CONVECTIVE HEAT TRANSFER ACROSS TRANSPARENT HONEYCOMB INSULATION MATERIALS

M. ARULANANTHAM, T. P. SINGH and N. D. KAUSHIKA†
Centre for Energy Studies, Indian Institute of Technology, Delhi, Hauz Khas, New Delhi-110016, India

(Received 18 September 1992; received for publication 24 June 1993)

Abstract—This article presents the mathematical theory of convective stability of a fluid (air) layer bounded by square honeycomb. The critical Rayleigh as well as post-critical Rayleigh regions are examined through the Nusselt number. The governing equations of square celled air temperature, vorticity, stream functions and velocity are solved by a finite difference method of explicit type, with central difference in space and forward difference in time, and the Nusselt number is calculated from known temperature and velocities. The effect of aspect ratio of cell, aspect ratio of wall and inclination of cell on suppression of convective motion are investigated. It is shown that the convective motion can be suppressed completely by varying the aspect ratio of the cell. In the post-critical Rayleigh region, thin walled honeycomb shows lower convective losses than thick walled devices.

Honeycomb Rayleigh number Transparent insulation material

NOMENCLATURE

\( g \) = Acceleration due to gravity
\( C_p \) = Specific heat
\( K_w, K_f \) = Thermal conductivity of wall and fluid
\( L \) = Height of honeycomb
\( S \) = Width of cell
\( W \) = Width of wall
\( A \) = Aspect ratio of cell = \( L/S \)
\( A_w \) = Aspect ratio of wall = \( L/W \)
\( Pr \) = Prandtl number = \( \mu C_p / K \)
\( Ra \) = Rayleigh number = \( g \rho_0 \beta \Delta T L^3 / \nu \)
\( RR \) = Ratio of wall conductivity to fluid conductivity = \( K_w / K_f \)
\( X \) = Distance from left side of wall
\( Y \) = Distance from bottom of honeycomb
\( x \) = Dimensionless distance = \( X/L \)
\( y \) = Dimensionless distance = \( Y/L \)
\( t \) = Time
\( U \) = Velocity in \( X \) direction
\( V \) = Velocity in \( Y \) direction
\( u \) = Dimensionless velocity = \( U/L \nu \)
\( v \) = Dimensionless velocity = \( V/L \nu \)
\( T \) = Dimensionless temperature = \( (\theta - \theta_e) / (\theta_s - \theta_e) \)
\( \dot{Q}_t \) = Total heat transfer at constant \( Y \)
\( N_a \) = Average Nusselt number at constant \( Y \) = \( \dot{Q}_t H / \kappa \Delta \theta \)

Greek letters
\( \gamma \) = Volume expansion coefficient
\( \sigma \) = Thermal diffusivity = \( K/\rho C_p \)
\( \rho \) = Density
\( \mu \) = Kinematic viscosity
\( \nu \) = Dynamic viscosity = \( \mu / \rho \)
\( \beta \) = Angle of inclination of cell
\( \theta \) = Temperature
\( t \) = Dimensionless time = \( \alpha t / L^2 \)
\( \psi \) = Dimensionless stream function
\( \omega \) = Dimensionless vorticity = \( \partial \psi / \partial x - \partial \theta / \partial y \)

†To whom all correspondence should be addressed.
INTRODUCTION

Transparent insulation materials (TIMs) represent a class of materials wherein air gaps are used to reduce unwanted heat losses. TIMs have similarities with conventional insulation materials insofar as the placement of the air gaps in the solid mass is concerned; they hold great promise of application in increasing the solar gain of outdoor thermal energy systems. In such applications, it is important to suppress natural convection in the air gap because the thermal transfer coefficient of the air gap increases with convective mass transport. Honeycomb cellular structures have been suggested [1] for many years, now as a means to suppress natural convection in enclosed air gaps. The fluid mechanical treatment of this type of problem has been presented, among others, by Ostrach and Pnueli [2], Catton [3], Edwards and Catton [4], who have applied the Malkus Veronis power integral technique to estimate the heat transfer in confined cells heated from below. They have expressed the results in terms of the critical Rayleigh number as a function of the aspect ratio of the air cell. The dimensionless parameter, Nusselt number, (characteristic of the ratio of convective heat flow to conduction) has also been investigated by a numerical solution [5], as well as by correlation equations [6, 7]. In the present article, a numerical approach, based on the finite difference solution of relevant two-dimensional fluid mechanical equations, is used to investigate

![Fig. 1. Schematic of square honeycomb.](image)

![Fig. 2. Effect of RK on Nusselt number. Ra = 50,000, A = 2, A_w = 10, β = 40°.](image)
the convective heat transport in an air celled array; both critical Rayleigh and post-critical Rayleigh regions are examined. The explicit emphasis is on the study of the effect of the wall aspect ratio on the natural convection phenomenon in the air cell.

PROBLEM STATEMENT AND MATHEMATICAL MODEL

Honeycombs of several cell shapes have, so far, been considered for suppression of natural convection in a fluid layer. For example, amongst others, hexagonal, circular, square and rectangular cells have been tried and tested. However, the square honeycomb device is in the most advanced state of development. Figure 1 shows a honeycomb of square cross section with finite side walls and isothermal upper and lower walls. The fluid inside the cell is air, in which the convection motion is started due to the difference in upper and lower temperatures. The convective motion is assumed to be two-dimensional[8]. The following further two assumptions were also made during the formulation of the governing equations:

(i) "Boussinesq approximation"—density of the fluid is constant except in the buoyancy term.
(ii) Coefficient of volumetric expansion of the fluid is independent of pressure.

Applying these assumptions to the governing equations of momentum, energy and continuity and introducing the concepts of stream function and vorticity, the completely non-dimensional equations are obtained

Fig. 3. Effect of $RK$ on temperature on left side of cell. $Ra = 50,000$, $A = 2$, $A_w = 10$, $\beta = 40^\circ$.

Fig. 4. Effect of $Ra$ on Nusselt number. $A = 10$, $ RK = 2$, $A_w = 10$, $\beta = 40^\circ$.
Fig. 5. Effect of aspect ratio of wall on Nusselt number. \( Ra = 50,000, \, \text{RK} = 2, \, A = 2, \, \beta = 40^\circ \).

\[
\frac{\partial T_I}{\partial \tau} + u \frac{\partial T_I}{\partial x} + v \frac{\partial T_I}{\partial y} = \frac{\partial^2 T_I}{\partial x^2} + \frac{\partial^2 T_I}{\partial y^2}
\]  
(1)

(ii) Momentum

\[
\frac{1}{Pr} \left[ \frac{\partial \omega}{\partial \tau} + u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} \right] = \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} + \text{Ra} \left[ \frac{\partial T_I}{\partial x} \cos \beta - \frac{\partial T_I}{\partial y} \sin \beta \right]
\]  
(2)

(iii) Continuity

\[
\omega = - \left[ \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right]
\]  
(3)

where

\[
u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}.
\]  
(4)

Fig. 6. Effect of aspect ratio of cell on Nusselt number. \( Ra = 50,000, \, A = 10, \, \text{RK} = 2, \, \beta = 40^\circ \).
For the total conductive and convective heat transfer \((Q_x)\) crossing a plane at constant \(y\), the average Nusselt number is
\[
\text{Nu}_y = \frac{Q_x L}{k \Delta \theta}
\] (5)
and
\[
Q_x = \rho C_p \int_0^s v \theta \, dx - K \int_0^s \frac{\partial \theta}{\partial y} \, dx.
\]
After non-dimensionalisation of \(Q_x\) and substituting it in \(\text{Nu}_y\), the Nusselt number is given by
\[
\text{Nu}_y = A \int_0^{\omega A} \left[ v(T_t - \frac{1}{A}) - \frac{\partial T_t}{\partial y} \right] \, dx.
\] (6)
The heat transfer through the wall is only through conduction, and the governing equation is
\[
\frac{\partial T_u}{\partial \tau} = \frac{\sigma}{\alpha} \left[ \frac{\partial^2 T_u}{\partial x^2} + \frac{\partial^2 T_u}{\partial y^2} \right]
\]
For finite wall boundaries, the temperature and heat flux distributions are matched for the left side of the wall with the right side of the cell and vice versa. The thermal boundary conditions for this are
\[
T_t(B, y) = T_u(0, y)
\]
\[
T_t\left(\frac{1}{A_u}, y\right) = T_u\left(\frac{1}{A_u}, y\right)
\]
\[
\frac{\partial T_t}{\partial x} (B, y) = RK \frac{\partial T_u}{\partial x} (0, y)
\]
\[
\frac{\partial T_t}{\partial x} \left(\frac{1}{A_u}, y\right) = RK \frac{\partial T_u}{\partial x} \left(\frac{1}{A_u}, y\right).
\]
The boundary conditions for the upper and lower plates are given by
\[
T_t(x, 0) = T_u(x, 0) = 1.0
\]
\[
T_t(x, 1) = T_u(x, 1) = 0.0
\]
\(u = v = \psi = 0\) on all boundaries of the cell.
METHOD OF SOLUTION

The governing equations are expressed in finite difference form using central differences in space and forward differences in time. The temperature equation, which is of parabolic type, is advanced from time step $n$ to time step $n + 1$ from the initial conditions. The vorticity equation, which is also of parabolic type is advanced from the step 'n' to $n + 1$. After knowing the vorticity, the stream function is calculated by solving the continuity equation, which is of elliptic type, by the successive over relaxation method. The velocities $u$ and $v$ are then calculated, and the procedure is repeated with new values of $u$, $v$ and $T$ at $n + 1$ for the time step $n + 2$ until steady state conditions are reached. The condition for steady state is defined as $[(\varphi^{n+1} - \varphi^n)/\varphi^{n+1}] < 10^{-3}$ where $\varphi$ is the stream function.

After knowing $u$, $v$, and $T$ at steady state, the Nusselt number is calculated from equation (6).

RESULTS AND DISCUSSION

Figure 2 shows the effect of $RK$ on the Nusselt number. The Nusselt number decreases with an increase in $RK$ and has a maximum value at $RK = 0$, which corresponds to adiabatic walls, and a minimum at $RK = \infty$, which corresponds to a linear variation of wall temperature. Figure 3 shows the effect of $RK$ on the temperature distribution of the air at the left side boundary of the cell. For low values of $RK$, the temperature distribution is non-linear and corresponds to adiabatic wall conditions, and for higher $RK$, it is equivalent to the linear temperature distribution. The effect of Rayleigh number on Nusselt number for the aspect ratio of the cell of 10 is shown in Fig. 4. The Nusselt number starts increasing after a particular Rayleigh number, which is the critical Rayleigh number, up to which the convection is completely suppressed. The effect of aspect ratio of the wall on Nusselt number, shown in Fig. 5, is that, with an increase in aspect ratio of the wall, the Nusselt number decreases. The effect of aspect ratio of the cell on Nu is depicted in Fig. 6. The Nusselt number decreases with an increase in aspect ratio of the cell and becomes constant (Nu = 1) after a particular aspect ratio of the cell, which shows that, beyond this aspect ratio of the cell, there is no convection. The effect of inclination on Nusselt number is given in Fig. 7. Nu first increases, reaches a maximum value at the inclination of 60° and then decreases.

CONCLUSION

From this study, we can conclude that the convective heat loss can be suppressed by varying the aspect ratio of the cell and by using thin walled honeycombs. From the materials point of view, materials with high $RK$ are preferable. The transmittance factor of the cellular array depends on the aspect ratio of the cell and the wall, and its higher values correspond to higher aspect ratio of the wall and lower aspect ratio of the air cell. It may be mentioned that the higher aspect ratio of the wall is in accordance with the higher transmittance factor, whereas the higher aspect ratio of the air cell is in opposition to the requirement of higher transmittance factor.

REFERENCES