hogan, 1977; Blumberg, 1977; Hamilton, 1990); a one equation model using algebraic mixing length (Johns & Oguz, 1990; Johns et al., 1991); or by higher order turbulence closure schemes using prognostic equations for mixing length and turbulence energy for three dimensional models (Blumberg & Mellor, 1987; Oey et al., 1985). Studies on the inter-comparison of a range of turbulence models have been done for shelf regions (Davies & Gerritsen, 1994; Xing & Davies, 1996a,b). In a series of five papers in 1998 the ASCE task committee on turbulence models in hydraulic computations documented many of the available techniques of turbulence modelling of surface water flow and transport. However, there have been very few attempts at the inter-comparison of different turbulence closure schemes in the context of estuarine circulation. Recently, Nunes Vaz and Simpson (1994) made a comparison of various closure schemes while Luyten et al. (1996) presented a family of turbulence closure models for stratified shallow water flows with application to the Rhine outflow region. Both deal with vertical one dimensional models of the stratification process.

The aim of the present study is to simulate the salinity field and vertical variation of currents by using two turbulence closure schemes. Hence, a laterally averaged multi-level model is developed and applied to Hooghly Estuary, along the east coast of India.
Hooghly Estuary (Figure 1) forms part of the Ganges and serves as a navigable waterway to the Calcutta and Haldia ports. The estuary communicates with the Bay of Bengal near Sagar island and is tidal for nearly 250 km. The vertical tide range at the mouth varies from 5-2 m during the springs to 1-83 m during neaps (Biswas, 1985). The associated peak tidal currents are usually 1-0-1-2 ms\(^{-1}\) but have been measured as exceeding 2-0 ms\(^{-1}\) at some places. The main component of the tide is semi-diurnal with \(M_2\) being the leading component. Normally there is a regularized freshwater discharge of about 1000 m\(^3\) s\(^{-1}\), found during the dry season. However, the discharge rises steeply to 3000 m\(^3\) s\(^{-1}\) during the monsoon period (June-Sept). Many tributaries also join the Hooghly, bringing a considerable amount of freshwater during monsoon season. The estuary is well-mixed for the major part of the year, but during monsoon season the salinities in the upper layer decrease compared to salinities in the deeper layer. This is due to the large run-off, making the estuary partially stratified. Considerable variation in the vertical stratification is noticed along the length of salt intrusion. It has been found that neap-spring tidal difference is not very sensitive to the vertical mixing. In view of the strategic location and the large population living around the estuary, the river has been a subject of detailed study for a long time.

Prior to this multi-level model, Sinha et al. (1995; 1996a) developed two-dimensional depth averaged models to study the circulation and salinity in the estuary. Subsequently, Sinha et al. (1996b) coupled this model with a two dimensional storm surge model to study the effect of the surge generated at the mouth for different cyclonic storms in the adjacent sea. The models are in the major part of the estuary able to produce water levels within 2 cm and 0.5° in phase. The depth averaged currents and salinity are also in reasonable agreement during low discharge conditions. However, during high discharge period, the estuary loses much of the salt due to large run off. Also during this period, salt transport is largely due to tidal diffusion, advection and gravitational circulation. These results support the use of a multi-level channel model to stimulate the tidal circulation and salinity fields. The available observations and the model results have shown that the salinity intrusion is mainly confined to 40 km from the mouth.

Two turbulence closure schemes are used to parameterize the diffusion of momentum and salt in the vertical. In one case, the vertical transfer of momentum and salinity has been parameterized using mixing length expressions with Richardson number dependent damping functions. Results from this model are compared with a one equation model, which uses a prognostic equation for turbulence energy and specified algebraic mixing length. Surface elevations, current profiles and salinity field are compared with the available observations. Numerical experiments have been carried out with two closure schemes during low and high discharge periods for neap and spring tidal periods. Computation were also done to study the residual circulation and salt balance in the estuary.

Model formulation

Basic equations

The Hooghly Estuary is modelled by a channel having a variable depth and breadth. Bathymetry for the present study has been taken along the track as shown in Figure 1. The origin O, of the longitudinal axis Ox is placed at the landward end of the estuary and Oz points vertically upwards from the equilibrium level of the water surface at \(z=0\). At position x and time t, the location of the free surface is given by \(z=\zeta(x,t)\). The impermeable floor of the channel corresponds to \(z=-h(x)\) and its breadth is denoted by \(b(x)\).
After invoking hydrostatic pressure and Boussinesq approximations, the equations for continuity, momentum and salt conservation in breadth averaged form can be written as continuity:

\[
\frac{\partial}{\partial x} (bu) + \frac{\partial}{\partial z} (bu) = 0
\]  

(1)

or

\[
b \frac{\partial H}{\partial t} + \frac{\partial}{\partial x} (bhu) = 0
\]  

(2)

Momentum conservation:

\[
\frac{\partial}{\partial t} (bu) + \frac{\partial}{\partial x} (bu^2) + \frac{\partial}{\partial z} (buw) = -gb \frac{\partial \zeta}{\partial x} - \frac{\partial}{\partial z} \left( \frac{\partial (\rho p)}{\partial x} \right) + \frac{1}{H} \frac{\partial}{\partial x} \left( bN \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial z} \left( K_m \frac{\partial (bu)}{\partial z} \right)
\]  

(3)

Salt conservation:

\[
\frac{\partial}{\partial t} (bs) + \frac{\partial}{\partial x} (bws) + \frac{\partial}{\partial z} (bws) = \frac{\partial}{\partial z} \left( K_s \frac{\partial (bs)}{\partial z} \right)
\]  

\[
+ \frac{\partial}{\partial x} \left( bN \frac{\partial s}{\partial x} \right)
\]  

(4)

where

\[
\bar{u} = \int_{-H}^{0} u \, dz
\]

and \( u, w \) - Reynold’s averaged components of fluid velocity; \( H \) - total depth, \((\zeta + h)\); \( N \) - horizontal eddy coefficient; \( K_m, K_s \) - vertical eddy coefficients for momentum and salt; \( \rho \) - fluid density; \( \rho_o \) - reference density.

Here, density is solely determined by salinity (s) using an equation of state

\[
\rho = \rho_o \left(1 + \Gamma s\right)
\]  

(5)

where \( \Gamma = 7.5 \times 10^{-4} \)

**Parameterization of turbulent processes**

**Scheme I: first order closure.** Initially, we consider the parameterization of eddy coefficients using a first order turbulence closure based on a mixing length approach. Following Perrels and Karelse (1981), we take

\[
K_m = l_m^2 \frac{\partial u}{\partial z} f(R_i)
\]  

(6)

\[
K_s = l_s^2 \frac{\partial s}{\partial z} g(R_i)
\]  

(7)

where the mixing length \( l_m \), is defined by

\[
l_m = \kappa (z + z_r) \quad \text{if} \quad 0 \leq z \leq 0.25H
\]  

\[
l_m = \kappa (0.25H + z_r) \quad \text{if} \quad 0.25H \leq z \leq H
\]  

(8a)

(8b)

and \( \kappa \) is Von Karman’s constant (0.4) and \( z_r \) is a measure of roughness length. The Richardson number \( R_i \), which is a measure of stability, is given by

\[
R_i = \frac{-g \frac{\partial p}{\partial z}}{\rho_o \left( \frac{\partial u}{\partial z} \right)^2}
\]  

(9)

Depending upon the stability of the flow, many simulations are performed using different damping functions (Munk & Anderson, 1948; Hamilton, 1990). Reasonable results are obtained by using:

\[
f(R_i) = \exp (-4 R_i)
\]

\[
g(R_i) = \exp (-15 R_i)
\]  

(10)

when the flow is stably stratified. In an unstable stratification \( (R_i < 0) \), Blumberg (1977) used a scheme where he replaced the averaged salinity of the adjacent layers into unstable layers. However, this approach was found to be unsuitable with regard to the present study and the formula given by Charnock (1967) is used for unstable conditions. In the neutral stability case, the damping functions would be absent. The effective horizontal mixing is largely due to the cross-channel shear induced dispersion, and accordingly, following Boerrie and Hogan (1977), we take

\[
N = C_d \, b(s) \, \bar{u}
\]  

(11)

where \( C_d \) is a disposable parameter.

**Scheme II: turbulent kinetic energy closure.** Here, a single equation is used for turbulence energy \( E \), and a specified function for the mixing length \( l \), based on the work of Johns and Oguz (1990), given by:

\[
\frac{\partial}{\partial t} (bE) + \frac{\partial}{\partial x} (buE) + \frac{\partial}{\partial z} (bwsE) = bK_m \left( \frac{\partial u}{\partial z} \right)^2
\]

\[
+ bN \left( \frac{\partial u}{\partial x} \right)^2 \quad \frac{g}{\rho_o} bK_s \frac{\partial \rho}{\partial x} \frac{\partial u}{\partial z} \left( K_E \frac{\partial (bE)}{\partial z} \right)
\]

\[
+ \frac{\partial}{\partial x} \left( bN \frac{\partial bE}{\partial x} \right) - b \epsilon
\]  

(12)

where \( K_E \) and \( K_s \) are the exchange coefficients for turbulent energy and salt, respectively.
Unlike scheme 1, this method is valid even when the stratification becomes locally unstable because the buoyancy term in Equation 12 acts as a source of turbulence energy thus increasing the value of $E$ and enhancing the vertical mixing.

The various eddy coefficients appearing in the formulation are expressed in terms of the turbulent energy density. In the present work we chose

$$K_M=K_S=K_B=K$$

(13)

where $K=C^4 E^{1/3}$

(14)

Here, $C=0.08$ is an empirical constant (Launder & Spalding, 1972) and vertical mixing length, $l$, is given by

$$l=l_0 \exp \left(-0.25 \beta (z+h)\right)$$

(15)

Following Johns (1978), $l_0$ is determined by

$$l_0 = \frac{K E^{1/3}}{d \left( \frac{E^{2/3}}{l_0} \right)}$$

(16)

$$l_0 = \kappa z_s \text{ at } z = -h$$

Here the coefficient $\beta$ determines the shape of the mixing length profile.

The dissipation term $\varepsilon$ is parameterized as:

$$\varepsilon = \frac{C^2 E^2}{l_D}$$

(17)

where the dissipation length scale, $l_D$, is given by:

$$l_D = l_0 \exp \left(0.75 \beta (z+h)\right)$$

(18)

The horizontal eddy coefficient, $N$, is parameterized on the basis of dimensional grounds (Stacy & Zedel, 1986; Johns & Oguz, 1990) as:

$$N = a a x E^{1/3}$$

(19)

where the grid increment, $a$, is taken as a horizontal mixing length scale and $a$ is a disposable parameter.

**Boundary and initial conditions**

The above equations are accompanied by appropriate boundary conditions. At the floor of the channel, no-slip condition is used. Further, no diffusive flux of salinity across the floor of the estuary is considered. At the free surface, zero surface stress condition and diffusive salt flux are appropriate. For tidally induced flow at the free surface, turbulent kinetic energy flux is zero. At the floor of the channel it is assumed that there is no diffusive flux of turbulent energy across the floor (alternative approaches are suggested by Baumert & Radach, 1992; Xing & Davies, 1996a).

Thus the fluid motion is solely driven by the boundary forcing to be applied at the extremities of the channel. In the present study, a tidal boundary condition for $M_2$ component has been specified as:

$$\zeta = 0.85 \cos (\omega t + 185^\circ) \text{ for neap}$$

(20a)

$$\zeta = 2.21 \cos (\omega t + 140^\circ) \text{ for spring}$$

(20b)

At the landward end ($x=0$), a radiation condition is used, which takes care of the outward transmission of the disturbance in the form of a simple progressive wave. Following Johns and Oguz (1990) the freshwater discharge through the estuary can be achieved by prescribing

$$\bar{u} + \left( \frac{g}{h} \right)^{1/2} \zeta = 2u_0$$

(21)

where $u_0$ is a constant velocity that determines the strength of the freshwater flow. The salinity boundary conditions to be applied at the open boundaries is such that the salinity of inflowing water is prescribed and the outflowing water is determined by using Equation (4).

To start the time stepped solution to equations described in the previous section, we prescribe all dependent variables as being zero when $t=0$. We have also chosen $s=s_0$ (mean salinity for low discharge period) all along the estuary when $t=0$. We then integrate with the steady tidal forcing and a constant value of $u_0$ till the steady state is achieved.

**Co-ordinate transformation and method of solution**

A non-dimensional vertical co-ordinate, $\sigma$, is introduced to facilitate the representation of the boundary conditions accurately at $z=-h(x)$ and $z=\zeta(x, t)$ by taking $\sigma=(z+h)/(\zeta+h)$, so that $\sigma$ increases monotonically from $\sigma=0$ at the floor to $\sigma=1$ at the free surface. Recently, Haung and Spalding (1996) pointed out that over the steep bottom slopes, the conventional definition of horizontal diffusion in the sigma co-ordinate system causes spurious diffusive transport. They presented a new formula which produces almost no spurious diffusion. The formula suggested by Haung and Spalding (1996) for horizontal diffusion was used for testing. Numerical results have revealed no major differences between this formula
and the one used in the present study, due to lack of abrupt variations of bathymetry in the channel.

The predictive equations are solved numerically in the transformed co-ordinates. Computations are carried out on a staggered grid in the horizontal and unstaggered grid in the vertical. The space derivatives in momentum equation, turbulent energy equation and continuity equation are described by central differences. However, the salinity transport equation uses an upstream scheme similar to Johns and Oguz (1990). Although vertical diffusion terms, dissipation and suppressive terms are treated semi-implicitly, horizontal diffusion is treated explicitly. Finally, forward difference is used for time stepping. The complete details of the numerical method is presented in Rao (1996).

**Numerical experimentation**

Numerical experiments are conducted by integrating the governing set of equations using appropriate initial and boundary conditions as discussed in the model formulation section and the boundary and initial conditions section. For the purpose of comparison data from a special observational programme conducted by Calcutta Port Trust, India during December, 1992 for spring and neap tides has been used. The salinity data for a few stations, collected by Central Pollution Control Board, India, as also utilized for the same period. Although some of the data are not synoptics they are assumed to be representative of the conditions in the estuary for low discharge conditions. During monsoon period the availability of data in this region is very limited. However, under varying discharge conditions (2500 m$^3$ s$^{-1}$ to 4000 m$^3$ s$^{-1}$) vertical salinity and current data were obtained at Gangra, Haldia and Balari for a spring tide.

Figures 2, 3 and 4 depict the measured vertical salinity profiles along the estuary. During the low discharge period the salinity data are available for both neap and spring tides at six stations for two tidal cycles. Figure 2(a,b) delineates averaged flood and ebb salinity profiles, respectively, over these two cycles for the spring tide under low discharge conditions. The freshwater discharge during this period varied from 1000 m$^3$ s$^{-1}$ to 1100 m$^3$ s$^{-1}$. Partial stratification is evident in the first 10 km from the head of the estuary. Here top to bottom salinities varied by as much as 1-0 (using the practical salinity scale). The maximum salinity observed during the flood is 23-2 and 19-5 during the ebb. Figure 3(a,b) represents salinity profiles during flood and ebb, respectively, for spring tide and are obtained from the very limited available observations during July 1980 data. The freshwater discharge during this period varied from 2300 m$^3$ s$^{-1}$ to 3400 m$^3$ s$^{-1}$. The salinity intrusion length during this period is confined to 38 km from the mouth of the estuary. Pronounced stratification is noticed in some parts of the estuary with top to bottom salinity variation being 1 to 2. However, the maximum salinity observed during this period is 13-1 for the flood and 11 during the ebb.

Figure 4(a,b) show the salinity distribution for flood and ebb period representing neap conditions of December 1992. The freshwater flow is slightly more than spring tide for the same period (1132 m$^3$ s$^{-1}$ to
1150 m³ s⁻¹). In this case the data is limited to only four stations and hence the profiles broadly represent the characteristics of the estuary. No major difference was noticed in salinity profiles from spring tide except for the available maximum salinity of 17.1 at the mouth during the flood and 15 during the ebb. The salt intrusion is confined to 42 km from the mouth of the estuary. The relatively low salinities all through the year for both spring and neap tides is due to the dilution of salt water by constant freshwater discharge to maintain the channel. We will confine our attention to these observations while comparing the model computed results.

The lower estuary of length 45 km from the seaward end at Sagar was chosen to study the tidal flow and salinity profiles. The variation of breadth and depth along the channel is shown in Figure 5. Here, the depth along the track shown by solid dark line in Figure 1 has been chosen as the representative bathymetry for the present study. The breadth of the estuary at each computational point has been measured from Calcutta Port Trust survey maps. The selection of 41 computational points in x-direction results in a horizontal grid distance of 1.12 km. The prescription of 11 computational levels in the vertical provides a vertical grid distance of 0.55 m in the shallow regions and 1.0 m in the deeper region. With the scheme of discretization, the attainment of computational stability is limited by gravity wave speed. For the present study a time step of 120 seconds is found to be consistent with computational stability.

In the experiments described here the right choice of $C_d$ (scheme I), $\beta$ and $a$ (scheme II) is essential. After many numerical experiments it was found by taking $C_d=0.22$, $\beta=0.2$ m⁻¹ and $a=10.0$ a satisfactory vertical and horizontal mixing is attained. The bottom of the estuary is mainly made up of mud and sand and hence the roughness length has been chosen as 0.07 cm ( Soulsby, 1983 ).

The model computations start from an initial state of rest and constant salinity with the appropriate boundary conditions. After 8 days of integration with low water at the seaward end, an effectively oscillatory steady state solution is achieved. The model results of the ninth day are analysed; the flood and ebb corresponds to 6 h and 12 h respectively, of a tidal cycle. Although the model is forced only with $M_2$ tidal amplitude, due to non-linearities higher harmonics may get generated. A Fourier analysis has been carried out in the form of the following equation:

$$\zeta = \langle \zeta \rangle + \sum_{k} \zeta_k \sin \left( \frac{2\pi ky}{T} + \Phi_k \right)$$

(22)

where $\langle \zeta \rangle$ is the mean value of $\zeta$ and $\zeta_k\ (s > \pi)$ are the amplitude of non-linearly generated higher harmonics. The amplitude and phase of first harmonic $M_2$ is presented at three stations in Table 1 for both spring and neap tides. It is evident from Table 1 that model predicted elevations (both scheme I and scheme II) are in good agreement with observations (within 2 cm at Haldia and Gangra). But at Balari the deviation in elevations are as much as 4 cm. Neap comparisons show a maximum deviation of about 8 cm at some places. The deviations in tidal phase are of the order of 5° to 15° at all the stations. The error at Balari is probably due to the closeness to the open boundary. In general an increase in elevations is observed from the seaward end to up estuary. Both schemes have reproduced the elevations very well, with scheme II showing slightly better results.
Comparison of velocity and salinity profiles

**Neap simulations**

*Low discharge period.* Initially the model was run for low discharge (1000 m³ s⁻¹) by prescribing observed neap elevation (Equation 20a) and salinity at the seaward end. The ninth day model results are compared with the observations described earlier. Figure 6 shows a comparison of observed and computed currents at three depths for the three stations shown in Figure 1. The three depths correspond to surface (S), middle (M) and bottom (B) which are at levels 11, 6 and 1 respectively. Broadly, the predicted currents are in reasonable agreement with observations (note that the phase is also in agreement). However, the under-prediction of bottom currents is evident by both the closure schemes. Under-estimation of currents at bottom suggests that the value of $z_o$ is not sufficient. However, this has not been increased in the simulations as it may dampen the elevations. Xing and Davies (1996b) also noticed a similar kind of problem in their simulation of tidal currents in shelf edge regions. The oscillations in computed currents at station 1 is probably due to the closeness to the open boundary—here a radiation boundary condition is used.

The salinity variation over a tidal cycle for the same three stations are depicted in Figure 7. Here, the
observations are plotted at every 2 h. The computed results show that the top to bottom salinities varied by 1. The partial stratification is evident during flood, and the salinity profiles obtained by scheme II are more reasonable than scheme I predictions. It is also noted that during the peak ebb period the water is more saline in the surface layers. It may be deduced from these simulations that intense mixing takes place at the time of ebb and just after ebb.

The along channel plots of depth dependent salinity and currents during high and low water computed by scheme I are shown in Figure 8(a,b), respectively. The current speeds varied from 0.7 m near the surface to 0.1 m at the bottom. It can be noticed from these figures that the major part of the estuary is well mixed and partial stratification is found near the head of the estuary. More pronounced stratification is obtained with scheme II and are shown in Figure 9(a,b). No major difference in current speeds and directions are noticed with scheme II. By comparing with the observed flood and ebb salinity profiles (Figure 4), it is found that the computed salinity profiles by scheme II are better.

**High discharge period.** Figures 10 and 11 delineate the salinity and velocity fields for a neap tide during high discharge period with scheme I and scheme II, respectively. The maximum salinity at the seaward end is reduced to 14.6 during flood and 12.2 during ebb. The limited salinity observations (available only for high tide and hence not presented in this paper) have actually shown that the estuary is free from salinity for the first 10 km from the head. Both the schemes failed to reproduce this pattern. However,

![Figure 7](image_url)

**Figure 7.** Comparison of observed and computed salinity over a tidal cycle at the three stations shown in Figure 1 at surface (S), middle (M) and bottom (B) for a neap tide. Solid line: scheme I; dashed line: scheme II; dotted line: observed results.

![Figure 8](image_url)

**Figure 8.** Salinity and velocity fields during (a) flood and (b) ebb for low discharge conditions with scheme I for a neap tide.
the salinity profiles computed by scheme II are more reasonable than scheme I as scheme II is able to simulate the partial stratification. No significant differences in current speeds or directions are noticed in comparison with low discharge period. Comparison with observations is not possible due to the non-availability of continuous observations of salinity and velocities. However, the general characteristics are seemingly reproduced properly.

**Spring simulations**

**Low discharge period.** By prescribing $1000 \text{ m}^3 \text{s}^{-1}$ as freshwater discharge at the landward end; the observed elevation (Equation 20b) and salinity for spring tide at the seaward end, the model is therefore integrated for eight days to obtain steady state. The computed and observed velocities over a tidal cycle are shown in Figure 12. As noted in neap simulations, the bottom currents are grossly under-predicted by both the schemes. First order closure scheme has produced unrealistic oscillations in currents particularly at station 1. The simulated currents are in reasonable agreement (especially scheme II) with observations at the surface and middle levels. Peak current speeds (both flood and ebb) ranged between 2.4–2.6 ms$^{-1}$. Figure 13 depicts the salinity variation over a tidal cycle for the same three stations. Here, again the observations are plotted at every two hours. Scheme I produced unrealistic stratification in salinity profiles with as much as 4 difference in salinity of the water from top to bottom. The salinity profiles computed by scheme II are more in agreement with observations. In general the salinity predictions are within 15% of the observations.

The along channel plots of depth dependent salinity and currents computed by schemes I and II are shown in Figures 14 and 15, respectively. The current speeds varied from 2.2 ms$^{-1}$ at the surface to 0.10 ms$^{-1}$ at the bottom with slightly stronger currents by scheme II. The scheme I produced partially stratified salinity profiles during flood and well-mixed profiles during
Figure 12. Comparison of observed and computed currents over a tidal cycle at the three stations shown in Figure 1 at surface (S), middle (M) and bottom (B) for a spring tide. Solid line: scheme I; dashed line: scheme II; dotted line: observed results.

Although scheme II has also produced slight stratification, it is not very prominent and confined to the lower part of the estuary. Again during the ebb well mixed profiles are visible with salt intrusion being limited to 35 km from the mouth. The computed flood salinity profiles due to scheme II are in better agreement with observations (Figure 2), where scheme I generated ebb salinity profiles are in reasonable agreement with observations. The salinity computations have shown intense mixing during ebb time in shallower zones. The main features of the spring tide, as compared with the neap tide for low discharge period, are much higher salinity (the salinity increased by 4 at the mouth) and stronger currents. However, the variation in vertical salinity profiles (particularly when computed by scheme II) is very small and supports the observational evidence. In general the mean salinity profiles (not presented to save space) have produced well-mixed profiles during spring tide and partially stratified profiles during neap tide.

High discharge period. By increasing the freshwater discharge to 3000 m$^3$ s$^{-1}$, the estuary characteristics are simulated during spring tide for monsoon period. Figures 16 and 17 show the computed salinity and current fields along the channel by schemes I and II, respectively. Scheme I produced either well-mixed or partial stratification in salinity profiles, where as scheme II predictions show stratification during both flood and ebb periods. Both the schemes have overpredicted the salinity in the estuary. The observations (Figure 3) have shown much less salinity and more pronounced stratification. It is difficult to achieve the exact matching as observations may have been affected by higher freshwater influx and for longer periods than the integration period. The spring currents varied as much as 2.0 m s$^{-1}$ at the top to 0.2 m s$^{-1}$ near the bottom. The high tidal velocities compared to neap tide will allow larger tidal excursion. There is no marked difference in both stratification and intrusion from neap to spring tide.

Comparisons of observations and computations

To compare the performance of the two closure schemes, variances ($\sigma_o$ for observations and $\sigma_c$ for computations), root mean square error (E), root mean square error after removing uniform bias ($E_{nu}$) and skill of the schemes (Pielke, 1984) have been calculated for both currents and salinity. Holt and Sethuraman (1988) pointed out that root mean square errors are not very good indicators in predicting the overall structure of a variable. However, they provide an overview of the absolute accuracy of computations.
Figure 13. Comparison of observed and computed salinity over a tidal cycle at the three stations shown in Figure 1 at surface (S), middle (M) and bottom (B) for a spring tide. Solid line: scheme I; dashed line: scheme II; dotted line: observed results.

Figure 14. Salinity and velocity fields during (a) flood and (b) ebb for low discharge conditions with scheme I for a spring tide.

Figure 15. Salinity and velocity fields during (a) flood and (b) ebb for low discharge conditions with scheme II for a spring tide.

Table 2 gives these statistics for various computations of low and high discharge conditions. In general, it can be seen that the difference between the two schemes is very small. The root mean square errors of scheme II are slightly less than scheme I, however, the performance of scheme I in predicting neap velocities and spring salinities is marginally better. The prediction with TKE closure is better overall because it is able to simulate the mixing length and unstable stratification properly.
During neap tide, scheme I produced a maximum value of 18 cm$^2$s$^{-1}$ in the shallow region close to the lower layers. The exchange coefficient decreases fairly uniformly until the mid-depths and later becomes small in the surface layer. In the case of computations with scheme II, the maximum value is 30 cm$^2$s$^{-1}$ near the bottom which decreases upwards. It appears that the shear near the bottom is responsible for maximum eddy coefficients. The computed profiles performed by scheme I also show that the eddy diffusivity magnitude varied all along the estuary, but more uniform values are generated by scheme II.

During the spring tide the maximum value of eddy diffusivity due to scheme I is 38 cm$^2$s$^{-1}$ near the bottom in the deeper region and the value reduces in the surface layers. A stratified mixing is observed in this region which can also be seen from the salinity profiles. Again in the shallow region near the mouth of the estuary another peak of eddy coefficient (16 cm$^2$s$^{-1}$) is noticed in the lower layers which decreases from mid-depth upwards. This region had shown partial stratification in salinity profiles. Scheme II has produced fairly uniform magnitude in eddy coefficients, with a maximum value of 45 cm$^2$s$^{-1}$ near the bottom. Proceeding from the bottom to the surface, the value decreases uniformly until mid-depths and later decreases to small values near the surface. This may be the reason that scheme II generated partial stratification in many parts of the estuary. The better performance of TKE closure is attributed to its ability to stimulate the turbulence during unstable stratification. The high turbulence near the bed region is probably responsible for slightly stronger currents with this scheme. The algebraic form of mixing length in scheme II is able to produce a maximum value closer to the bed.

Comparison of eddy coefficients

To understand the reasons for the differences in the computations, it is essential to examine the eddy exchange coefficients. Figures 18 and 19 illustrate the eddy exchange coefficients computed by both closure schemes for low discharge periods during neap and spring tides. Here, the plots show the magnitude of the eddy exchange coefficients at the time of high water.

Computation of residual circulation and salt variability

The numerical model presented here is useful in understanding some of the dynamics that govern the circulation of the estuary. In the preceding section it was shown that the performance of TKE closure scheme is better, hence, the residual circulation is simulated with scheme II only. Earlier two-dimensional depth-averaged simulations of Sinha et al. (1995) have shown flood and ebb directed flows in different channel cross-sections. However, the vertical structure of the residual currents is presented in this section. Figure 20(a) shows one such pattern for the low discharge conditions based on the 16th day simulation. Here, the dashed (positive) contour indicates
TABLE 2. Error statistics for (a) various computations with scheme I for low discharge conditions; (b) various computations with scheme II for low discharge conditions and (c) salinity of spring tide computations with high discharge

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\sigma_o$</th>
<th>$\sigma_c$</th>
<th>$\epsilon$</th>
<th>$\epsilon_o$</th>
<th>$\epsilon_o/\sigma_o$</th>
<th>$\epsilon_c/\sigma_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neap</td>
<td>velocity</td>
<td>0.564</td>
<td>0.452</td>
<td>0.239</td>
<td>0.231</td>
<td>0.420</td>
</tr>
<tr>
<td></td>
<td>salinity</td>
<td>0.635</td>
<td>0.826</td>
<td>1.10</td>
<td>0.739</td>
<td>1.72</td>
</tr>
<tr>
<td>Spring</td>
<td>velocity</td>
<td>0.810</td>
<td>0.740</td>
<td>0.420</td>
<td>0.389</td>
<td>0.520</td>
</tr>
<tr>
<td></td>
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<td>1.07</td>
<td>1.91</td>
<td>1.82</td>
<td>1.45</td>
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<tr>
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<td>velocity</td>
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<td>0.364</td>
<td>0.288</td>
<td>0.280</td>
<td>0.510</td>
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<td>0.59</td>
<td>0.93</td>
</tr>
<tr>
<td>Spring</td>
<td>velocity</td>
<td>0.810</td>
<td>0.660</td>
<td>0.318</td>
<td>0.299</td>
<td>0.390</td>
</tr>
<tr>
<td></td>
<td>salinity</td>
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<td>2.17</td>
<td>2.00</td>
<td>1.89</td>
<td>1.43</td>
</tr>
<tr>
<td>Scheme I</td>
<td>$\sigma_o$</td>
<td>0.72</td>
<td>1.79</td>
<td>1.63</td>
<td>1.47</td>
<td>2.26</td>
</tr>
<tr>
<td>Scheme II</td>
<td>$\sigma_c$</td>
<td>0.72</td>
<td>1.70</td>
<td>1.46</td>
<td>1.40</td>
<td>2.02</td>
</tr>
</tbody>
</table>

Figure 18. Eddy diffusivity (cm$^2$ s$^{-1}$) for low discharge conditions (a) with scheme I and (b) with scheme II for a neap tide.

Figure 19. Eddy diffusivity (cm$^2$ s$^{-1}$) for low discharge conditions (a) with scheme I and (b) with scheme II for a spring tide.

magnitude of the ebb directed flow and solid (negative) contour indicates the flood directed currents. The two-layer structure is evident in the major portion of the estuary and this is confined to middle and surface layers. Intuitively, the existence of two-layer structure has implications in the transport of pollutants, nutrients and sediment. To establish the reason for this residual flow, numerical experiments were conducted. Initially, by stopping the freshwater flow, the residual circulation was calculated. The results do not show significant variation either in the structure or magnitude (surface value decreased slightly) of the residual currents. In the second experiment, by keeping 1000 m$^3$ s$^{-1}$ freshwater flow, the horizontal density gradients were omitted in the momentum equation. The computed flow is shown in Figure 20(b). The absence of any inward flow is noted all along the estuary and the ebb directed current strength is slightly decreased. These computations confirm the importance of horizontal density gradients in the maintenance of the two-layer structure of the residual currents.

In Figure 21, the variability and magnitude of various terms in the salt balance Equation 4 is shown. The time variability of salt is essentially balanced by the sum of horizontal advection and diffusion. The
variability of advective or diffusive fluxes over the cross-section may be important, but it is not possible to take care in the present model framework. The contribution from vertical advection is not very important. Before the slack-water period, turbulent fluxes become significant and vertical diffusive flux always contributes positively in the maintenance of the salt in the estuary. In a tidally averaged sense a balance is seen between advective salt flux and diffusive fluxes.

Conclusions

In the present work a breadth averaged multi-level numerical model is presented for the Hooghly Estuary, along the east coast of India. Two turbulent closure schemes are used to parameterize the sub-grid scale processes. Observations made by Calcutta Port Trust and Central Pollution Control Board, India are utilized. The models are able to stimulate many of the observed features in the estuary. The comparison of TKE closure and first order closure based on Richardson number dependent damping functions show slightly better prediction in elevations by TKE model. The variation in tidal current profiles by both the schemes is insignificant. However, TKE closure scheme is seemingly doing better in the prediction of salinity profile. The models are also tested for neap-spring variations for low and high discharge conditions. Both the schemes have under-estimated the bottom currents, and need to be appropriately taken care of in further studies. The ability of TKE closure in unstable stratification is fairly evident, and can be seen in the computation of eddy exchange coefficients. The estuary varies between well-mixed and partially stratified for various tidal phases and discharge conditions. The horizontal density gradient in the estuary is responsible for the maintenance of residual currents. Results also indicate that a salt balance exists between advective and turbulent fluxes. To study the transverse variation along with vertical variation in currents and salinity, a three dimensional model would be more appropriate. Nevertheless, the present model is a first step and it is useful for understanding many of the estuarine processes. Further work is aimed at using higher order closure schemes and the development of three dimensional model for the estuary.

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