Effect of a large amplitude Langmuir wave on the Weibel instability

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Abstract

The effect of a large amplitude Langmuir wave on the Weibel instability in a plasma with an electron beam is studied. The electromagnetic perturbation couples to the Langmuir wave to give rise to two Langmuir sideband waves, which in turn produce a current enhancing the electromagnetic perturbation. The Langmuir wave enhances the growth over its linear value. The enhancement is larger at shorter wavelengths. For \( \nu_0/c \sim 0.15 \), the growth rate is enhanced by a factor of 3/2.

Recent experiments with an intense short pulse laser deal with plasmas produced via tunnelling ionization with a controllable density and with an anisotropic electron velocity distribution function. In these plasmas the transverse temperature is greater than the longitudinal one [1]. Hence, one may have a streaming component of hot electrons. This system is unstable to the Weibel instability and can give rise to magnetic field generation through the amplification of an electromagnetic perturbation with a growth rate \( \omega_b \nu_b/c \), where the subscript \( b \) stands for beam.

In this Letter we investigate the effect of a large amplitude Langmuir wave of frequency \( \omega_0 \sim \omega_p \) on the Weibel instability. The Langmuir wave may be a result of a parametric process, e.g. Raman scattering, or could be produced by a laser wave via linear mode conversion.

The electromagnetic perturbation couples to the pump wave to give rise to two sideband waves which in turn produce a nonlinear current enhancing the electromagnetic perturbation.

We consider a plasma with electron density \( n_0^0 \) and electron temperature \( T_e \). An electron beam of density \( n_0^b \) propagates through it with velocity \( \nu_b \). A large amplitude Langmuir wave exists in the plasma with electrostatic potential

\[
\phi_0 = \phi_0 e^{-i(\omega_0 t - k_0 z)} ,
\]

where \( \omega_0 = \omega_{pp}(1 + k_0^2 \nu_b^2/\omega_{pp}^2)^{1/2} \), \( \omega_{pp} = (4\pi n_0^0 e^2/m)^{1/2} \), \( e \) and \( m \) are the electronic charge and mass.

The Langmuir wave gives rise to an oscillatory velocity to electrons and causes density perturbations of plasma and beam electrons. For beam electrons, the oscillatory velocity and density are

\[
\nu_{0b} = - \frac{\phi_0 k_0 e}{m(\omega_0 - k_0 \nu_b)^2} ,
\]

\[
n_{0b} = - \frac{n_0^b e k_0^2 \phi_0}{m(\omega_0 - k_0 \nu_b)^2} .
\]
The plasma electron velocity, \( u_0 \), and density, \( n_0 \), can be obtained from Eqs. (2) by dropping the subscript \( b \) and taking \( u_s = 0 \).

Now we perturb the equilibrium by an electromagnetic perturbation,

\[
E = E e^{-(\omega_E - k \cdot x)}, \quad B = \frac{c}{\omega} k \times E,
\]

\( B \) is primarily polarized along \( \hat{y} \). The nonlinear coupling between this perturbation and the pump produces sideband waves whose electrostatic potentials can be written as

\[
\phi_1 = \phi_1 e^{-i(\omega_1 t - k_1 \cdot x)}, \quad \phi_2 = \phi_2 e^{-i(\omega_2 t - k_2 \cdot x)},
\]

where \( \omega_{1,2} = \omega \pm \omega_0 \), and \( k_{1,2} = k \hat{x} \pm k_0 \hat{z} \). The linear response of beam electrons to \( \phi_1 \) can be written as

\[
\nu_{1b} = -\frac{e k \phi_1}{m(\omega_1 - k_1, v_s)}, \quad n_{1b} = -\frac{n_{0b} e k^2 \phi_1}{m(\omega_1 - k_1, v_s)^2},
\]

The beam response to \( \phi_2 \) can be obtained from Eqs. (4) by replacing 1 by 2. The linear response of plasma electrons to \( \phi_1 \) is,

\[
\nu_1 = -\frac{e \phi_1}{m \omega_1}, \quad n_1 = \frac{k_1^2}{4 \pi e} \chi_1 \phi_1,
\]

where \( \chi_1 = -(\omega_{pp}^2 + k_1^2 v_s^2)/\omega_1^2 \) is the plasma electron susceptibility. The pump and the sideband waves produce a nonlinear current density at \( (\omega, k) \),

\[
J_{NL} = -\frac{1}{2} n_1 e\nu_0 - \frac{1}{2} n_2 e\nu_0 - \frac{1}{2} n_{1b} e\nu_{0b} - \frac{1}{2} n_{2b} e\nu_{0b}^*.
\]

The linear current density, \( J^L \), due to the \( E \) and \( B \) fields can be obtained as

\[
J^L = -n_0 e\nu - n_{0b} e\nu_b - n_b e\nu_b^*.
\]

Solving the equation of motion and continuity,

\[
\nu = \frac{eE}{m_i \omega_1} \hat{z}, \quad \nu_b = \frac{eE}{m_i \omega} \left( \frac{k v_s}{\omega} + \hat{z} \right),
\]

\[
n_b + n_{0b} e k^2 v_s^2 E, \quad J^L = -\frac{e^2 E}{m_i \omega} \left( \frac{n_0^0 + n_{0b}^0 + n_{0b}^0}{\omega^2} \right) \hat{z}
\]

where we have taken \( E \parallel \hat{z}, \ J^L \) can be written as

\[
J^L = -\frac{e^2 E}{m_i \omega} \left( \frac{k v_s}{\omega} \right) E \hat{z}.
\]

The magnetic field of the \( (\omega, k) \) perturbation couples to the oscillatory velocity to produce ponderomotive forces \( F_{p_{1,2}} \) at \( (\omega_{1,2}, k_{1,2}) \),

\[
F_{p_1} = -\frac{e}{2c} \nu_0 \times B = i e k \phi_{p_1}, \quad F_{p_2} = -\frac{e}{2c} \nu_0 \times B = i e k \phi_{p_2},
\]

where \( \phi_{p_1} = (1/2i \omega) \nu_0^* \cdot E \) and \( \phi_{p_2} = -(1/2i \omega) \nu_0^* \cdot E \).

The ponderomotive potentials can produce the electron density perturbation,

\[
n_{1NL} = \frac{k^2}{4 \pi e} \chi_1 \phi_{p_1}, \quad n_{2NL} = \frac{k^2}{4 \pi e} \chi_2 \phi_{p_2},
\]

\[
n_{1bNL} = \frac{k^2}{4 \pi e} \chi_{b1} \phi_{pb_1}, \quad n_{2bNL} = \frac{k^2}{4 \pi e} \chi_{b2} \phi_{pb_2},
\]

where \( \phi_{pb_1} = (1/2i \omega) \nu_{0b}^* \cdot E \) and \( \phi_{pb_2} = (1/2i \omega) \nu_{0b}^* \cdot E \).

Using the density perturbations in Poisson’s equation, one obtains

\[
e_1 \phi_1 = \left( \chi_1 \phi_{p_1} + \chi_{b1} \phi_{pb_1} \right) \frac{k_1^2}{k_1^2}, \quad e_2 \phi_1 = \left( \chi_2 \phi_{p_2} + \chi_{b2} \phi_{pb_2} \right) \frac{k_2^2}{k_2^2},
\]

where we have used \( e_1 = \frac{1}{2i \omega}, \ e_2 = \frac{1}{2i \omega} \).
Fig. 2. Normalized growth rate $\gamma/\omega_{pp}$ plotted versus normalized oscillatory velocity $|v_0|/c$.

where

$$\epsilon_{1,2} = 1 - \frac{\omega_{pp}^2 + k_1^2 v_{th}^2}{\omega_{1,2}^2} \left( \frac{\omega_{1,2} - k_1 v_0}{\omega_{1,2}^2} \right)^2.$$  

Using Eqs. (6) and (9) along with the wave equation for $E$,

$$\nabla \times \nabla \times E = \frac{4i}{c^2} \left( J^L + J^{NL} \right) + \frac{\omega^2}{c^2} E,$$

one finds,

$$k^2 - \frac{\omega^2}{c^2} + \frac{\omega_{pp}^2}{c^2} + \frac{\omega_{pb}^2}{c^2} \left( 1 + \frac{k^2 v_s^2}{\omega^2} \right)$$

$$= - \frac{k^2}{4c^2} |v_0|^2 \Psi(\omega)$$

where

$$\Psi(\omega) = \frac{1}{\epsilon_1} \left( \frac{\chi_1 + \chi_{1b}}{1 - k_0 v_s/\omega_0} \right)^2$$

$$+ \frac{1}{\epsilon_2} \left( \frac{\chi_2 + \chi_{2b}}{1 - k_0 v_s/\omega_0} \right)^2$$

For $k_0 \sim 3\omega_0/c$, $\frac{1}{2}mv_s^2 \sim 1$ keV, and $\omega_{pp} \gg \omega_{pb}$, one gets

$$k^2 + \frac{\omega_{pp}^2}{c^2} + \frac{\omega_{pb}^2}{c^2} + \frac{k^2 v_s^2}{c^2}$$

$$= \frac{|v_0|^2}{4c^2} \omega_0 \Delta - \frac{k^2}{\omega^2 - \Delta^2},$$

where $\Delta = \omega_0 - (\omega_{pp}^2 + k_1^2 v_{th}^2)^{1/2}$, and $k v_{th} \ll 0.3 \omega_{pp}$. For large values of $k$, the roots are

$$2 \omega^2(\infty) = \left( \frac{\omega_{pp}^2 v_s^2}{c^2} + \frac{|v_0|^2}{4c^2} \omega_0 \Delta \right)$$

$$\pm \left[ \left( \frac{\omega_{pp}^2 v_s^2}{c^2} + \frac{|v_0|^2}{4c^2} \omega_0 \Delta \right)^2$$

$$+ \frac{4 \Delta^2 \omega_{pb}^2 v_s^2}{c^2} \right]^{1/2}.$$  

In case $|v_0|^2 = 0$ (no pump wave) or $\Delta = 0$, the growth rate defined by the root with minus inside and plus outside, is $\gamma \sim \omega_{pp} v_s/c$. It might be noted that had $\Delta$ been positive we would have suppression of the Weibel instability.

To conclude, in laser produced plasmas, drifting hot electrons are produced. One may model them as a beam propagating through a plasma. The system is unstable to purely growing electromagnetic perturbation (Weibel instability) on a time scale of $\gamma^{-1} = c/\omega_{pp} v_s$.

For $n_b \sim 4\% n_p$ and $\frac{1}{2}mv_s^2 \sim 1$ keV, $\gamma^{-1} \sim 40$ fs. Under the same conditions, where a large amplitude Langmuir wave exists in the system the magnetic perturbation grows faster.

For $|v_0|/c \sim 0.15$, we find $\gamma^{-1} \sim 25.7$ fs. It might be noted that had we taken two beams of densities $\omega_{pp} = 2\omega_{pb}$ and drift velocities $v'_s = v_s/2$ we would have found the same growth rate for the Weibel instability when a large amplitude Langmuir wave is taken into account. We may note also that when $v_s = 0$ we still have an instability which is similar to the electromagnetic two stream instability studied by Tripathi and Liu [2].

References