Properties of bright solitons in averaged and unaveraged models for SDG fibres

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Abstract

Using the slowly varying envelope approximation and averaging over the fibre cross-section the evolution equation for optical pulses in semiconductor-doped glass (SDG) fibres is derived from the nonlinear wave equation. Bright soliton solutions of this equation are obtained numerically and their properties are studied and compared with those of the bright solitons in the unaveraged model.

1. Introduction

As shown by Kaplan [1,2] bistable or multistable solitons, if they exist could be of great technological utility because one would then be able to develop an ultrafast fibre-optic switching device for applications in signal processing. In this connection [3–9] several authors have studied soliton propagation and soliton properties including switching dynamics in fibres made of semiconductor-doped glasses (SDG) which shows nonlinear saturation of refractive index at not too high light intensities [10,11].

Usually the pulse dynamics in SDG fibre is modelled with the following functional form of the nonlinear refractive index,

\[ n_{NL} = \frac{n_2 J}{1 + I/I_s}, \tag{1} \]

where \( n_2 \) is the usual Kerr coefficient, \( I \) is the light intensity and \( I_s \) is the intensity at which saturation occurs. In this connection two kinds of models are possible: one in which the model equation is obtained from the usual nonlinear Schrödinger equation by replacing the Kerr term by \( n_{NL} \) given above [5] and the second in which the model equation is derived from the nonlinear wave equation using the slowly-varying envelope approximation (SVEA) and averaging over the fibre cross-section. The averaging procedure is necessary because the transverse field distribution in a fibre mode is not uniform over the entire fibre cross-section. For the polynomial form of nonlinear permittivity, as in the case of Kerr nonlinearity, the averaging process results in a constant factor [12] before the nonlinear term. As a result the functional form of the nonlinear refractive index in the final equation coincides with the functional form of the nonlinear permittivity. However, for the nonpolynomial form of the nonlinear permittivity the averaging process leads to a different form of nonlinear refractive index in the resulting dynamical equation. As a result the averaged and unaveraged equations differ and might lead to different quantitative and qualitative results for the same physical phenomenon, as it has been shown by Lyra et al. [13] for the modulational instability. This drives one to think that for nonlinear fibre systems with saturating nonlinearity the simple replacement of the Kerr term by a
saturating term in the evolution equation governing pulse dynamics is not desirable. One should, instead, derive the nonlinear propagation equation from the Maxwell's equations or the equivalent nonlinear wave equation, with an appropriate saturating form of nonlinear permittivity, employing the standard method of slowly varying envelope approximation (SVEA) and averaging over the fibre cross-section.

In the given paper we model the saturating form of the nonlinear permittivity \( \varepsilon_{NL}(I) \) for SDG fibres with

\[
\varepsilon_{NL}(I) = \frac{\varepsilon_2 I}{1 + I/I_s},
\]

and derive the averaged nonlinear differential equation for the complex envelope amplitude of an optical pulse. \( \varepsilon \) in Eq. (2) is the Kerr coefficient for the nonlinear permittivity which is related to the usual Kerr coefficient for the refractive index \( n_z \) through \( \varepsilon = 2n_0 n_z/\varepsilon_0 \), \( n_0 \) being the linear refractive index. We then carry out a comparative study of the properties of the fundamental bright solitons in this model with those in the model based on \( \varepsilon_{NL}(I) \) given by Eq. (1).

2. Derivation of the averaged equation

Consider a monomode isotropic fibre with a circular cross-section. Let \( z \) be the axis along the fibre. The nonlinear wave equation inside the fibre can be written as

\[
\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 D^L}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 D^NL}{\partial t^2},
\]

where \( E \) is the electric field vector of the wave, \( c \) is the speed of light and

\[
D^L = \int_0^\infty \varepsilon(t') E(t-t') \, dt',
\]

\[
D^NL = \varepsilon_{NL}(I) E = \varepsilon_2 \frac{|E|^2E}{1 + |E|^2/I_s}
\]

are the linear and the nonlinear parts respectively of the electric induction vector \( D \). Here \( \varepsilon \) is the linear permittivity.

If we assume the solitary wave inside the fibre to be supported entirely by the \( HE_{11} \) mode of the fibre far from cut-off then the electric field is confined inside the fibre core and its major component is transverse and linearly polarized. Under these conditions the electric field vector \( E \) can be represented as

\[
E(r, z, t) = e R(r) A(t, z)
\]

\[
\times \exp\{-i(\omega t - \beta z)\},
\]

where \( e \) is the unit vector in the direction of polarization, \( \beta \) is the propagation constant, \( R(r) \) is the mode function describing the transverse distribution of the electric field in the mode over the fibre cross-section and \( A(t, z) \) is the slowly varying complex envelope amplitude. Here \( r \) is a vector with components \( x \) and \( y \) in the transverse \( (x, y) \) plane. By slow variation of \( A(t, z) \) we mean the following,

\[
\omega^2 \frac{\partial^2 A}{\partial t^2} \ll \omega^{-4} \frac{\partial A}{\partial t} \ll |A|,
\]

\[
\beta^2 \frac{\partial^2 A}{\partial z^2} \ll \beta^{-4} \frac{\partial A}{\partial z} \ll |A|.
\]

Usually the temporal dispersion is assumed to be small so that \( \varepsilon(t') \) is a sharply peaked function. Then \( E(r, z, t-t') \) can be expanded, for small times \( t' \), into a Taylor series

\[
E(t-t') = E(t) - t' E_t(t) + \frac{1}{2} t'^2 E_{tt}(t) - \frac{1}{12} t'^3 E_{ttt}(t) + \ldots.
\]

From here onwards a suffix stands for the partial derivative with respect to it unless stated otherwise. Substituting for \( E(t-t') \) from Eq. (9) in Eq. (4) we get

\[
\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 D^L}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 D^NL}{\partial t^2}
\]

\[
\times \sum_{n=0}^\infty \frac{i^n}{n!} \frac{\partial^n \varepsilon(\omega)}{\partial \omega^n} \frac{\partial^n A}{\partial t^n},
\]

where we have taken into account that

\[
\varepsilon(\omega) = \int_0^\infty \varepsilon(t') \exp(i\omega t') \, dt',
\]

\[
\frac{\partial^n \varepsilon(\omega)}{\partial \omega^n} = i^n \int_0^\infty t'^n \varepsilon(t') \exp(i\omega t') \, dt'.
\]

For weak dispersion it is sufficient to keep only the first four terms of the series in Eq. (10). Keeping this in
mind we differentiate \( D^L(t) \) twice with respect to time to obtain

\[
D^L_\parallel = e R(t) \exp\left(-i(\omega t - \beta z)\right)
\times \left[ -\omega^2 eA(t, z) - i(\omega \epsilon_{\omega} + 2\epsilon)A_t
+ f(\omega) A_{tt} + ig(\omega) A_{ttt} \right],
\]

where

\[
f(\omega) = \epsilon(\omega) + 2\omega \epsilon_{\omega} + \frac{1}{2} \omega^2 \epsilon_{\omega\omega},
\]

\[
g(\omega) = \epsilon_{\omega} + \omega \epsilon_{\omega\omega} + \frac{1}{2} \omega^2 \epsilon_{\omega\omega\omega}.
\]

Similarly differentiating \( D^{NL} \) twice with respect to \( t \) and dropping the higher-order terms we obtain

\[
D^{NL}_{\parallel} = -\frac{\epsilon_2 \omega^2 |R|^2 R |A|^2 A}{1 + (|R|^2 |A|^2)/I_s}
- 2i\epsilon_2 \omega \left| R \right|^2 R (|A|^2)A_t
\times \left( A_{\perp} R + R A_{\perp} + 2i\beta R A_{\parallel} - \beta^2 R A \right),
\]

where

\[A_{\perp} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\]

we get from the wave equation (3)

\[A_{\perp} R + R A_{\perp} + 2i\beta R A_{\parallel} - \beta^2 R A \]

\[+ \frac{\omega^2}{c^2} eAR + i\frac{\omega}{c^2} R(2\epsilon + \omega \epsilon_{\omega})A_t
- f(\omega) \frac{\partial}{\partial t} R A_{tt} - ig(\omega) \frac{\partial}{\partial t} A_{ttt} \]

\[= -\frac{\epsilon_2 \omega^2 |R|^2 R |A|^2 A}{c^2 \left[ 1 + (|R|^2 |A|^2)/I_s \right]}
- 2i\epsilon_2 \omega \left| R \right|^2 R (|A|^2)A_t
\times \left( A_{\perp} R + R A_{\perp} + 2i\beta R A_{\parallel} - \beta^2 R A \right),
\]

Assuming the intensity to be much below the threshold for self-focusing effects, we can treat the transverse field distribution in the mode in a linear fashion. In other words the function \( R(r) \), \( r = \sqrt{x^2 + y^2} \) is assumed to satisfy the linear mode equation

\[\Delta R + (\omega^2 / c^2 - \beta^2) R = 0.
\]

Eq. (20), subject to the boundary condition at \( r = a \) where \( a \) is the core radius, has a known solution. Because of the fact that the difference in the refractive indices of the core and the cladding is very very small the functional form of \( R(r) \) is approximated, to a good degree of accuracy, by a Gaussian function \([14]\), i.e.

\[R(r) \sim \exp(-r^2/a_0^2).\]

Adopting this approximation we average Eq. (19) over the cross-section of the fibre and obtain

\[\begin{align*}
&iA_{\parallel} + \frac{i\omega}{2\beta c^2} (2\epsilon + \omega \epsilon_{\omega})A_t - f(\omega) \frac{\partial}{\partial t} A_{tt}
+ \frac{1}{2\beta} A_{ttt} - i \frac{g(\omega)}{2\beta c^2} A_{ttt} \\
&= -\left[ \frac{\epsilon_2 \omega^2 |A|^2 A}{2\beta c^2} + i \frac{\epsilon_2 \omega}{\beta c^2} (|A|^2)A_t \right]
\times \left[ \frac{1}{|A|^2/I_s} - \frac{\ln(1 + |A|^2/I_s)}{|A|^4/I_s^2} \right].
\end{align*}\]

For mode \( HE_{11} \), \( \beta = \omega \sqrt{\epsilon/c} = k \) where \( k \) is the wave number. As a result we get

\[\begin{align*}
&\frac{\omega}{2\beta c^2} (2\epsilon + \omega \epsilon_{\omega}) = k_\omega = \frac{1}{v_g}, \\
f(\omega) = c^2 (k_\omega^2 + k_{\omega\omega}),
\end{align*}\]

\[g(\omega) = k^2 (k_{\omega\omega} + \frac{1}{2} k k_{\omega\omega\omega}),\]

where \( v_g \) is the group velocity. Taking this into account Eq. (22) can be rewritten as

\[\begin{align*}
iA_{\parallel} + \frac{1}{v_g} A_t + \frac{1}{2k} \left( A_{tt} - \frac{1}{v_g} A_{ttt} \right)
- k_{\omega\omega} A_{ttt} - \frac{i k_{\omega\omega}}{2k v_g} A_{ttt} = -i \frac{k_{\omega\omega}}{k c^2} (|A|^2)A_t
\times \left[ \frac{1}{|A|^2/I_s} - \frac{\ln(1 + |A|^2/I_s)}{|A|^4/I_s^2} \right].
\end{align*}\]

To simplify Eq. (24) further we note that in Fourier space \( A_{\perp} - (1/v_b^2) A_{\parallel} \) reads \( (k^2 - \omega^2/v_b^2) \hat{A} \); \( \hat{A} \) being the Fourier transform of \( A \). Now
Since the difference between the phase and group velocities of a pulse in a monomode fibre is negligibly small, for all practical purposes we can assume them to be equal. Then we obtain

\[-\left(k^2 - \frac{\omega^2}{v_g^2}\right)A = -\left(k + \frac{\omega}{v_g}\right)\left(k - \frac{\omega}{v_g}\right)A\]

\[= -k \left(1 + \frac{\omega/k}{v_g}\right)\left(k - \frac{\omega}{v_g}\right)A. \quad (25)\]

Going back to the coordinate space we obtain

\[-\left(k^2 - \frac{\omega^2}{v_g^2}\right)A \approx -2k\left(k - \frac{\omega}{v_g}\right)A\]

\[= -\frac{2\omega}{v_g}\left(k - \frac{\omega}{v_g}\right)A. \quad (26)\]

If we now determine \(A + (1/v_g)A_t\) from Eq. (24) and use Eqs. (27) along with Eqs. (7) and (8) we obtain

\[i\left(A_t + \frac{1}{v_g}A_t\right) = \frac{k_{\text{nw}}}{2}A_t + \frac{n_2\omega^2}{k^2c^2} |A|^2 A\]

\[\times \left[\frac{1}{|A|^2/I_s} - \frac{\ln(1 + |A|^2/I_s)}{|A|^4/I_s}\right]\]

\[= i\frac{\omega n_2}{2k^2c^2} |A|^2 A\]

\[\times \left[\frac{1}{|A|^2/I_s} - \frac{\ln(1 + |A|^2/I_s)}{|A|^4/I_s}\right], \quad (28)\]

where we have made the substitution \(\epsilon_2 = 2n_2\). To nondimensionalize this equation we introduce

\[q = \frac{A}{\sqrt{I_s}}, \quad \xi = \frac{\omega n_2 I_s}{c} z,\]

\[\tau = \sqrt{\frac{\omega n_2 I_s}{(-k_{\text{nw}})c}} \left(t - \frac{z}{v_g}\right), \quad (29)\]

where we have assumed that we are working in the anomalous dispersion regime for which \(-k_{\text{nw}}>0\). A simple algebra leads to the following dimensionless form of Eq. (28),

\[iq_t + \frac{1}{2}q_{\tau\tau} + q \left(1 - \frac{\ln(1 + |q|^2)}{|q|^2}\right)\]

\[= i\alpha q_{\tau\tau} + i\sigma q \left(1 - \frac{\ln(1 + |q|^2)}{|q|^2}\right), \quad (30)\]

where

\[\alpha = \frac{1}{6} \frac{k_{\text{nw}}}{(-k_{\text{nw}})c} \sqrt{\frac{\omega n_2 I_s}{(-k_{\text{nw}})c}} ,\]

\[\sigma = \frac{1}{v_g} \sqrt{\frac{\omega n_2 I_s}{(-k_{\text{nw}})k^2c}} ,\]

\[\delta = 2 \sqrt{\frac{n_2 I_s}{(-k_{\text{nw}})c}} , \quad (31)\]

In the absence of the perturbation term on the right hand side of Eq. (30) the pulse dynamics is governed by

\[iq_t + \frac{1}{2}q_{\tau\tau} + q \left(1 - \frac{\ln(1 + |q|^2)}{|q|^2}\right) = 0. \quad (32)\]

In terms of the earlier introduced dimensionless variable the unaveraged model based on \(n_{\text{NL}}(I)\) given by Eq. (1) is described by the following differential equation,

\[iq_t + \frac{1}{2}q_{\tau\tau} + q \frac{|q|^2}{1 + |q|^2} = 0. \quad (33)\]

In what follows we shall study the properties of the fundamental bright solitons.

3. Soliton solution (lossless case)

We look for the fundamental bright soliton solutions of Eqs. (32) and (33) satisfying

\[\lim_{\tau \to \pm \infty} q(\xi, \tau) = \lim_{\tau \to \pm \infty} q_0(\xi, \tau) = 0 \quad (34)\]

and the condition of stationarity in \(\xi\). We put

\[q(\xi, \tau) = \sqrt{\psi(\xi, \tau)} \exp\{i\phi(\xi, \tau)\}. \quad (35)\]
Then from Eqs. (32), (33) and (35), we get

\[ \frac{1}{\psi} \psi_x + \phi_x + \frac{1}{\psi} \psi_x \phi_x = 0 \quad \text{(both models)} \]  

(36)

\[ \phi_x + \frac{1}{4 \psi} \psi_{\tau\tau} + \frac{1}{8 \psi^2} \phi_x^2 + \frac{1}{2} \phi_x^2 \psi_x = 0 \quad \text{(unaveraged model)} \]  

(37)

\[ - \phi_x + \frac{1}{4 \psi} \psi_{\tau\tau} - \frac{1}{8 \psi^2} \phi_x^2 - \frac{1}{2} \phi_x^2 \psi_x + \left( 1 - \frac{\ln(1 + \psi)}{\psi} \right) = 0 \quad \text{(averaged model)} \]  

(38)

where \( \psi(\xi, \tau) \) satisfies

\[ \lim_{\tau \to +\infty} \psi(\xi, \tau) = \lim_{\tau \to -\infty} \psi(\xi, \tau) = 0 . \]  

(39)

As usual the stationarity in \( \xi \) gives \( \psi_{\xi} = 0 \) and we get from Eq. (36)

\[ \psi(\tau) \phi_x = c(\xi) . \]  

(40)

As it is known [15] the conditions of stationarity in \( \xi \) and localization in \( \tau \) can be satisfied simultaneously only if \( c(\xi) = 0 \). As a result we get

\[ \phi = \theta \xi + \phi_0 , \]  

(41)

where \( \phi_0 = \phi(\xi = 0) \) and \( \theta \) is a constant that represents the nonlinear addition to the propagation constant. Now substituting for \( \phi \) from Eq. (41) into Eqs. (37) and (38) we obtain

\[ \frac{1}{4 \psi} \psi'' - \frac{1}{8 \psi^2} (\psi')^2 - \frac{\psi}{1 + \psi} - \theta = 0 \quad \text{(unaveraged model)} \]  

(42)

\[ \frac{1}{4 \psi} \psi'' - \frac{1}{8 \psi^2} (\psi')^2 - \frac{\ln(1 + \psi)}{\psi} + (1 - \theta) = 0 \quad \text{(averaged model)} \]  

(43)

where the prime stands for ordinary derivative with respect to \( \tau \). In order to determine soliton solutions we must integrate Eqs. (42) and (43) numerically. For that we need an appropriate value of \( \theta \) for each model since not for all but only for some particular values of \( \theta \) (for a given input amplitude \( \psi_0 \)) Eqs. (42) and (43) can have bright soliton solutions. To determine \( \theta \) we multiply each of the above equations by \( \psi' \) and use the boundary conditions (39) to obtain

\[ \frac{1}{8 \psi} \frac{d\psi}{d\tau} + (1 - \theta) \psi - \ln(1 + \psi) = 0 \quad \text{(unaveraged model)} \]  

(44)

\[ \frac{1}{8 \psi} \frac{d\psi}{d\tau} + (1 - \theta) \psi - F(\psi) = 0 \quad \text{(averaged model)} \]  

(45)

where

\[ F(\psi) = \int_0^\tau \frac{\ln(1 + \psi)}{\psi} \, dt . \]  

(46)

Since we are looking for \( N = 1 \) bright soliton solutions with a maximum \( q_0 = \sqrt{\psi_0} \) at \( \tau = 0 \) we obtain from Eqs. (44) and (45)

\[ \theta = 1 - \frac{\ln(1 + \psi_0)}{\psi_0} \quad \text{(unaveraged model)} \]  

(47)

\[ \theta = 1 - \frac{F(\psi_0)}{\psi_0} \quad \text{(averaged model)} . \]  

(48)

4. Numerical results and comparison of results

For a given value of \( \psi_0 \) we first determine the corresponding value of \( \theta \) for each of the models and then integrate Eqs. (42) and (43) to determine \( \psi(\tau) \) and hence the soliton solution. The results of our study are depicted in Figs. 1–6. Note that in all our figures the

![Graph](image-url)
solid line corresponds to the unaveraged model while the broken line to the averaged model. In Fig. 1 we have plotted the soliton width $\tau_0$ while in Fig. 2 we have $\theta$ as a function of the dimensionless energy $\varepsilon = \int_0^\infty p(\tau) d\tau$, $p(\tau)$ being the soliton power. Fig. 3 contains the soliton width as a function of the nonlinear addition to the propagation constant $\theta$. If we analyze these figures we notice that none of the models studied here admits bistable solitons as envisaged by Kaplan [1,2] since $\varepsilon$ is not a double-valued function of $\theta$. However as shown in Fig. 3 the soliton width $\tau_0$ is a double-valued function of $\theta$ and hence from Figs. 1–3 we conclude that the given models contain two-state solitons in the sense that for given fibre parameters and a given value of the pulse width there exist two solitons with different energies and hence different peak powers and shapes. This property is similar to that of optical bistable systems since the models considered here have two output solitonic states (corresponding to two different peak powers) for each given input control parameter $\tau_0$. For illustration the two-state soliton solutions for $\tau_0 = 1.4$ have been depicted in Fig. 4. Note that from here onwards the solitons with lower values of energy will be called to be belonging to the lower branch while the solitons with higher value of energy to the upper branch.

Further, it is clear from Fig. 1 that the soliton width obtainable in both these models is bounded from below: $\tau_0 \geq \tau_{\text{im}}$ where $\tau_{\text{im}}$ is the minimum pulse width possi-

Fig. 2. Nonlinear propagation constant shift $\theta$ as a function of dimensionless energy $\varepsilon$. Unaveraged model: solid curve. Averaged model: broken curve.

Fig. 3. Soliton width $\tau_0$ as function of nonlinear propagation constant shift $\theta$. Unaveraged model: solid curve. Averaged model: broken curve.

Fig. 4. Soliton shapes for $\tau_0 = 1.4$. Unaveraged model: solid curve. Averaged model: broken curve.

Fig. 5. Soliton shapes for the dimensionless energy $\varepsilon = 5$. Unaveraged model: solid curve. Averaged model: broken curve.
Fig. 6. (a) Soliton peak amplitude $q_0$ and (b) soliton width $\tau_0$ as a function of propagation distance for $\epsilon = 5$ and $\Gamma = 0.0138$. Unaveraged model: solid curve. Averaged model: broken curve.

We get that $Q_{\text{max}} = 1.268$ for $\epsilon = 6.663$ in the unaveraged model and $Q_{\text{max}} = 1.347$ for $\epsilon = 8.193$ in the averaged model. For an approximate quantitative analysis let us use the following parameter values,

\[ \lambda_0 = 1.55 \text{ \mu m}, \quad -k_{\text{aww}}(\lambda_0) = 2.77 \times 10^{-26} \text{ s}^2/\text{m}, \]
\[ \omega = 10^{16} \text{ s}^{-1}, \quad n_2 = \Delta n_{\text{cut}} \approx 10^{-5}. \]

Then we obtain that $\tau_{\text{un}} \approx 14.5 \text{ fs}$ in the unaveraged model while $\tau_{\text{av}} \approx 16 \text{ fs}$ in the averaged model.

Fig. 1 also shows that for smaller values of energy each $\tau_0$ corresponds to a single value of $\epsilon$, i.e. that the models have usual one soliton solutions. This is understandable since for lower values of $|q|^2$ expanding the nonlinear functions into series in both the models and keeping only the leading order terms we recover the well known nonlinear Schrödinger equation which does not possess two-state solitons. Finally it is clear from Fig. 2 that solitons in both these models are stable under small perturbations since the stability criterion $\partial I / \partial \theta > 0$ is satisfied in the whole range of $\theta$ values.

If we compare solitons of a given energy then it is evident from Figs. 1 and 2 that for any given value of energy the soliton width predicted by the averaged model is larger and the corresponding peak amplitude (and hence the peak power) is smaller than those predicted by the unaveraged model. This is understandable if we realize from Fig. 2 that for a given value of energy the corresponding nonlinear shift in the propagation constant $\theta$, which is responsible for pulse compression in the anomalous dispersion regime, is larger in the unaveraged model than in the averaged model. As an example, we have depicted in Fig. 5, the soliton solutions in both models with $\epsilon = 5$ which clearly demonstrates the above statement. Further we notice from Fig. 1 that the unaveraged model has two-state solitons for a larger interval of the input control parameter compared to the averaged model and hence predicts a wider range of switching. Note that for $1.268 < \tau_0 < 1.366$ the averaged model has no soliton solutions.

Finally if we compare solitons of a given pulse width in these models then we notice from Fig. 1 that the solitons of both the lower as well as upper energy branches in the averaged model are wider (large pulse width) and smaller (lower peak amplitude or power) than the corresponding solitons in the unaveraged model.

5. Effect of fibre loss

The linear dissipation can be taken into account by phenomenologically adding a loss term $-i\Gamma \sigma$ to the right hand side of Eqs. (32) and (33) where the dimensionless loss parameter $\Gamma$ is given by

\[ \Gamma = \frac{c}{\alpha n_0 \lambda_0}, \quad (49) \]

where $\alpha$ is the phenomenological loss coefficient measured in decibels per kilometer. As a result the pulse dynamics is governed by the following model equations.
We have numerically integrated Eqs. (50) and (51) using the symmetrized split-step Fourier method. As initial conditions we took the soliton solutions of the lossless equations for a given energy $\varepsilon$. The result of the calculation for $\Gamma = 0.0138$ corresponding to a loss rate of 0.12 dB/km and initial energy $\varepsilon = 5$ are depicted in Fig. 6. Fig. 6a contains the peak amplitude $q_0$ and Fig. 6b the soliton width $\tau_0$ as functions of $\xi$. As it is visible from these figures the initial rate of increase in the soliton width is slower showing that the soliton property of the pulse is maintained. Subsequently, because of the considerable decrease in the soliton amplitude nonlinearity weakens and the pulse is dominated by dispersion and undergoes monotonic broadening. Hence the soliton behaviour under fibre dissipation is similar to that of solitons for the Kerr nonlinearity.

6. Conclusions

We have derived the averaged nonlinear partial differential equation governing pulse dynamics in SDG fibres with nonlinear saturation in the refractive index. The fundamental bright soliton solutions in both the averaged and unaveraged models have been numerically determined. Their physical characteristics have been calculated and compared. It has been shown that while the soliton properties in both the models are similar qualitatively they do differ quantitatively which might be useful for experimental observation of two-state solitons in SDG fibres. Which of the models considered here is closer to reality can be decided only by the experimental data.

References