Rotational Stability of Unreinforced and Reinforced Embankments on Soft Soils

Shenbaga R. Kaniraj
Civil Engineering Department, Indian Institute of Technology, New Delhi 110 016, India
(Received 18 May 1994; accepted 10 June 1994)

ABSTRACT

Solutions for the rotational stability analysis of unreinforced and reinforced embankments on soft soils have been developed. The general case of an embankment having a partial-height dry tension crack, a berm, and an excavation outside the berm has been considered. A limit equilibrium method assuming a circular slip surface and total stress analysis have been used. In reinforced embankments, three different directions of reinforcement force have also been considered. The solutions are presented in the form of simple equations. Using these equations, the location of the critical slip circle, and the minimum factor of safety can be calculated directly for unreinforced embankments, for a given limiting tangent. Similarly, for reinforced embankments, the equations can be used to calculate the location of the critical slip circle and the maximum required reinforcement force, for a given limiting tangent. The conditions that are to be satisfied to get valid solutions are described. The use of the solutions is explained with illustrative examples.

NOTATION

\( a \) Level of reinforcement above ground surface (m)
\( b \) Crest width (m)
\( \nu \) Cohesion of embankment soil (kPa)
\( \nu_{eq} \) Equivalent constant undrained cohesion of foundation soil in depth
\( D \) (kPa)
$D$ Depth of limiting tangent below ground surface (m)
$F$ Factor of safety
$H$ Height of embankment (m)
$H'$ Uncracked height of embankment $= H - H_c$ (m)
$H_c$ Height of tension crack (m)
$k_1$ Ratio of berm thickness to embankment height
$k_2$ Ratio of berm width to embankment height
$L_a$ Moment arm for reinforcement force $P$ about the centre of the slip circle (m)

$m$ Ratio of stability factor $N_1$ to $N_2$
$M_o$ Total overturning moment (kNm/m)
$M_{ob}$ Overturning moment due to soil mass in the berm (kNm/m)
$M_{oc}$ Overturning moment due to soil mass in the embankment in the zone of tension crack (kNm/m)
$M_{oe}$ Overturning moment due to soil mass in the embankment below the tension crack zone (kNm/m)
$M_{ox}$ Overturning moment due to soil mass in the excavation (kNm/m)
$M_r$ Total resisting moment (kNm/m)
$M_{re}$ Moment due to resisting forces in the embankment along the slip surface below the tension crack (kNm/m)
$M_{rf}$ Moment due to resisting forces in the foundation soil along the slip surface (kNm/m)
$M_{rr}$ Resisting moment due to reinforcement force $P$ (kNm/m)
$n$ Side slope ($n$ horizontal to 1 vertical)
$N_1$ Stability factor for foundation soil
$N_2$ Stability factor for embankment soil
$P$ Reinforcement force (kN/m)
$P_{max}$ Maximum required reinforcement force (kN/m)
$S_e$ Normalized embankment strength parameter
$S_f$ Normalized foundation strength parameter
$W$ Weight of excavation (kN/m)
$X_{1'}$ $X$ coordinate of point $1'$ of the slip circle (m)
$X_o$ $X$ coordinate of the centre of the slip circle (m)
$X_s$ Horizontal distance between the outer edge of the excavation and the toe of the embankment (m)
$X_{xs}$ Horizontal distance between the centroid of the excavation and the toe of the embankment (m)
$Y_o$ $Y$ coordinate of the centre of the slip circle (m)

$\alpha$ Inclination of the reinforcement force $P$ to the horizontal
$\beta$ Ratio of uncracked height to total height of embankment
$\gamma$ Unit weight of embankment soil (kN/m$^3$)
\[ \lambda \quad \text{Averaging coefficient for frictional stress in the embankment} \]
\[ \mu \quad \text{Berm factor} \]
\[ \phi \quad \text{Angle of shearing resistance of embankment material (°)} \]

1 INTRODUCTION

Generally, five modes of failure (namely, bearing capacity, sliding, spreading, foundation soil squeezing and rotational failures) are considered in the design of embankments on soft soils. Of these, the rotational failure is often the critical mode of failure and therefore governs the design. The embankments constructed on soft soils are usually not very high and are built very quickly. Conventionally, a total stress analysis is carried out using a limit equilibrium approach. The factor of safety of unreinforced embankments, at the end of construction, is evaluated assuming the failure surface to be an arc of a circle. From a consideration of the moment equilibrium of several trial failure surfaces, the critical failure surface and the corresponding minimum factor of safety are obtained. Because of the iterative process, the analysis nowadays is usually carried out using a computer program.

For unreinforced embankments without berm, Low (1989) has presented solutions for the location of the critical slip circle and the minimum factor of safety, for a given limiting tangent, in the form of simple equations. The overall minimum factor of safety can be calculated directly by considering different limiting tangents. Kaniraj and Abdullah (1993) have presented simple solutions with which the effect of berm and of a full-height dry tension crack on the stability of embankments can be evaluated quantitatively.

For reinforced embankments, Kaniraj and Abdullah (1992a, b, 1994) and Kaniraj and Panwar (1994) have presented solutions for the location of the critical slip circle and the maximum required reinforcement force, for a given limiting tangent. Using these solutions, the effect of berm and of a full-height dry tension crack on the reinforcement design can be evaluated quantitatively. Three different directions of reinforcement force (namely, horizontal, tangential to the failure plane, and bisectorial to the horizontal and tangential directions) have been considered in these solutions.

The general case of the embankment design problem is considered in this paper. The tension crack is assumed to be a partial height crack. The presence of a berm and an excavation outside the berm is also considered. In the case of reinforced embankments, the three different directions of reinforcement force, as before, are considered. Solutions for the location of the critical slip circle and the minimum factor of safety are presented first for unreinforced embankments. Following this, solutions are presen-
Fig. 1. Unreinforced embankment on soft soil.

ted for the critical slip circle and the maximum required reinforcement force, for reinforced embankments. The conditions to be satisfied in order to get valid solutions are also described. The use of the solutions is explained with illustrative examples.

2 UNREINFORCED EMBANKMENT

Figure 1 shows an arbitrary slip circle in an embankment tangential to a limiting tangent at depth D. The slip circle encloses an excavation and a berm, and terminates at the bottom of a partial-height dry tension crack of height $H_c$. The origin of the (X, Y) axes is assumed to be at the level of the limiting tangent, on a vertical line passing through the toe E of the embankment. $X_o$ and $Y_o$ are the coordinates of the centre of the slip circle. The dimensions of the berm are expressed in terms of the height of the embankment. The weight of the excavation, $W_X$, is equivalent to a force $W_X$ acting in the upward direction and this acts through the centre of gravity of the excavation at a horizontal distance $X_x$ from the toe.

2.1 Solutions for critical slip circle and minimum factor of safety

The details of the derivations of the solutions for the location of the critical slip circle and the minimum factor of safety are presented in Appendix 1.
For a given limiting tangent, the coordinates of the critical slip circle are given by

\[ \frac{X_0}{H} = \frac{n}{2} - k_1 k_2 + \frac{W_x}{\gamma H^2} \quad (1) \]

\[ \frac{Y_0}{H} = 1.564 \alpha_1 \quad (2) \]

where

\[ \alpha_1 = \frac{\alpha'_1}{\frac{D}{H} + \beta - \frac{\beta^2}{2}} \quad (3) \]

\[ \alpha'_1 = \beta^2 \left( 1 - \frac{2}{3} \beta - \frac{D}{H} \right) + 2\beta \frac{D}{H} + \left( \frac{D}{H} \right)^2 + \frac{n^2}{12} \]

\[ + \mu - \frac{W_x}{\gamma H^2} \left[ \frac{W_x}{\gamma H^2} + n + 2 \left( \frac{X_0}{H} - k_1 k_2 \right) \right] \quad (4) \]

\( \mu \) is a term called the berm factor and is given by

\[ \mu = k_1 k_2 (n + k_2) (1 - k_1) \quad (5) \]

The value of \( \mu \) increases as the size of the berm increases and \( \mu = 0 \) for embankments without berm. \( \beta \) is the ratio of the uncracked height to the total height of the embankment and is given by

\[ \beta = 1 - \frac{H_c}{H} \quad (6) \]

For a full-height tension crack, \( \beta = 0 \); if there is no tension crack and the failure surface passes through the full height of the embankment, \( \beta = 1 \). Other notation is explained in Fig. 1. For a given limiting tangent, the minimum factor of safety is expressed as

\[ F = S_f N_1 + S_e N_2 \quad (7) \]

\( S_f \) and \( S_e \) are called the normalized foundation strength parameter and embankment strength parameter, respectively. The values of \( S_f \) and \( S_e \) are given by

\[ S_f = \frac{C_d}{\gamma H} \quad (8) \]

\[ S_e = \frac{C}{\gamma H} + \lambda \tan \phi \quad (9) \]
\( N_1 \) and \( N_2 \) are called the stability factors. These are given by

\[
N_1 = 5.55 \times 10^{0.47} \frac{(D)^{0.53}}{\left(\frac{D}{H} + \beta - \frac{\beta^2}{2}\right)}
\]

(10)

\[
N_2 = mN_1
\]

(11)

where

\[
m = 0.5 \left[ \left(1 + \frac{\beta}{D}\right)^{0.53} - 1 \right]
\]

(12)

Table 1 gives the values of \( m \) for different values of \( D/H \) and \( \beta \). Other notation is explained in Appendix 1.

3 REINFORCED EMBANKMENT

Figure 2 shows an arbitrary circular failure plane tangential to a limiting tangent at depth \( D \). The reinforcement is at a distance \( a \) above the ground surface. \( \alpha \) is the inclination of the direction of the reinforcement force \( P \) to the horizontal. Three different directions of reinforcement force—namely, horizontal (\( \alpha = 0 \)), tangential (\( \alpha = \theta = \angle MQV \)), and bisectorial (\( \alpha = \theta/2 = \angle MQV/2 \))—have been considered in the analysis. The details of the derivation of the solutions for the location of the critical slip circle and the maximum required reinforcement force are presented in Appendix 2.

3.1 Solutions for critical slip circle and maximum reinforcement force

3.1.1 Horizontal reinforcement force

For a given limiting tangent, the coordinate \( X_0 \) of the critical slip circle is given by eqn (1) and \( Y_0 \) is obtained by solving

\[
\left(\frac{Y_0}{H}\right)^{1.47} - 3.128 \left(\frac{D + a}{H}\right) \left(\frac{Y_0}{H}\right)^{0.47} - 2.128 \left(\frac{F_2}{F_1}\right) = 0
\]

(13)

where

\[
F_1 = A_1 S_f + B_1 S_e
\]

(14)
Table 1

Values of $m$ (eqn (12))

<table>
<thead>
<tr>
<th>$D/H$</th>
<th>$\beta = 0.1$</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.0507</td>
<td>0.0976</td>
<td>0.1414</td>
<td>0.1828</td>
<td>0.2220</td>
<td>0.2594</td>
<td>0.2952</td>
<td>0.3297</td>
<td>0.3629</td>
<td>0.3950</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0259</td>
<td>0.0507</td>
<td>0.0746</td>
<td>0.0976</td>
<td>0.1199</td>
<td>0.1414</td>
<td>0.1623</td>
<td>0.1828</td>
<td>0.2026</td>
<td>0.2220</td>
</tr>
<tr>
<td>2.0</td>
<td>0.0131</td>
<td>0.0259</td>
<td>0.0384</td>
<td>0.0507</td>
<td>0.0628</td>
<td>0.0746</td>
<td>0.0862</td>
<td>0.0976</td>
<td>0.1088</td>
<td>0.1199</td>
</tr>
<tr>
<td>3.0</td>
<td>0.0088</td>
<td>0.0174</td>
<td>0.0259</td>
<td>0.0343</td>
<td>0.0426</td>
<td>0.0507</td>
<td>0.0588</td>
<td>0.0667</td>
<td>0.0746</td>
<td>0.0824</td>
</tr>
<tr>
<td>4.0</td>
<td>0.0066</td>
<td>0.0131</td>
<td>0.0195</td>
<td>0.0259</td>
<td>0.0322</td>
<td>0.0384</td>
<td>0.0446</td>
<td>0.0507</td>
<td>0.0568</td>
<td>0.0628</td>
</tr>
<tr>
<td>5.0</td>
<td>0.0053</td>
<td>0.0105</td>
<td>0.0157</td>
<td>0.0208</td>
<td>0.0259</td>
<td>0.0310</td>
<td>0.0360</td>
<td>0.0409</td>
<td>0.0458</td>
<td>0.0507</td>
</tr>
</tbody>
</table>

$m = 0$, for all $D/H$ when $\beta = 0$. 
Fig. 2. Reinforced embankment on soft soil.

\[ A_1 = 3.06 \left( \frac{D}{H} \right)^{0.53} \]  \hspace{1cm} (15)

\[ B_1 = 1.53 \left[ \left( \frac{D}{H} + \beta \right)^{0.53} - \left( \frac{D}{H} \right)^{0.53} \right] \]  \hspace{1cm} (16)

\[ F_2 = F'_2 - F \frac{D + a}{H} \left( \frac{D}{H} + \beta - \frac{\beta^2}{2} \right) \]  \hspace{1cm} (17)

\[ F'_2 = F \left[ \frac{1}{2} \left( \frac{D}{H} \right)^2 + \frac{\beta^2}{2} - \frac{\beta^3}{3} + \frac{n^2}{24} + \frac{\beta}{D} \frac{D}{H} - \frac{\beta^2}{2} \frac{D}{H} ight. \]

\[ + \frac{\mu}{2} - \frac{W_s}{\gamma H^2} \left( \frac{X_s}{H} + \frac{N}{2} - k_1 k_2 + \frac{1}{2} \frac{W_s}{\gamma H^2} \right) \]  \hspace{1cm} (18)

The values of \( Y_o/H \) in eqn (13) have been presented by Kaniraj and Abdullah (1992a, b) in a table and a chart. For a given limiting tangent, the maximum required horizontal reinforcement force \( P_{\text{max}} \), to achieve a target factor of safety \( F \), is expressed as

\[ \frac{P_{\text{max}}}{\gamma H^2} = \frac{\alpha_2}{L_o} \]  \hspace{1cm} (19)
where
\[
\alpha_2 = F \left( \frac{D}{H} + \beta \right) - \frac{\beta^2}{2} \left( \frac{Y_o}{H} \right) - F_2 - F_1 \left( \frac{Y_o}{H} \right)^{1.47} \tag{20}
\]
\[
\frac{L_\theta}{H} = \frac{Y_o}{H} - \frac{D + a}{H} \tag{21}
\]

### 3.1.2 Tangential reinforcement force

For a given limiting tangent, $X_o$ of the critical slip circle is given by eqn (1) and $Y_o$ is given by
\[
\frac{Y_o}{H} = 1.672 \left( \frac{F'_2}{F_1} \right)^{0.68} \tag{22}
\]

The maximum required tangential reinforcement force for a given limiting tangent is given by eqn (19) in which
\[
\frac{L_\theta}{H} = \frac{Y_o}{H} \tag{23}
\]

### 3.1.3 Bisectorial reinforcement force

For a given limiting tangent, $X_o$ of the critical slip circle is given by eqn (1) and $Y_o$ is obtained by solving
\[
\left( \frac{Y_o}{H} \right)^{2.47} - 1.032 \frac{D + a}{H} \left( \frac{Y_o}{H} \right)^{1.47} - \frac{F_3 Y_o}{F_1} + \frac{F_4}{F_1} = 0 \tag{24}
\]
where
\[
F_3 = 0.532 F \left[ \frac{a}{H} \left( \frac{\beta^2}{2} - \beta - \frac{D}{H} \right) + \frac{D}{H} \left( \frac{D}{H} - \frac{3}{2} \beta^2 + 3\beta \right) + \frac{n^2}{6} \right.
\]
\[
+ 2\mu + 2\beta^2 \frac{4}{3} \beta^3 - 4 \frac{W_x}{\gamma H^2} \left( \frac{X_x}{H} + \frac{n}{2} - k_1 k_2 + \frac{1}{2} \frac{W_x}{\gamma H^2} \right) \right] \tag{25}
\]
\[
F_4 = 0.266 F \frac{D + a}{H} \left[ \left( \frac{D}{H} \right)^2 + \frac{n^2}{12} + \mu + \beta \left( \beta - \frac{2}{3} \beta^2 + 2 \frac{D}{H} - \beta \frac{D}{H} \right) \right.
\]
\[
- \frac{W_x}{\gamma H^2} \left( \frac{X_x}{H} + \frac{n}{2} - k_1 k_2 + \frac{1}{2} \frac{W_x}{\gamma H^2} \right) \right] \tag{26}
\]
The maximum required bisectorial reinforcement force for a given limiting tangent is given by eqn (19) in which

\[
\frac{L_a}{H} = \frac{Y_o}{H} \sqrt{1 - \frac{1}{2} \frac{D + a}{Y_o} \frac{H}{H}}
\]  

(27)

4 CONDITIONS FOR THE VALIDITY OF THE SOLUTIONS

In order that the equations for the location of the critical slip circle, the minimum factor of safety, and the maximum required reinforcement force give valid solutions, three conditions must be satisfied. The three conditions are as follows.

4.1 Condition 1

The centre of the slip circle must lie at a level at or above the bottom of the tension crack (i.e. at a level at or above 1' in Figs 1 and 2). This condition is expressed by

\[
\frac{Y_o}{H} \geq \beta + \frac{D}{H}
\]  

(28)

4.2 Condition 2

In the analysis it has been assumed that the entire berm and the excavation lie within the failure plane, i.e. in Figs 1 and 2, NE \( \geq \) SE. This condition is given by

\[
\frac{Y_o}{H} \geq 0.5 \left[ \frac{D}{H} + \frac{\left( \frac{n}{2} - k_1k_2 + \frac{W_x}{ \gamma H^2} + \frac{X_s}{H} \right)^2}{\frac{D}{H}} \right]
\]  

(29)

where \( X_s \) is the horizontal distance of the outer edge of excavation (point S) from toe, i.e. \( X_s = SE \).

The following additional points about eqn (29) should also be noted:

(a) The slip circle should not cut through the excavation. Therefore, it is also necessary to draw the slip circle to check this.
(b) When there is a berm but no excavation, \( W_x/\gamma H^2 = 0 \), and \( X_s/H = k_2 \).
(c) When there is an excavation but no berm, \( k_1 = k_2 = 0 \).
(d) When there is no berm and no excavation, \( W_x/\gamma H^2 = 0 \); \( k_1 = k_2 = 0 \); and \( X_s/H = 0 \).

4.3 Condition 3

As shown in Figs 1 and 2, the terminal point \( I' \) of the slip circle in the embankment should lie below the crest and not below either of the two slopes. This condition can be stated as

\[
    nH \leq X_{1'} \leq nH + b
\]

where

\( X_{1'} = X \) coordinate of point \( I' \), and
\( b = \) crest width.

The condition stated in eqn (30) is expressed by eqns (31) and (32).

\[
    \frac{Y_o}{H} \geq 1/2 \left[ \frac{\left( \frac{n}{2} + k_1k_2 - \frac{W_x}{\gamma H^2} \right)^2}{\beta + \frac{D}{H}} + \beta + \frac{D}{H} \right] \tag{31}
\]

\[
    \frac{Y_o}{H} \leq 1/2 \left[ \frac{\left( \frac{n}{2} + k_1k_2 - \frac{W_x}{\gamma H^2} + \frac{b}{H} \right)^2}{\beta + \frac{D}{H}} + \beta + \frac{D}{H} \right] \tag{32}
\]

5 ILLUSTRATIVE EXAMPLES

The use of the solutions is illustrated by two examples. Figures 3 and 4 show the details for the first and second examples, respectively. First, the minimum factor of safety of the embankments needs to be calculated. If the factor of safety is less than 1.3, then the maximum required reinforcement force to achieve this value is determined.
5.1 Solution for Example 1 (Fig. 3)

5.1.1 Unreinforced embankment

From eqn (6)

\[ \beta = 1 - \frac{4}{6} = \frac{1}{3} \]

The equivalent cohesion, \( c \), in the 2-m height of the embankment below the tension crack, is calculated using the principle of weighted average as (Low, 1989)

\[ c = \frac{0.5 \times 30 + 1.5 \times 20}{2} = 22.5 \text{ kPa} \]

\( \mu = 0 \) and \( W_x = 0 \), as no berm or excavation are present. The normalized foundation strength parameter is calculated as \( S_t = 0.185 \).

The undrained strength of the foundation soil is uniform with depth. Therefore, the depth of limiting tangent for the overall minimum factor of safety is 3 m. Using the equations presented in previous sections, the following values can be calculated for \( D = 3 \) m:

\[ \frac{Y_o}{H} = 1.5 \]
\[ X_o = 9 \text{ m} \]
\[ \alpha_1 = 1.754 \]
\[ \frac{Y_o}{H} = 2.743 \]
\[ Y_o = 16.46 \text{ m} \]
\[ N_1 = 6.435 \]
\[ m = 0.155 \]
\[ S_e = 0.208 \]

Using these values, the minimum factor of safety of the embankment is calculated as 1.4.

- According to eqn (29), 2.743 \( \geq \) 0.83 — OK
- According to eqn (30), 2.743 \( \geq \) 2.50 — OK
- According to eqn (31), 2.743 \( \geq \) 1.77 — OK

Since all the three conditions are satisfied, the solutions are valid solutions. The failure surface is shown in Fig. 3.

Low (1991) has solved this example using a computer program called STABR (Duncan & Wong, 1984) and has computed the minimum factor of safety as 1.38. The computer program STABR incorporates both
Bishop's simplified method and the ordinary method of slices. It is evident that the solutions presented in the present paper give good results.

5.1.2 Reinforced embankment

Since the minimum factor of safety is more than 1.3, the embankment is stable and no reinforcement is required.
5.2 Solution for Example 2 (Fig. 4)

5.2.1 Unreinforced embankment

From eqn (6)

\[ \beta = 1 - \frac{3}{6} = 0.5 \]

Using the saturated unit weight above the water level and the buoyant unit weight below the water level, the weight of the excavation is calculated as \( W_e = 36.75 \) kN/m. The berm factor and the normalized foundation strength parameter are calculated as \( \mu = 0.5625 \) and \( S_f = 0.167 \), respectively.

For reasons explained in Section 5.1, the depth of limiting tangent for the overall minimum factor of safety is 6 m. Using the equations presented in the present paper, the following values are calculated for \( D = 6 \) m:

\[
\begin{align*}
X_o/H & = 0.801 \\
X_o & = 4.81 \text{ m} \\
\alpha_1 & = 1.855 \\
Y_o/H & = 2.901 \\
Y_o & = 17.41 \text{ m} \\
N_1 & = 5.396 \\
m & = 0.12 \\
S_e & = 0.274
\end{align*}
\]

Using these values, the minimum factor of safety of the embankment is calculated as 1.08.

- According to eqn (29), \( 2.901 \geq 1.50 \) — OK
- According to eqn (30), \( 2.901 \geq 5.15 \) — not OK
- According to eqn (31), \( 2.901 \geq 1.23 \) — OK
- According to eqn (32), \( 2.901 \leq 7.60 \) — OK

Since all the conditions are not satisfied, the solutions are not valid. Condition 2 is not satisfied, which means that the slip circle does not fully enclose the berm and the excavation. The slip circle is shown in Fig. 4 as 1-1-1. It can be seen that the slip circle cuts through the excavation which has been fully taken into consideration in the analysis. The result obtained is therefore conservative and the actual minimum factor of safety is more than the calculated value of 1.08.

Now, the analysis is repeated disregarding the presence of the excava-
tion altogether. In this case, the following values are calculated for 
\[ D = 6 \text{ m} \]
\[ X_0 / H = 0.75 \]
\[ X_0 = 4.5 \text{ m} \]
\[ \alpha_1 = 2.045 \]
\[ Y_0 / H = 3.199 \]
\[ Y_0 = 19.19 \text{ m} \]
\[ N_1 = 5.65 \]
\[ m = 0.12 \]

The minimum factor of safety of the embankment is calculated as 1.13.

- According to eqn (29), 3.199 \( \geq \) 1.50 — OK
- According to eqn (30), 3.199 \( \geq \) 2.03 — OK
- According to eqn (31), 3.199 \( \geq \) 1.27 — OK
- According to eqn (32), 3.199 \( \leq \) 7.75 — OK

Since all the conditions are satisfied, the solutions are valid solutions. The slip circle corresponding to this solution is shown in Fig. 4 as 2-2-2. It can be seen that the slip circle cuts through the excavation which has not been taken into consideration in the analysis. The result obtained therefore overestimates the factor of safety and the actual minimum factor of safety is less than the calculated value of 1.13.

From the above results the following conclusions can be made:

1. The factor of safety of the embankment lies between 1.08 and 1.13, which for practical purposes gives a good estimate of the factor of safety because of the closeness of the range.
2. The entire berm is effective in providing stability to the embankment.
3. The excavation is sufficiently away from the embankment that it does not influence/decrease the stability of the embankment significantly.
4. The solutions can be used to select the location of the excavations for borrow material, drainage, etc., such that they do not affect the stability of the embankment.

5.2.2 Reinforced embankment

The reinforcement force is generally assumed to act in the horizontal direction as this gives a conservative estimate of the required reinforcement force. Calculations for the horizontal direction reinforcement force
only are therefore presented here. When the excavation is considered in
the analysis, the following values can be calculated for $D = 6 \text{ m}$:

\[
\begin{align*}
X_o/H &= 0.801 \\
X_o &= 4.81 \text{ m} \\
Y_o/H &= 2.85 \\
Y_o &= 17.11 \text{ m} \\
P_{\text{max}} &= 229 \text{ kN/m}
\end{align*}
\]

Condition 2 is again not satisfied. The critical slip circle is very close to the
slip circle 1-1-1 in Fig. 4 and cuts through the excavation.

When the excavation is disregarded in the analysis, the following values

\[
\begin{align*}
X_o/H &= 0.75 \\
X_o &= 4.5 \text{ m} \\
Y_o/H &= 3.21 \\
Y_o &= 19.26 \text{ m} \\
P_{\text{max}} &= 169 \text{ kN/m}
\end{align*}
\]

All the conditions are satisfied. The critical slip circle is very close to the
slip circle 2-2-2 in Fig. 4 and cuts through the excavation.

For reasons explained in the case of unreinforced embankment, the

\[
\text{value of } P_{\text{max}} \text{ is in the range 169–229 kN/m. The critical slip circles again}
\]

show that the excavation does not influence the stability of the embank-
ment significantly. Therefore, it is more appropriate to consider a $P_{\text{max}}$

value slightly more than 169 kN/m in the design.

It should be noted here that if the excavation is very large (a large value of

$W_x$) or if the excavation is situated very far from the embankment (a large

value of $X_1$), no meaningful solutions may be obtained because of the high

negative value associated with the last term of eqns (4) and (18). If the exca-

vation is very deep, then the stability of the embankment will be influenced

by the stability of the cut made for the excavation itself. In such cases, it may

be appropriate to use conventional methods like the Bishop’s simplified

method and the ordinary method of slices for rotational stability analysis.

\section{6 Conclusions}

Solutions for the rotational stability analysis of unreinforced and re-

inforced embankments on soft soils have been presented. The general case of

the embankment having a partial-height dry tension crack, a berm, and an
excavation outside the berm has been considered. A limit equilibrium method assuming circular slip surface and total stress analysis have been used. In the case of reinforced embankments, three different directions of reinforcement force (namely, horizontal, tangential and bisectorial) have been considered. Solutions for the location of the critical slip circle and the minimum factor of safety for a given limiting tangent have been presented for unreinforced embankments first. Following this, solutions for the location of the critical slip circle and the maximum required reinforcement force for a given limiting tangent have been presented for reinforced embankments. In both the cases, solutions are expressed in the form of simple equations. The conditions that are to be satisfied for the results to be valid have also been identified. The use of the solutions has been explained using example problems. The results obtained for an unreinforced embankment using the equations presented in the paper and a computer program which incorporates both Bishop’s simplified method and the ordinary method of slices have been compared. The solutions presented in the paper give good results. The solutions can also be used to locate the excavations properly such that they do not affect the stability of the embankment.

REFERENCES


APPENDIX 1

DERIVATION OF THE EQUATIONS FOR UNREINFORCED EMBANKMENT

For the arbitrary slip circle shown in Fig. 1 tangential to the limiting tangent at depth \( D \), the factor of safety \( F \) is given by

\[
F = \frac{M_r}{M_o}
\]  
(A1.1)

where

\( M_r = \) total resisting moment,
\( M_o = \) total overturning moment.

The resisting moment consists of two components and can be written as

\[
M_r = M_{rf} + M_{re}
\]  
(A1.2)

where

\( M_{rf} = \) moment due to resisting forces in the foundation soil along the slip surface NMJ,
\( M_{re} = \) moment due to the resisting forces in the embankment along the slip surface II'.

\( M_{rf} \) is expressed as (Low, 1989)

\[
M_{rf} = 3.06 \, c_a \, D^{0.53} \, \gamma_o^{1.47}
\]  
(A1.3)

where

\( c_a = \) equivalent constant undrained cohesion within depth \( D \),
\( D = \) depth of limiting tangent below ground surface,
\( H = \) height of embankment,
\( \gamma = \) unit weight of embankment soil.

The soil in \( G'GII' \) is assumed to act as surcharge pressure on \( G'I' \). The effect of this on the frictional resistance along \( II' \) is taken into account by writing the expression for \( M_{re} \) as

\[
M_{re} = 1.53 \, (c + \lambda \gamma H \tan \phi) \, [(D + H')^{0.53} - D^{0.53}] \, \gamma_o^{1.47}
\]  
(A1.4)

where

\( c = \) cohesion of embankment soil,
\( H' = H - H_c \),
\( H_c = \) height of tension crack,
\( \lambda = \) averaging coefficient for frictional stress in the embankment,
\( \phi = \) angle of shearing resistance of embankment soil.
\(\lambda\) is given by (Low, 1989)

\[
\lambda = 0.19 + \frac{0.02n}{\frac{D}{H}} \quad \text{for} \quad \frac{D}{H} \geq 0.5 \quad (A1.5)
\]

The total overturning moment consists of four components and can be written as

\[
M_o = M_{oc} + M_{oc} - M_{ob} + M_{ox} \quad (A1.6)
\]

where

- \(M_{oc}\) = moment due to the soil mass \(EG'1'J\) in the embankment,
- \(M_{oc}\) = moment due to the soil mass \(G'GIH'\) in the zone of tension crack,
- \(M_{ob}\) = moment due to the soil mass \(ABCE\) in the berm,
- \(M_{ox}\) = moment due to the soil mass in the excavation.

\(M_{oc}\) is expressed as

\[
M_{oc} = \frac{\gamma H'}{2} \left[ X_o(nH' - X_o) - \frac{(nH')^2}{3} + 2Y_o \left( D + \frac{H'}{2} \right) \right.
\]

\[
\left. - \left( D + \frac{H'}{2} \right)^2 \frac{H'^2}{12} \right] \quad (A1.7)
\]

\(M_{oc}\) is expressed as

\[
M_{oc} = \frac{\gamma(H - H')}{6} \left[ 6Y_o(D + H') - 3(D + H')^2 - 3X_o^2 \right.
\]

\[
\left. + 3nX_o(H + H') - n^2(H^2 + H'^2 + HH') \right] \quad (A1.8)
\]

\(M_{ob}\) is given by

\[
M_{ob} = k_1k_2\gamma H^2 \left[ \frac{(k_2 - nk_1)H}{2} + X_o \right] \quad (A1.9)
\]

\(M_{ox}\) is given by

\[
M_{ox} = W'_{x}(X_x + X_o) \quad (A1.10)
\]

where

- \(W'_{x}\) = weight of soil mass in the excavation,
- \(X_x\) = horizontal distance between the centroid of the excavation and the toe of the embankment.

Thus, all the components of the resisting and overturning moments are
expressed by equations in terms of $X_0$ and $Y_0$. The expressions for $M_r$ and $M_e$ are substituted in eqn (A1.1). Partial derivatives of the equation with respect to $X_0$ and $Y_0$ are obtained and equated to 0. This gives two equations, the solution of which gives eqns (1) and (2) for the centre of the critical slip circle tangential to the limiting tangent. Using these equations in eqn (A1.1), the expression for the minimum factor of safety given in eqn (7) can be obtained.

APPENDIX 2

DERIVATION OF THE EQUATIONS FOR REINFORCED EMBANKMENT

For the arbitrary failure surface shown in Fig. 2 tangential to the limiting tangent at depth $D$, the factor of safety $F$ is given by eqn (A1.1). The total resisting moment in this case consists of three components and is given by

$$M_r = M_{rf} + M_{re} + M_{rt} \quad \text{(A2.1)}$$

where $M_{rt} = $ moment due to reinforcement force $P$.

$M_{rf}$ and $M_{re}$ are given by eqns (A1.3) and (A1.4), respectively. $M_{rt}$ is given by

$$M_{rt} = PL_a \quad \text{(A2.2)}$$

where

$L_a = $ moment arm of $P$ about the centre of the slip circle,

$= Y_o - D - a$, for horizontal reinforcement force,

$= Y_o$, for tangential reinforcement force,

$= Y_o \left[ 1 - \frac{(D + a)/2Y_o}{0.5} \right]$, for bisectorial reinforcement force.

The four components of the total overturning moment and their expressions are the same as in the case of the unreinforced embankment and are given in Appendix 1.

Substituting for $M_r$ and $M_{rt}$ from eqns (A2.1) and (A2.2), respectively, in eqn. (A1.1) and rearranging

$$P = \frac{M_oF - M_{rf} - M_{re}}{L_a} \quad \text{(A2.3)}$$

The expressions for $M_{re}$, $M_{rf}$, $M_o$, and $L_a$ are substituted in eqn (A2.3). Partial derivatives of the equation with respect to $X_0$ and $Y_0$ are obtained and equated to 0. This gives two equations, the solution of which gives the equations for the centre of the critical slip circle. Substituting these expressions for $X_0$ and $Y_0$ in eqn (A2.3), the corresponding expressions for $P_{\text{max}}$ given in the paper can be obtained.