A NOVEL COMPUTER SOFTWARE FOR CALCULATING OPTIMUM LOCATION AND RATINGS OF A SHUNT CAPACITOR FOR LOSS REDUCTION IN DISTRIBUTION FEEDERS

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Abstract—The different methods of analysis for the problem of installing shunt capacitors on distribution systems have been discussed, and a computer software is developed for calculation of the optimum size and location of a shunt capacitor on a distribution feeder for maximum reduction in power loss from standard distribution system parameters. This program can be easily used for a practical system consisting of non-uniform loads and sections of different wire sizes.

Optimum Capacitor Location Loss reduction

INTRODUCTION

The practice of placement of shunt capacitors on distribution feeders for improvement of power factor and voltage profile has been there for a long time. The assumption of uniform load distribution did not produce correct results. Later, representation of the feeder by a number of segments and combination of concentrated and uniformly distributed load gave a more realistic approach. Here also, the assumption of uniform wire sizes all along the feeder still gave inaccurate results.

In this paper, the computer program is developed considering the Normalization Technique as given by Grainger and Lee [1]. The various methods of analysis of the problem from the uniform feeder to the Normalization Technique are also explained.

UNIFORMLY LOADED SINGLE FEEDER WITH CONCENTRATED LOAD AT INTERVALS

The \( P^2R \) loss reduction due to placement of a shunt capacitor can be explained by considering a segment of the distribution feeder as given in Fig. 1. The active and reactive components of the \( P^2R \) loss are \( (I \cos \theta)^2 R \) and \( (I \sin \theta)^2 R \). When a capacitor is placed with current \( I_c \), then the new line current will be given by

\[
I^2_c R = (I \cos \theta)^2 R + (I \sin \theta - I_c)^2 R.
\]  

(1)

The reduction in loss due to adding the capacitor is \( \Delta L \)

\[
\Delta L = I^2 R - I^2_c R = 2(I \sin \theta)I_c R - I^2_c R.
\]  

(2)

From the above, it can be concluded that we need to consider only the reactive component of the line current for loss reduction calculation.

Figure 1 shows a combination of uniformly distributed load with concentrated loads at intervals where \( I_1 \) = reactive current flowing in the segment, \( I_2 \) = reactive current at the end of the segment.

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$I_c$ = capacitor current, $R$ = resistance of the segment and $a$ = fraction of line segment length to capacitor location.

We have the $I^2R$ loss without the capacitor as

$$L = [I_1^2 + I_1 I_2 + I_2^3]R.$$  \hfill (3)

The $I^2R$ loss after adding the capacitor is

$$L' = 3 \int_0^1 [I_1(I_1 - I_2)X]^2R \, dx + 3 \int_0^1 [I_1(I_1 - I_2)X]^2R \, dx + 3 \int_0^1 [I_1(I_1 - I_2)X]^2R \, dx$$

$$= [I_1^3 + I_1 I_2 + I_2^3]R + 3a[(2 - a)I_1 I_c + aI_2 I_c - I_2^3]R.$$ \hfill (4)

The $I^2R$ loss reduction due to the capacitor is

$$\Delta L = 3a[(2 - a)I_1 I_c + aI_2 I_c - I_2^3]R.$$ \hfill (5)

If $p = I_2/I_1$ and $q = I_c/I_1$

$$\Delta L = 3[(p - 1)q^2 + 2aq - aq^2]I_1^2R.$$ \hfill (6)

The reduction in loss per unit loss due to the capacitor is

$$\delta = \frac{\Delta L}{L} = \frac{3a[(2 - a)I_1 I_c + aI_2 I_c - I_2^3]R}{[I_1^3 + I_1 I_2 + I_2^3]R}$$

$$= kaq[(2 - a) + ap - q]$$ \hfill (7)

where

$$k = \frac{3}{(1 + p + p^2)}.$$ \hfill (8)

**OPTIMUM VALUES OF CAPACITOR AND LOCATION**

For optimum loss reduction, the location at which a given capacitor should be placed can be found by taking

$$\frac{\partial^2(\Delta L)}{\partial a^2} = 3(2q(p - 1)a + 2q - q^2)I_1^2R = 0$$

$$\frac{\partial^2(\Delta L)}{\partial a^2} = 6q(p - 1)I_1^2R < 0$$

$$a = \frac{1 - 1/2q}{1 - p} \quad 0 \leq a \leq 1$$ \hfill (9)

$$\delta = K(1 - p)a^2q.$$ \hfill (10)

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**Fig. 1.** Uniformly loaded feeder with concentrated loads at intervals.
For the value of the capacitor at a given location for optimum loss reduction

\[
\frac{\partial (\Delta L)}{\partial q} = 3a(\alpha p + 2 - \alpha - 2q)l_1^2R = 0
\]

\[
\frac{\partial^2 (\Delta L)}{\partial q^2} = -6aI_1^2R < 0
\]

\[
q = 1 - \frac{1}{2}(1 - \alpha)\alpha
\]

\[
\delta = K\alpha q^2.
\]

From the above two conditions, we have, for optimal loss reduction,

\[
a = \frac{2}{3(1 - \alpha)} \quad 0 \leq \alpha \leq 1 \quad \alpha \neq 1
\]

\[
q = \frac{2\alpha}{3} \quad \alpha \leq 1/3
\]

\[
= 1 - \frac{1}{2}(1 - \alpha) \quad \alpha \geq 1/3
\]

\[
\delta = \frac{8K}{27(1 - \alpha)} \quad \alpha \leq 1/3
\]

\[
= \frac{K}{4}(1 + \alpha)^2 \quad \alpha \geq 1/3.
\]

Fig. 3. Locations and current ratings of shunt capacitors installed for loss reduction.
From these equations, we can derive corresponding conditions for optimum loss reduction in various cases of uniformly distributed load or concentrated load.

**PEAK LOSS AND ENERGY LOSS REDUCTION AND USE OF SWITCHED CAPACITORS**

Maintenance of correct voltage level is one of the fundamental requirements of an electrical distribution system. The installation of shunt capacitors in a distribution feeder always improves the voltage levels at the farther end of the feeder, but it is likely that, at light loads, there can be a rise in voltage at the farther end of the feeder due to the presence of these capacitors. This can be overcome by the use of switched capacitors. The switched capacitors can be connected to the feeder depending upon the reactive load on the feeder. A discrete set of capacitor ratings, with increment corresponding to the smallest unit considered for a feeder, may be located at various points...
to fulfill such a requirement. The optimum size and location of the capacitor can be determined on the basis of maximum energy loss reduction within the upper and lower bounds of values.

Hence, in the distribution feeder given in Fig. 1, the peak loss reduction in a line segment due to adding a fixed capacitor is given by

$$\Delta L_n = 3a[(2-a)I_{in}I_n + aI_{2n}I_c - I_c^2] \frac{R_n}{1000} \text{ kW}$$  \hspace{1cm} (16)$$

and the corresponding energy loss reduction is

$$\Delta E_{L_n} = 3a[(2-a)I_{in}I_cF_{LD} + aI_{2n}I_cF_{LD} - I_c^2] \frac{R_nT}{1000} \text{ kWh}$$  \hspace{1cm} (17)$$

where

$I_{in} =$ peak reactive current in amperes at the start of line segment before adding the capacitor
$I_{2n} =$ peak reactive current at the end of line segment
$I_c =$ capacitor current
$a =$ fraction of line segment length to capacitor location
$R_n =$ resistance of line segment in ohms
$F_{LD} =$ reactive load factor
$T =$ time in hours fixed capacitor is connected to feeder.

Similarly, the peak power loss and energy loss reduction due to adding a switched capacitor are

$$\Delta L'_n = 3b[(2-b)I'_{in}I'_n + bI_{2n}I'_c - (I'_c)^2] \frac{R_n}{1000}$$  \hspace{1cm} (18)$$

$$\Delta E_{L'_n} = 3b[(2-b)I'_{in}I'_cF'_{LD} + bI_{2n}I'_cF'_{LD} - (I'_c)^2] \frac{R_nT}{1000} \text{ kWh}$$  \hspace{1cm} (19)$$

where

$b =$ fraction of line segment length to switched capacitor location
$I'_{in} =$ peak relative current in amperes at start of line segment before switched capacitor is added
$I_{2n} =$ peak reactive current at the end of the line segment
$I'_c =$ switched capacitor current
$T' =$ time that switched capacitor is connected to feeder
$F_{LD} =$ reactive load factor during the time that switched capacitor is connected to feeder.

THE NORMALIZATION TECHNIQUE

In the above methods of analysis of distribution feeders, the wires are taken to be of uniform size, whereas in a practical situation, neither the reactive load nor the wire sizes are the same. We have different wire sizes depending upon the loads on the feeder, and the wire sizes decrease as we reach the end of the feeder. So, conversion of an actual feeder into an equivalent uniform feeder will make the analysis more realistic.

Figure 2(a) shows a feeder consisting of several sections with reactive loads and wire sizes as given in Table 1.

For converting the physical length of the feeder to its equivalent uniform feeder of unit length, we take $r_j$, the resistance in ohms/km of the $j$th section, as the resistance of the equivalent uniform feeder.

The uniform length of the other sections will be

$$L_{un} = \frac{L_i r_i}{r_j} \hspace{1cm} i = 1, 2, \ldots, k. \hspace{1cm} (20)$$

The total length in km of the equivalent feeder is defined by

$$L_n = \sum_{i=1}^{k} \frac{L_i r_i}{r_j}. \hspace{1cm} (21)$$
The uniform resistance in ohms per normalized unit length of the feeder is given by

\[ r = \sum_{j=1}^{k} L_j r_j. \]

In Table 1 given above, we take \( r_j = r_4 \) for calculation of the normalized feeder. The equivalent lengths of the sections are given in km and also in p.u. Figures 2(a), (b) and (c) give the representation of the physical feeder, equivalent feeder and the normalized uniform feeder in p.u.

From the above, it is apparent that the section lengths of the physical feeder (relative to each other) are expanded, contracted or unaltered by the normalizing transformation depending upon the corresponding \( r_j/r_4 \) ratios. Thus, a physical feeder of different wire sizes, assumed to have uniform load distribution, becomes non-uniform on the equivalent feeder; thus, bringing out the significance of this method of analysis.

**OPTIMUM LOCATION AND CAPACITOR BANK SIZES**

For the placement of shunt capacitors on the distribution feeder, we can directly apply this method to actual load distributions found in practice; as a result, capacitors can be specified to satisfy the actual compensation levels.

Considering the three-phase non-uniformly distributed reactive load, such as that represented by Fig. 2(a), the normalized current density function is defined as

\[ f(x) = \frac{I(x)}{I_t} \]  \hspace{1cm} (22)

where \( I_t \) is the peak reactive current injected in the feeder at the starting of the feeder, \( x \) is the distance measured along the normalized equivalent feeder from the same end and \( I(x) \) is the reactive current density at \( x \).

The normalized feeder reactive current function is

\[ F(x) = \sum_{j=1}^{k} F(x). \]  \hspace{1cm} (23)

For optimizing the loss reduction in the distribution feeder, we need to place \( n \) fixed shunt capacitors at locations, say \( h_i (i = 1, 2, \ldots, n) \), and of sizes \( I_{a,i} (i = 1, 2, \ldots, n) \) as represented in Fig. 3.

The power loss reduction by the single capacitor \( I_{a,i} \) is

\[ LP_i = 2rI_{a,i} \int_{0}^{h_i} \left( I(x) - \sum_{k=1}^{i-1} I_{a,k} \right) dx - r h_i I_{a,i}^2. \]  \hspace{1cm} (24)
The total power loss reduction is

\[ LP = \sum_{i=1}^{n} LP_i. \]  

(25)

Similarly, the total energy loss reduction is

\[ LE = \sum_{i=1}^{n} LE_i \]  

(26)

where

\[ LE_i = \int_{0}^{T} \left[ 2r I_{ci} \int_{0}^{h_i} I_i F(x) - \sum_{k=1}^{i-1} I_{ck} \right] dx - r I_{ci}^2 \]  

(27)

and the total savings are given by

\[ S = K_p LP + K_e LE - K_c \sum_{i=1}^{n} I_{ci} \]  

(28)

where \( K_p, K_e, \) and \( K_c \) are the respective constants to convert power loss savings, energy loss savings and capacitor bank ratings to dollars.

**OPTIMUM LOCATIONS FOR CAPACITORS**

To find the location at which maximum savings occur, we differentiate equation (28) given above. Thus,

\[ \frac{\partial S}{\partial h_i} \bigg|_{h_i = h} = \frac{\partial LP}{\partial h_i} + K_e \frac{\partial LE}{\partial h_i} = 0 \]  

(29)

\[ \frac{\partial LP}{\partial h_i} = \frac{\partial LP_i}{\partial h_i} = 2r I_{ci} I_i \left( F(h_i) - \sum_{k=1}^{i-1} \frac{I_{ck}}{I_i} \right) - r I_{ci}^2 \]  

(30)

from which, we get

\[ \frac{\partial LE}{\partial h_i} = \frac{\partial LE_i}{\partial h_i} = T \left[ 2r I_{ci} I_i \left( F(h_i) - \sum_{k=1}^{i-1} \frac{I_{ck}}{I_i} \right) - r I_{ci}^2 \right] \]  

(31)

\[ F(h_i)^* = \frac{(k_p + k_e T)}{(k_p + K_e T L_i)} \left( \frac{I_{ci}}{2I_i} + \sum_{k=1}^{i-1} \frac{I_{ck}}{I_i} \right) \]  

(32)

The steps involved in the computer solution approach developed in FORTRAN are given in Fig. 4. The first major step involves the normalization of the feeder from the given data. The solution gives the optimum values of the locations for the capacitors.

**OPTIMUM CAPACITOR VALUES**

In a similar way, as done in finding the formula for optimum locations, we differentiate equation (28) for finding values of capacitors to be installed at predesignated locations along the feeder. So, we get

\[ \frac{\partial S}{\partial I_{ci}} = K_p \frac{\partial LP}{\partial I_{ci}} + K_e \frac{\partial LE}{\partial I_{ci}} - K_c \bigg|_{I_{ci} = I_{ck}} = 0. \]  

(33)

For any \( i \), we have the following

\[ \frac{\partial LP_i}{\partial I_{ci}} = 2r \int_{0}^{h_i} I_i F(x) \, dx - 2r I_i \sum_{k=1}^{i} I_{ck} \]  

(34)

\[ \frac{\partial LE_i}{\partial I_{ci}} = -2r I_i I_{ci} \quad \text{for} \quad k > i \]  

(35)
\[
\frac{\partial L_{P_k}}{\partial I_{i}} = 0 \quad \text{for} \quad k < i \quad \text{(36)}
\]
\[
\frac{\partial L_{E_i}}{\partial I_{i}} = T \left[ 2r L_{i} \int_{0}^{h_i} I_{i} F(x) \, dx - 2r h_i \sum_{k=1}^{i} I_{k} \right] \quad \text{(37)}
\]
\[
\frac{\partial L_{E_k}}{\partial I_{i}} = -2r h_k I_{k} T \quad \text{for} \quad k > i \quad \text{(38)}
\]
\[
\frac{\partial L_{E_k}}{\partial I_{i}} = 0 \quad \text{for} \quad k < i \quad \text{(39)}
\]

Substituting from equations (34) to (39) in equation (33),
\[
\frac{\partial S}{\partial I_{i}} = k_p \sum_{k=1}^{n} \frac{\partial L_{P_k}}{\partial I_{i}} + k_e \sum_{k=1}^{n} \frac{\partial L_{E_k}}{\partial I_{i}} - k_c = 0
\]
\[
\left( \begin{array}{cccc}
  h_1 & h_2 & \ldots & h_n \\
  h_2 & h_3 & \ldots & h_n \\
  h_1 & h_1 & \ldots & h_{i+1} \\
  h_n & h_n & \ldots & h_n \\
\end{array} \right)
\]
\[
\left( \begin{array}{c}
  I_{i}^{*} \\
  I_{i}^{*} \\
  I_{i+1}^{*} \\
  I_{n}^{*} \\
\end{array} \right) = \left( \begin{array}{c}
  \frac{(k_p + k_e TL_{i})}{(k_p + k_e T)} I_{wq}(h_{i+1}, h_i) - \sum_{i=1}^{i-1} I_{i} - \frac{k_z}{2r(k_p + k_e T)} I_{i}(0, h_i) - \frac{k_c}{2r(k_p + k_e T)} I_{i}(0, h_i) - \frac{k_z}{2r(k_p + k_e T)} I_{i}(0, h_i) - \frac{k_c}{2r(k_p + k_e T)} I_{i}(0, h_i) - \frac{K_{c}}{2r(k_p + k_e T)} I_{i}(0, h_i) - \frac{K_{c}}{2r(k_p + k_e T)} \\
\end{array} \right) \quad \text{(41)}
\]

where
\[
I(0, h_i) = \int_{0}^{h_i} I_{i} F(x) \, dx
\]

when \( k_c = 0 \), i.e. when the cost of capacitors is neglected, the solution to equation (41) can be given by
\[
I_{i}^{*} = \frac{(k_p + k_e TL_{i})}{(k_p + k_e T)} I_{wq}(h_{i+1}, h_i) - \sum_{i=1}^{i-1} I_{i} \quad \text{for} \quad i = 1, 2, 3, \ldots, n
\]
\[
\text{(42)}
\]

where
\[
I_{wq}(h_{i+1}, h_i) = \frac{1}{h_i - h_{i+1}} \int_{h_i}^{h_{i+1}} I_{i} F(x) \, dx
\]

and \( h_{n+1} \) = the beginning of the feeder.

The flowchart for the computer program developed in FORTRAN for finding the value of the capacitor for predesignated locations is given in Fig. 5. The major steps involve the conversion of locations into normalized p.u. values and finding values \( I_{i1}, I_{i2}, \ldots, I_{in} \) from the normalized reactive current function \( F(x) \) and then calculating the corresponding capacitor ratings.

**CONCLUSION**

In this paper, we have done a brief study of the various methods previously used for loss reduction on distribution feeders and optimizing the loss reduction by suitable placement of shunt capacitors. The computer program can be easily used for a practical system. The optimum location for available capacitor ratings and optimum capacitor ratings for suitable locations can be easily calculated using this program. Further extension may include incorporating voltage control problems and use of switched capacitors along the feeder.

**REFERENCES**