SOLAR THERMAL ANALYSIS OF TRANSPARENT-HONEYCOMB-INSULATED GROUND COLLECTOR-STORAGE SYSTEM

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Abstract—This paper presents an analysis of heat transfer processes in a transparent-honeycomb-insulated solar collector made of such low-energy materials as soil/sand/concrete, which also acts as a heat storage. The analysis assumes solar intensity and atmospheric temperature as well as the resultant temperature in the ground and concrete/sand region to be periodic. An explicit expression for the heat flux that can be extracted at constant mass flow rate and at constant heat extraction temperature is derived. Numerical computations corresponding to typical months of June, September and December at New Delhi are presented. The solar conversion efficiency of 30-60% corresponding to a collection temperature in the range of 40-70°C is reported. The solar gain and heat storage characteristics of the proposed system are found to be of the right order of magnitude for solar water-heating applications.

Keywords—Solar thermal engineering, transparent insulation material, integrated collector-storage system, solar water heating, periodic thermal analysis.

NOMENCLATURE

\( C_c \) specific heat of concrete/sand \((\text{J/kgK})\)
\( C_s \) specific heat of the ground \((\text{J/kgK})\)
\( c_w \) specific heat of fluid \((\text{J/kgK})\)
\( D_1 \) inside diameter of heat extraction pipe \((\text{m})\)
\( d \) honeycomb cell width \((\text{m})\)
\( F_R \) heat removal factor
\( h_c \) heat transfer coefficient from concrete/sand to fluid \((\text{W/m}^2\text{K})\)
\( K_c \) thermal conductivity of concrete/sand \((\text{W/mK})\)
\( K_s \) thermal conductivity of the ground \((\text{W/mK})\)
\( K_f \) thermal conductivity of fluid \((\text{W/mK})\)
\( K \) extinction coefficient of glass \((1/m)\)
\( L \) depth of honeycomb cell \((\text{m})\)
\( m_w \) mass flow rate of fluid \((\text{kg/s})\)
\( n \) refractive index of glass cover
\( P \) perimeter of heat extraction pipe \((\text{m})\)
\( Q(t) \) heat flux \((\text{W/m}^2)\)
\( S_i \) solar intensity \((\text{W/m}^2)\)
\( S_n \) steady-state component of solar intensity
\( S_m \) amplitude of \(m\)th harmonic of solar intensity
\( T_A(t) \) ambient air temperature at time \(t\) \((\text{K})\)
\( T(\theta) \) transmittivity honeycomb panel
\( T'(r) \) transmittivity of honeycomb cover
\( T_1(r) \) total heat loss coefficient of honeycomb panel \((\text{W/m}^2\text{K})\)
\( T_2(r) \) depth of concrete/sand \((\text{m})\)

Greek letters
\( \omega \) solar angle
\( \alpha_{hr} \) effective absorptivity-transmittivity product
\( \epsilon \) emissivity of honeycomb array
\( \rho_c \) density of concrete/sand material \((\text{kg/m}^3)\)
\( \rho_g \) density of the ground \((\text{kg/m}^3)\)
INTRODUCTION

Solar thermal systems in which the same configuration serves as a collector as well as for storage have an obvious advantage over the conventional systems having the collector and storage in separate units. Such systems are referred to as integrated collector-storage systems (ICS). Low-energy materials, such as soil/concrete/sand, may be used as collector/storage materials in such systems [3, 4]. The ICS are inherently simple in their technology but are characterized by low thermal conversion efficiencies (10–15% at 40–50°C). In recent years, air-filled honeycomb devices have been suggested [5, 6] to reduce the heat losses and increase the solar gain of ground ICS. This paper presents an analysis of solar heat transfer processes in a ground ICS. The analysis is based on recent accurate determinations of the criteria of convection suppression, solar transmittance and thermal loss reduction characteristics of the honeycomb devices [7]. The heat conduction in the collector/storage zone is evaluated by the periodic solution of a one-dimensional Fourier heat conduction equation.

SCHEMATIC AND ANALYSIS

The schematic of the proposed system is shown as an exploded view of its components in Fig. 1. The outer surface of the system is exposed to incident solar radiation and atmospheric air. A part of incidental solar radiation is reflected from the surface cover and a small fraction is absorbed in it. The remainder entering the honeycomb array is attenuated by the cellular matrix before it reaches the blackened absorber (x = 0), where a small part of it is reflected and the rest is absorbed. All the transmissions and absorptions are functions of the solar angle of incidence which, for a particular location and day, is a function of the time of the day. The sensible heat stored in the concrete/sand region is transferred to fluid (water/air) through a network of pipes embedded in between the concrete/sand and the ground. The ground offers excellent insulation characteristics for the concrete/sand region and increases the heat retention duration in the concrete/sand slab. The material used for heat extraction pipes should have a high thermal conductivity as well as being non-corrosive; metallic and plastic pipes are suitable for such an application.

Fig. 1. Schematic diagram of honeycomb ICS: (a) exploded view, (b) parallel flow pipe network.
The solar radiation absorbed by the plane $x = 0$ is given by

$$S'(t) = (\alpha t)_{\text{abs}}(t).S(t).$$

The effective transmittance-absorptance product of the honeycomb cover system may be expressed as

$$(\alpha t)_{\text{eff}}(t) = T(\theta).T1(t).T2(t).s(t).$$

Formulations for the calculation of solar beam and global radiation transmittance, as well as the transmittance-absorptance product of honeycomb cover systems, have been reported by, amongst others, Hollands et al. [8], Symons [10] and Arulanandham and Kaushika [9] and have been adopted in the present paper.

The temperature distribution in the concrete/sand and the ground region is governed by the Fourier heat conduction equation:

$$K_i \frac{\partial^2 T_i(x,t)}{\partial x^2} = \rho_i C_i \frac{\partial T_i(x,t)}{\partial t}$$

$$K_s \frac{\partial^2 T_s(x,t)}{\partial x^2} = \rho_s C_s \frac{\partial T_s(x,t)}{\partial t} .$$

The corresponding boundary conditions are

$$-K_i \frac{\partial T_i(x,t)}{\partial x} \bigg|_{x=0} = S'(t) - Q_L(t)$$

at $x = 0$,

$$-K_s \frac{\partial T_s(x,t)}{\partial x} \bigg|_{x=x_i} = S'(t) - U_i(T(x = 0, t) - TA(t))$$

at $x = x_i$.

The continuity of temperature at the interface $x = x_i$ may be expressed as

$$T_i(x = x_i, t) = T_s(x = x_i, t),$$

thus

$$T_i(x = x_i, t) = T_i(t)$$

$$T_s(x = x_i, t) = T_s(t).$$

The boundary condition for the ground is

$$T_s(x, t) \text{ is finite as } x \to \infty .$$

Heat is transferred through the honeycomb array by one or more of the following modes:

1. conduction and radiation through the solid cellular media;
2. convection, conduction and radiation across the air cell.

The air-filled honeycomb is designed to be non-convective, therefore, the convective heat losses through it may be assumed as zero and the heat loss from the surface cover to the ambient is due to wind forced convection and radiation. Based on these consideration, $Q_L$ was calculated for steady-state conduction [11]; the calculations were based on an iterative process using the Newton–Raphson method. The value of $U_i[U_i = Q_L(T(x = 0, t) - TA(t))]$ was then evaluated and its variation for range of $TA(t)$ and $T_i(x = 0)$ for several depths of honeycomb was studied and it was recommended that, for a fixed depth of honeycomb, $U_i$ may be considered as invariant with $TA$ and $T_i(x = 0)$ and, hence, with the time of the day in equation (5).

Energy balance at the interface $(x = x_i)$ is given by

$$-K_s \frac{\partial T_s(x_i,t)}{\partial x} \bigg|_{x=x_i} = -K_s \frac{\partial T_s(x_i,t)}{\partial x} \bigg|_{x=x_i} + Q(t).$$
As solar intensity and atmospheric temperature are periodic functions of time, the resultant temperature in the concrete/sand region and the heat flux will also be periodic functions of time. These periodic functions may be represented by a Fourier series; six harmonics are considered to be sufficient in this representation [12]:

\[ S(t) = S_0 + \sum_{m=1}^{6} S_m e^{im\omega t} \]  
\[ TA(t) = TA_0 + \sum_{m=1}^{6} TA_m e^{im\omega t} \]  
\[ T_s(t) = T_s_0 + \sum_{m=1}^{6} T_s_m e^{im\omega t} \]

and

\[ Q(t) = Q_0 + \sum_{m=1}^{6} Q_m e^{im\omega t} \]

The temperature distribution in the concrete/sand and the ground zones are given by the periodic solution of equations (3) and (4), respectively, as follows:

\[ T_1(x = x_1,t) = A_0 + B_0 x_1 + \sum_{m=1}^{6} (Ame^{im\omega t} + Bme^{-im\omega t})e^{im\omega t} \]  
\[ T_2(x = x_1,t) = C_0 + D_0 x_1 + \sum_{m=1}^{6} (Cme^{im\omega t} + Dme^{-im\omega t})e^{im\omega t} \]

where

\[ a_m = \left( \frac{im \rho_{c} C_p}{K_i} \right)^{1/2} \quad \text{and} \quad b_m = \left( \frac{im \rho C_p C_3}{K_i} \right)^{1/2} \]

Two modes of heat extraction from the systems are considered: (i) Heat extraction with constant flow of fluid; this will cause the temperature of the heat extraction fluid to vary. (ii) Heat extraction by keeping the temperature of the heat extraction zone constant. This will obviously require a variation in the flow rate.

**Constant-temperature heat extraction**

For constant-temperature heat extraction

\[ T_1(x = x_1,t) = T_{\infty} \]  
\[ T_2(x = x_1,t) = T_{\infty} \cdot \]

So, we have from equation (10)

\[ Q(t) = K_d D_0 - K_s B_0 + \sum_{m=1}^{6} (- K_d Dm \beta e^{-\beta mX} - K_s (Am a_m e^{im\omega t} - Bm a_m e^{im\omega t}))e^{im\omega t}. \]

Equations (11)–(16) specify the temperature and heat flux distributions in the system. Constants involved in these equations were obtained by substituting the values of \( T_1(x = x_1,t) \), \( T_2(x = x_1,t) \), \( T_s(t) \), \( TA(t) \) and \( S(t) \) in the boundary conditions specified by equations (5), (9) and (17)–(18) and
considering time-dependent and independent parts separately. This yields two sets of linear simultaneous equations illustrated by the following matrix equations:

\[
\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} [k_1] = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{20}
\]
\[
\begin{bmatrix} x_2 \\ x_2 \end{bmatrix} [k_2] = \begin{bmatrix} y_2 \end{bmatrix} . \tag{21}
\]
Values of the matrices are given in Appendix I.

Solution of equations (20) and (21) enables the determination of constants \( A_0, B_0, C_0, D_0 \) and \( A_m, B_m, C_m, D_m \) \((m = 1.6)\), which may subsequently be used for computation of \( Q(t) \).

**Heat extraction at constant mass flow rate**

If the heat is extracted by the heat exchange fluid (air/water) flowing at a constant rate, then

\[ Q(t) = m_w c_w (T_a(t) - T_f(t)) \tag{22} \]

The temperature of the fluid inside the heat extraction pipe \( T_f(l, t) \) can be expressed in terms of the temperature of the plane of heat extraction, \( T_\delta(t) \), by considering the heat balance of an element between \( l \) and \( l + \delta l \) along the length of heat extracting pipe as follows:

\[ m_w c_w \frac{dT_f(l, t)}{dl} \delta l = h_f [T_\delta(t) - T_f(l, t)] P \delta l \tag{23} \]

or

\[ \frac{dT_f(l, t)}{T_f(l, t) - T_\delta(t)} = - \frac{h_f P \delta l}{m_w c_w} \tag{24} \]

which on integration yields

\[ C_f' [T_f(l, t) - T_\delta(t)] = \exp \left( - h_f P l / m_w c_w \right) . \]

The constant of integration \( C_f' \) is obtainable from the initial condition that \( T_f(0, t) = T_\delta(t) \).

so, we have

\[ T_f(l, t) = T_\delta(t) - [T_\delta(t) - T_f(l, t)] \exp \left( - h_f P l / m_w c_w \right) \tag{25} \]

\[ T_f(l = L, t) = T_a(t) = T_\delta(t) F_R + T_f(t)(1 - F_R) \tag{26} \]

or

\[ T_\delta(t) = T_f(t) + (T_f(t) - T_\delta(t)) F_R \tag{27} \]

where

\[ F_R = 1 - \exp \left( - h_f P \delta l / m_w c_w \right) \tag{28} \]

is called the heat removal factor; \( h_f \) is the heat transfer coefficient from the concrete/sand to the fluid.

The flow of fluid (air/water) in the heat extraction pipe is considered to be laminar fully developed over a considerable length [2]:

\[ N_d = \frac{h_f D}{k_f} = 4.36 \tag{29} \]

\[ h_f = k_f 1.436 / D \]

In the constant mass flow mode of heat extraction, the temperature of the heat extraction plane varies with time and it may be determined from the boundary condition represented by equation (10), as follows:

\[ T_f(x = x_i, t) = T_\delta(t) \tag{30} \]

\[ T_\delta(x = x_i, t) = T_\delta(t) \tag{31} \]

\[ m_w c_w F_R (T_f(t) - T_A(t)) = K_s \frac{\partial T_f(x, t)}{\partial x} \mid x = x_i - K_s \frac{\partial T_f(x, t)}{\partial x} \mid x = x_i , \tag{32} \]
on substitution of the values of the terms on the right-hand side and simplification this results in

\[ T_d(t) = \frac{1}{m_n c_n F_R} [(K_e D_o - K_b O) + TA_e + \sum_m^{+}] \\
[K_d(C_m \beta m e^{D_m} - D_m \beta m e^{-D_m}) - K_e(A_m \beta m e^{A_m} - B_m \beta m e^{-B_m}) + TA_m e^{A_m}]. \] (33)

The constants involved in these equations are obtained by solving the following matrix equations:

\[ [A1][C1] = [B1] \] (34)

\[ [A2][C2] = [B2]. \] (35)

Values of the matrices are given in Appendix II.

The solution of equations (34) and (35) enables the determination of constants \( A_o, B_o, C_o, D_o \) and \( A_m, B_m, C_m, D_m \) \((m = 1, 6)\) for a constant mass flow rate, which may subsequently be used for computation of \( Q(t) \) and \( T_d(t) \).

RESULTS AND DISCUSSION

Numerical computations for the heat flux that can be extracted from the system were carried out on an ICL 3980 computer at IIT Delhi using a Fortran 77 program. Heat was extracted by the flow of fluid through a network of PVC pipes. The hourly solar intensity, \( S(t) \), and atmospheric temperature, \( T_A(t) \), data corresponding to typical months of June, September and December at New Delhi (28° 35' N, 77° 12' E) during 1978 were taken [1]. \( S'(t) \) was obtained from the corresponding series of \( S(t) \) and \( \alpha(t) \). The series of \( S'(t) \) and \( T_A(t) \) were subjected to harmonic analysis and corresponding amplitudes of harmonics of \( S'(t) \) and \( T_A(t) \) were obtained. Lexan honeycomb of square cell cross-section (3.5 mm \( \times \) 3.5 mm) was considered; the dimensions corresponded to a commercially available honeycomb cellular matrix and were suitable to stabilize a bound air layer to be non-convective. The values of the thermophysical parameters used in the calculations were as follows:

(i) Glass:

\( t_g = 3 \text{mm}, n = 1.526, K = 5/m. \)

(ii) Honeycomb:

\( t_c = 0.025 \text{mm}, d = 3.5 \text{mm}, \varepsilon = 0.28 \)

\( L = -0.05, 0.10, 0.15 \text{ and } 0.175 \text{ m}. \)

\( U_c = 2.964, 1.872, 1.388 \text{ and } 1.232 \text{ W/m}^2. \)

(iii) Concrete/sand region:

\( K_c = 1.750 \text{ W/mK}, \rho_c = 2242 \text{ kg/m}^3, C_i = 880 \text{ J/kgK} \)

\( x_1 = 0.0 \text{ m}, 0.10 \text{ m}, 0.15 \text{ m}, 0.20 \text{ m and } 0.30 \text{ m}. \)

(iv) Ground region:

\( K_g = 0.560 \text{ W/mK}, \rho_g = 2050 \text{ K/gm}^3, C_g = 1840 \text{ J/kgK}. \)

(v) Heat extraction fluid:

\( m_e = 0.001 \text{ kg/min}, 0.008 \text{ kg/min} \text{ and } 0.01 \text{ kg/min}, k_t = 0.6396 \text{ W/mK}. \)

(vi) Collection

\( T_i = 50, 60 \text{ and } 70^\circ C. \)

(vii) Pipe:

\( D_i = 17 \text{ mm}, D_o = 20 \text{ mm}. \)

The heat flux, \( Q(t) \), that can be extracted at a constant temperature varies with the time of the day, the month of the year, the depth \( x_i \) of the concrete/sand zone and the depth \( L \) of the honeycomb panel. The diurnal curves of \( Q(t) \) depicting the variations of \( Q(t) \) as a function of \( x_i \) and \( L \) are shown in Fig. 2(a–c) and Fig. 3(a–c), respectively. The months of June, September and December have been chosen as characteristic of summer, equinox and winter seasons at New Delhi. The magnitude of \( Q(t) \) increases with \( L \) and the rate of increase drops at values of \( L \) higher than 0.10 m. The maximum of the diurnal increase of the heat flux appears later than the maximum of solar intensity and the shift is a function of \( x_i \), which indicates that the thermal storage capability of the system increases with \( x_i \). However, the daily average efficiency of heat extraction decreases with \( x_i \), as illustrated in Fig. 4(a) and (b). The chosen value of \( x_i \) is therefore a compromise between the thermal storage characteristics and the efficiency of the system. In general, the efficiency values predicted in Fig. 4(a) and (b) are higher than the corresponding values reported by Sodha et al. [3]
Fig. 2. Hourly variation of heat flux for different values of $x_1$ for three typical months: (a) June, (b) September, (c) December.

for a ground collector/storage system with simple glazing as the cover system. The average improvement in efficiency is about 75%.

The heat extraction at a constant flow rate involves the variation of fluid temperature at the outlet. The variation depicted in Fig. 5 for three typical months which correspond to $F_a = 0.99$ has been considered. The variation of the daily average efficiency of heat extraction with mass flow rate and depths of the concrete/sand zone is illustrated in Fig. 6.
COMPARATIVE PERFORMANCE STUDY

The proposed system with a collector/storage area of 1 sq.m is estimated to provide 46 kg of hot water ($\Delta T = 40^\circ C$) on a yearly average basis at New Delhi (28°N). Cost data of major components of the system are outlined in Table 1. The cost of a conventional solar water heater (thermosyphon type) of the same characteristics is Rs 8050.00, which shows that the proposed

![Diurnal variation of heat flux for different honeycomb depths: (a) June, (b) September, (c) December.](image-url)
system is cost effective; the system has an added advantage of using low-energy materials as the collector/storage unit.

Locally available means of heating water in domestic and industrial sectors include electric geyser and oil-fired boilers. A comparative study of the performance of the proposed system with these
means of heating water ($\Delta T = 40^\circ C$) has been carried out. Three options: solar ICS water heater, electric geyser and oil-fired boiler, were considered. The water-heating capacities of installations varied with the type of application. The costs of options have, therefore, been considered on a per unit capacity basis. The non-recurring costs, fuel and non-fuel recurring costs were taken into account in an economic analysis based on nett present values [13]. The anticipated lifespan of the systems was taken to be 10 years, over which the discount rate was assumed to be 14%. Costs of electricity (Rs 2 per kWh) and high-speed diesel fuel (Rs $\approx 7.50$ per litre) were assumed to escalate at 10 and 13%, respectively. In options 2 and 3, the non-fuel recurring costs were assumed to escalate at 10 and 5%, respectively. In accordance with practical reality option 1 was not considered maintenance-free; it needs less maintenance as compared to other systems. For example, it needs periodic cleaning to remove scaling from inner tubes and dust from the glazing. The cost estimates based on our practical experience have, therefore, been incorporated. The results of the analysis are summarised in Table 2, which exhibits a preference for the proposed system.

**CONCLUSION**

A transparent-honeycomb-insulated ground integrated collector-storage system was analysed for a solar water-heating application. The solar conversion efficiency of 30–60% corresponding to a collection temperature in the range of 40–70°C is reported. The analysis assumes solar intensity and atmospheric temperature as well as the resultant temperature in the ground and the concrete/sand region to be periodic. A study of the comparative performance of the system with other locally available means of heating water, such as electric geyser and oil-fired boilers, has been

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<th>Component</th>
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<td>1000</td>
<td>Civil Engg Dept, IIT, Delhi</td>
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<tr>
<td>Pipe network</td>
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<td>Local manufacturers</td>
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<tr>
<td>Total (Rs)</td>
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</table>


<table>
<thead>
<tr>
<th>Item</th>
<th>Nett present cost (NPC)</th>
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</thead>
<tbody>
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<td></td>
<td>Option 1</td>
</tr>
<tr>
<td>Non-recurring costs (Rs)</td>
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<tr>
<td>Recurring fuel costs (Rs)</td>
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<tr>
<td>Non-fuel recurring costs (Rs)</td>
<td>47</td>
</tr>
<tr>
<td>Total nett present costs (Rs)</td>
<td>164</td>
</tr>
</tbody>
</table>
carried out using net present cost (NPC) analysis. The solar gain and heat storage characteristics, as well as the cost of the proposed system, are found to be of the right order of magnitude for practical application.

REFERENCES


APPENDIX 1

CONSTANT-TEMPERATURE HEAT EXTRACTION

\[ [x1] = [y1] \]  \hspace{1cm} (17)

\[ [x2] = [y2] \]  \hspace{1cm} (18)

Time dependent

\[
[x1] = \begin{bmatrix}
U_1 - K_{am} & U_1 + K_{am} & 0 & 0 \\
e^{m_1} & e^{-m_1} & 0 & 0 \\
0 & 0 & e^{m_2} & e^{-m_2} \\
0 & 0 & 0 & 0
\end{bmatrix}
\]  \hspace{1cm} (A1)

\[
[k1] = \begin{bmatrix}
A_0 \\
B_0 \\
C_0 \\
D_0
\end{bmatrix}
\]  \hspace{1cm} (A2)

\[
[y1] = \begin{bmatrix}
S_0 + U_0 T_0 A_0 \\
U_0 & 0 & 0
\end{bmatrix}
\]  \hspace{1cm} (A3)

Time independent

\[
[x2] = \begin{bmatrix}
U_1 - K_0 & 0 & 0 \\
1 & x_1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 1
\end{bmatrix}
\]  \hspace{1cm} (A4)

\[
[k2] = \begin{bmatrix}
A_0 \\
B_0 \\
C_0 \\
D_0
\end{bmatrix}
\]  \hspace{1cm} (A5)

\[
[y2] = \begin{bmatrix}
S_0 + U_0 T_0 \\
T_0 & T_0 & 0
\end{bmatrix}
\]  \hspace{1cm} (A6)
APPENDIX II

HEAT EXTRACTION AT CONSTANT MASS FLOW RATE

\[ [A1][C1] = [B1] \]
\[ [A2][C2] = [B2] \]

\[ [A1] = \begin{bmatrix}
U_l - K \rho m \\
\exp\left(1 + \frac{K \rho m}{m_c \epsilon F_k}\right) U_l + K \rho m \\
\exp\left(-\frac{K \rho m}{m_c \epsilon F_k}\right) 0 \\
0
\end{bmatrix} \begin{bmatrix}
U_l + K \rho m \\
\exp\left(1 - \frac{K \rho m}{m_c \epsilon F_k}\right) \left(\frac{-K \rho m}{m_c \epsilon F_k}\right) \\
\exp\left(-\frac{K \rho m}{m_c \epsilon F_k}\right) \left(\frac{-K \rho m}{m_c \epsilon F_k}\right) \\
\exp\left(1 + \frac{K \rho m}{m_c \epsilon F_k}\right) \left(\frac{K \rho m}{m_c \epsilon F_k}\right)
\end{bmatrix} \begin{bmatrix}
0 \\
0 \\
1 \\
0
\end{bmatrix} \]

\[ [C1] = \begin{bmatrix}
A_m \\
B_m \\
C_m \\
D_m
\end{bmatrix} \]

\[ [B_1] = \begin{bmatrix}
S_n + U_l T_{a, n} \\
T_{a, n} \\
0 \\
T_{a, n}
\end{bmatrix} \]

\[ [A_2] = \begin{bmatrix}
U_l \\
1 \left(x_l + \frac{K_s}{m_c \epsilon F_k}\right) \\
0 \\
0
\end{bmatrix} \begin{bmatrix}
- K_s \\
0 \\
\left(\frac{-K_s}{m_c \epsilon F_k}\right) \\
\left(x_l - \frac{K_s}{m_c \epsilon F_k}\right)
\end{bmatrix} \begin{bmatrix}
0 \\
0 \\
1 \\
0
\end{bmatrix} \]

\[ [C_2] = \begin{bmatrix}
A_o \\
B_o \\
C_o \\
D_o
\end{bmatrix} \]

\[ [B_2] = \begin{bmatrix}
S_o + U_l T_{a, o} \\
T_{a, o} \\
T_{a, o} \\
0
\end{bmatrix} \]