COMPUTER SIMULATION OF A COMBINED CYCLE POWER PLANT

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Abstract—This paper presents the simulation procedure developed to predict the performance of a combined cycle power plant from given performance characteristics of its main components. In order that the procedure could be validated, the simulation technique has been applied to a typical combined cycle power plant (having a dual pressure bottoming cycle) manufactured by a prominent company. The characteristics of the standard equipment like the air compressor, steam and gas turbines, various pumps, etc. have been taken from the manufacturer’s catalogues and converted into appropriate equations based on theoretical understanding. The performance of various heat exchangers (like economizers, superheaters, evaporators, etc.) has been determined by using the effectiveness concept after evaluating the overall heat transfer coefficient by using appropriate correlations from literature. The strategy of system simulation is obtained by judiciously interlinking the information flow diagrams of various components and thus the task is finally reduced to that of solving nine non-linear equations for nine variables. The predicted performance of the system is seen to be in good agreement with its rated performance.

NOMENCLATURE

\( a, b \) relative transverse and longitudinal pitch
\( A \) surface area, \( m^2 \)
\( A_f \) fin area, \( m^2 \)
\( A_{ff} \) free flow area in compact heat exchanger, \( m^2 \)
\( A_h \) heat exchanger frontal area, \( m^2 \)
\( c_p \) specific heat at constant pressure, \( J/kg \cdot K \)
\( d \) tube diameter, \( m \)
\( DPB \) boiler gas side pressure drop, bar
\( f \) friction factor
\( G \) gas mass velocity, \( kg/s \cdot m^2 \)
\( g \) gravitational acceleration, \( m/s^2 \)
\( h \) convective heat transfer coefficient, \( W/m^2 \cdot K \); specific enthalpy, \( J/kg \cdot K \)
\( h_f \) height of fin, \( m \)
\( h_v \) latent heat of vaporization, \( J/kg \)
\( HP, LP \) high and low pressure
\( Ja \) Jakob number
\( k \) thermal conductivity, \( W/m \cdot K \)
\( L \) length, \( m \)
\( M \) mass flow rate, \( kg/s \)
\( n \) number of moles
\( N_c \) compressor speed
\( n_f \) number of fins/m
\( N_L, N_T \) number of tube in longitudinal and transverse directions
\( Nu \) Nusselt number
\( P \) pressure, bar
\( Pr \) Prandtl number
\( Q \) heat transfer rate, \( W \)
\( Re \) Reynolds number
\( R_f \) fouling factor, \( m^2 \cdot K/W \)
\( s \) fin pitch, \( m \)
\( S_D, S_L, S_T \) diagonal, longitudinal and transverse pitch of a tube bank, \( m \)
\( t_f \) thickness of fin, \( m \)
\( T \) absolute temperature, \( K \)
\( U \) overall heat transfer coefficient, \( W/m^2 \cdot K \); fluid velocity, \( m/s \)
\( u \) mass average fluid velocity, \( m/s \)

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**INTRODUCTION**

With the twin crises of energy resources depletion and pollution increasingly engulfing our civilization, it has become very crucial to develop more efficient and less polluting thermal power plants, which are capable of effectively utilizing fuels like coal and natural gas, etc.

Gas turbines (GT) have, by now, proven to be very compact and reliable types of power plants having a very small gestation period and low capital cost. However, because of low efficiency due to high exhaust temperatures, these plants did not hitherto find wide application. Despite this, by using the exhaust heat of this gas turbine cycle to generate steam for a bottoming Rankine cycle, it is now possible to achieve much higher thermal efficiency than conventional steam power plants. Such combined cycle power plants are now clearly emerging as the most favored technology for electric power generation, not only because of their increased efficiency but also due to many other operational and environmental advantages discussed in the literature [1].

The design of such combined cycle (CC) power plants is obviously much more involved, especially because of the coupling between two different types of power-producing cycles and the need to identify the optimal distribution of power production between them. There is therefore a need for developing computer simulation techniques which would enable evaluation of various possible design options and also permit prediction of off-design performance of the system.

This paper presents the details of a simulation procedure which has been developed for predicting the performance of a typical CC power plant involving a gas turbine coupled to a dual-pressure bottoming cycle through a waste heat recovery boiler. The procedure has been validated by comparing its prediction to the rated performance of a typical 800 MW, two module combined cycle power plant, each module of which consists of two gas turbines with two waste heat recovery boilers feeding a single steam turbine (Fig. 1).

**COMPUTER SIMULATION STRATEGY**

The task of computer simulation involves predicting the operating conditions of the system (pressures, temperatures, energy and fluid flow rates) at which various mass and energy balances, all equations of state of working substances and the performance characteristics of the individual components are satisfied [2]. Therefore, the availability of performance characteristics of the various components constituting the system is a pre-requisite for system simulation. The strategy
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of system simulation is strongly dependent on the manner in which the characteristics of various components are available. For the purpose of system simulation, these characteristics are best represented by an information flow diagram, which is essentially a block diagram indicating that the output variables are known functions of the input variables. Often, it is possible to re-arrange the functional relationships and therefore the choice of input and output variables is to some extent arbitrary. It is therefore possible (and necessary) to choose the input and output variables judiciously to arrive at an optimal simulation strategy.

**COMPONENT SIMULATION**

The main components of a combined cycle power plant system are the compressor, turbines, liquid/vapour heat exchangers, pumps, throttling valves and the condenser. The simulation of a component implies prediction of the desired output information given the input information and physical dimension of the component.

**Compressor**

The characteristics of a gas-turbine compressor are usually given in the form of a graphical relationship between three dimensionless numbers, namely the compressor mass parameter (CMP), compressor pressure ratio (CPR) and the efficiency ($\eta_c$). Knowing the mass flow rate handled by the compressor and air inlet temperature, and the characteristic curves (or the equation representing these), the work input to the compressor, the compressor discharge pressure ($P_2$) and temperature ($T_2$) can be computed as follows by using basic thermodynamic relationships [3]:

$$w_c = CMPc_p \frac{AM}{r_c}$$

$$\eta_c = f_3\left(\frac{M \sqrt{T_i}}{P_i}, \frac{N_c}{T_c}\right)$$

$$\Delta T_{2i} = T_i \left[\left(\frac{P_f}{P_i}\right)^{\gamma-1} - 1\right]$$

$$W_c = CMPc_p \sqrt{T_i P_i (CPR^{1/\gamma} - 1)}.$$

**The combustion process**

The fuel used in the combustion chamber is natural gas composed of different constituents. According to the first law for a steady-state steady flow, adiabatic combustion process, the total enthalpy of products and the reactants is equal [4], i.e.

$$\sum_r n_r [h_{r0} + \Delta h_r] = \sum_r n_x [h_{x0} + \Delta h_x].$$

The adiabatic flame temperature (TIT) is determined iteratively so that the above equation is satisfied.

**Turbines**

The characteristics of turbines are also given by three dimensionless numbers, namely the turbine mass parameter (TMP), turbine pressure ratio (TPR) and its efficiency ($\eta_t$). The simulation of a turbine implies determination of TMP, $\eta_t$, exhaust gas temperature and the power output, for specified values of TPR and turbine inlet temperature (TIT). The basic thermodynamic relations used in the turbine calculation are [3]:

$$\frac{M_t \sqrt{T_i}}{P_i} = f_3\left(\text{TPR}, \frac{N_t}{\sqrt{T_i}}\right)$$

$$\eta_t = f_4\left(\text{TPR}, \frac{N_t}{\sqrt{T_i}}\right)$$

$$\Delta T_{34} = \eta_t T_i \left[1 - \left(\frac{1}{\text{TPR}}\right)^{1/\gamma}\right]$$
\[ W_I = \eta T \sqrt{\frac{1}{c_P T_0}} \left[ 1 - \left( \frac{1}{\text{TPR}} \right)^{\frac{1}{\eta'}} \right]. \] (9)

**Pumps**

The pumps mass parameter (PMP) and its efficiency (\( \eta_p \)) are primarily functions of the pump pressure ratio (PPR). Using pump characteristics, the work input and outlet temperature are computed from basic thermodynamic equations:

\[
P_{\text{MP}} = f_5(\text{PPR})
\]

\[
\eta_p = f_6(\text{PPR})
\]

\[
W_p = f_7(h_i, \text{PPR})
\]

\[
h_2 = h_i + W_p
\]

\[
T_2 = f_8(h_2, W_p).
\]

**Condensate/feed water control valves**

The steam cycle is fitted with a number of control valves to maintain the liquid levels in various drums and the pressure in the deaerator constant under varying load conditions. The pressure drop/flow rates through these valves under steady state conditions are therefore determined so as to ensure the mass and energy balance of the drums downstream. Thus the pressure drop across the condensate feed control valve (\( \Delta P_{v1} \)) should be such as to ensure the mass balance of the deaerator. Similarly, the pressure drop \( \Delta P_{v2} \) and \( \Delta P_{v3} \) are calculated so that the mass balance of HP- and LP-drums are satisfied. The pressure drop \( \Delta P_{v4} \) is estimated on the basis of the energy balance across the deaerator tank, so as to ensure a constant pressure in it.

**Economizer/superheater heat exchangers**

Economizers and superheaters are basically cross-flow heat exchangers of different configuration provided at various sections of the boiler to raise water temperature and superheat the steam before entry into the HP- and LP-turbines. The simulation of these heat exchangers involves determination of the pressures and temperatures of outgoing streams for given pressures, temperatures and flow rates of incoming streams. This is most conveniently done using the effectiveness concept:

\[
Q = cC_{\text{m}}(T_{g} - T_{w}). \tag{15}
\]

Now the effectiveness of any heat exchanger can be determined once the NTU and heat capacity ratio of the two fluids are known. The main task involved in heat exchanger simulation is thus, the estimation of overall heat transfer coefficient \( U \), which determines the value of NTU. This, in turn, requires estimation of the gas-side and water/steam-side heat transfer coefficients, the efficiency of fins employed on the gas-side and various fouling factors. The heat transfer correlations used for simulation are given in Appendix A, and the procedure for calculation of various pressure drops is given in Appendix B.

**THE SYSTEM SIMULATION PROCEDURE**

The system simulation strategy is obtained by suitably combining the information flow diagrams of individual components of the system. This combination is effected through the interlinking variables which appear as the output from one component and are used as the input in the other component. The information flow diagram for the complete system, obtained by combining these component information flow diagrams, is shown in Fig. 2.

It is obvious from the multinested nature of Fig. 2, that an iterative solution strategy would be imperative. This necessitates assumption of suitable initial values to start the simulation. The best strategy, as identified from the information flow diagram is to assume appropriate values of the following variables and then the simulation of various components can be carried out sequentially:

1. air flow rate through air compressor (\( M_{air} \));
2. gas-side pressure drop across boiler (DPB);
As indicated, assuming $M_{in}(V_1)$, and knowing the condition of air at inlet to the compressor and its characteristics, the compressor pressure ratio (CPR) and its discharge temperature ($T_2$) and pressure ($P_2$) are computed from the simulation procedure. These are the input conditions to the combustion chamber. From the heat balance across the combustion chamber, knowing the heat of formation of each constituent, the temperature of gases at the inlet to the turbine is iteratively computed, so that the total enthalpy of the products equals the enthalpy of the reactants. The pressure drop across combustion chamber is taken as 2.5% of compressor outlet pressure. Thus turbine inlet pressure can also be calculated. As the pressure drop across the boiler is assumed ($V_2$),
the pressure ratio across the turbine can be calculated and using it the turbine mass parameter (TMP) and exhaust temperature ($T_4$) are determined.

Since the deaerator pressure is kept constant, assuming HP- and LP-pump exit pressures ($V_1$, $V_4$), their mass flow rates ($M_{HP}$, $M_{LP}$) and efficiencies can be determined from the pump characteristics. The WHRB calculations are now started from the HP-superheater, since that is the first piece of equipment which meets the hot gases leaving the gas turbine in the system under consideration (Fig. 1). Knowing the pressure, temperature and the mass flow rates of incoming streams and the design of the superheater, the pressures and temperatures of streams going to the HP-turbine ($P_{MST}$, $T_{MST}$) and the HP-evaporator ($T_{X1}$) and the pressure drop inside and across the superheater $[(\Delta P_{HPS})_0$ and $(\Delta P_{HPS})_1]$ are determined using heat exchanger simulation procedure.

The analysis of the HP-evaporator is carried out by assuming the drum pressure ($V_3$). The condition of liquid–vapour mixture (vapour quality $x_t$), hot gases leaving the evaporator ($T_m$) and the pressure drop across tube bundles $[(\Delta P_{CPH})_0$, can then be found. The HP-economizer 2 is next analyzed, by assuming economizer inlet temperature ($V_4$) and using the information already available, as indicated in the information flow diagram (Fig. 2).

Similarly, the simulation of the LP-superheater (bare-tubes) and the LP-evaporator are carried out using the effectiveness concept after evaluating the overall heat transfer coefficient by using appropriate correlations. The HP-economizer-1 and the LP-economizer are arranged in parallel. The outgoing gas condition are therefore determined by writing the heat balance of these two together. The condensate preheater, which is placed at the rear section of WHRB, receives hot water from the condenser at condensate temperature ($T_{COND}$). The preheater outlet temperature ($T_{CPH0}$), stack temperature ($T_{STACK}$) and the pressure drops inside and across the preheater $[(\Delta P_{CPH})_0, (\Delta P_{CPH})_1]$ are determined from the simulation procedure. The pressure of water at the inlet to the condensate preheater ($P_{CPH}$) can also be calculated, as indicated in the information flow diagram.

The analysis of the HP-turbine is then carried out using the pressure and temperature conditions, as calculated in the HP-superheater simulation, which are the inlet condition to the turbine and the outlet pressure of steam leaving the LP-superheater, which forms the exit condition of the HP-turbine. The turbine mass flow rate and exit temperature ($M_{MST}$, $T_{EX}$) can now be calculated from the turbine characteristics. Using the basic mixing rule of thermodynamics, the condition of steam at the entry to the LP-turbine is calculated. As the condenser pressure ($V_8$) is assumed, LP-turbine analysis is conducted and the mass flow and steam exit conditions ($M_{SS}$, $T_{EX}$) are determined.

The condenser simulation can also be carried out once the condensate pressure ($V_4$) and the flow and temperature of the cooling water ($M_{CW}$, $T_{CW}$) are specified. The heat transfer in the condenser ($Q_C$) is thus calculated.

The pressure drop across the condensate feed control valve ($V_9$) is also an assumed variable. Using this along the value of condensate preheater inlet pressure ($P_{CPH}$), the pressure at the exit of the condensate feed pump can be determined. The performance ($M_{FP}$, $W_{FP}$) of this pump can now be determined as the inlet pressure, $P_{COND}$, had already been assumed.

The analysis of all the components of the system is thus computed, and checks are made to ensure that the values of various assumed variables satisfy the following compatibility conditions:

2. Energy balance of HP-drum.

### Table 1.

<table>
<thead>
<tr>
<th>Fuel flow rate (kg/s)</th>
<th>Compressor inlet temp. (°C)</th>
<th>Condenser cooling water inlet temp. (°C)</th>
<th>Gas cycle computed</th>
<th>Power output (MW)</th>
<th>Steam cycle computed</th>
<th>Combined cycle computed</th>
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<tbody>
<tr>
<td>6.23</td>
<td>27</td>
<td>32</td>
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<td>109.0</td>
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<td>50</td>
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<td>392.0</td>
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<td>27</td>
<td>32</td>
<td>259.9</td>
<td>150.9</td>
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<td>27</td>
<td>32</td>
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<td>158.6</td>
<td>439.6</td>
<td>441.4</td>
</tr>
<tr>
<td>Fuel flow (kg/s)</td>
<td>Compressor pressure ratio</td>
<td>Compressor inlet temp. (°C)</td>
<td>Main steam flow (kg/s)</td>
<td>Secondary steam flow (kg/s)</td>
<td>Secondary steam pressure (bar)</td>
<td>Main steam pressure (bar)</td>
</tr>
<tr>
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<tr>
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<td>47.2</td>
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<td>107.14</td>
<td>54.6</td>
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<td>9.01</td>
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<td>412.3</td>
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<td>107.44</td>
<td>69.8</td>
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<td>478.2</td>
<td>1080.0</td>
<td>108.01</td>
<td>70.7</td>
<td>5.48</td>
</tr>
</tbody>
</table>
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(3) Mass balance across HP-pump and HP-turbine.
(4) Energy balance of LP-drum.
(5) Temperature matching at HP-Econ 2 inlet with HP-Econ 1 outlet.
(6) Mass balance of various streams at exit of HP-turbine and inlet to LP-turbine.
(7) Energy balance across condenser.
(8) Mass balance of deaerator tank.
(9) Pressure drop balance across boiler.

Consequently, the whole task of system simulation reduces to that of obtaining appropriate values of nine variables so that the nine compatibility equations are satisfied.

SOLUTION TECHNIQUE

It is obvious that these equations are highly complicated functions of the variables and, therefore, it is extremely important to adopt an appropriate solution technique to achieve convergence to the correct solution in the minimum computation time.

A review of various methods available shows that Warner’s method [5] is best suited for such complicated non-linear equations. The most important feature of this method is that only one set of error values needs to be calculated to determine the successive approximations to the correct solution.

VALIDATION OF PROGRAM

The validity of the procedure developed has been tested by comparing the predicted performance of the system with the rated performance for a system designed by a reputed power plant engineering company. Table 1 shows the comparison of power output at various loads. It is observed that the predicted and rated values of both the gas and steam cycles match to within 1%.

A comparison of the corresponding operating conditions is given in Table 2.

CONCLUSIONS

The present work describes the general procedure for the simulation of a dual pressure gas-steam combined cycle power plant. The method has been tested by comparing its predictions with data of a typical combined cycle; a very good correspondence between the predicted and the actual performance has been obtained. With a slight modification, this approach can be adopted to suit similar configurations encountered in practice.

REFERENCES

APPENDIX A

The effectiveness of cross-flow exchangers can be calculated from the standard expression [6]:

For both fluids unmixed
\[ \epsilon = 1 - \exp\left( \frac{1}{C_i} \left[ \text{NTU} \right] \right) \]
(A1)

For \( C_{\text{max}} \) (mixed) and \( C_{\text{min}} \) (unmixed)
\[ \epsilon = \left( \frac{1}{C_i} \right) \left[ 1 - \exp\left( -C_i \right) \right] \]
(A2)

For all exchangers when \( C_i = 0 \)
\[ \epsilon = 1 - \exp\left( -\text{NTU} \right) \]
(A3)

where NTU is number of transfer units and \( C_i \) is the heat capacity ratio and these are given by

\[ C_i = \frac{C_{\text{min}}}{C_{\text{max}}} \]
(A4)

\[ \frac{1}{\text{NTU}} = \frac{R_{\text{f}}}{(\sigma h \mu C_i)} + \ln\left( \frac{d_i}{d_i} \right) + \frac{R_{\text{f}}}{(\sigma h \mu C_i)} + \frac{1}{(\sigma h \mu C_i)} \]
(A5)

where \( C \) and \( h \) refer to cold and hot fluids, and \( R_{\text{f}} \) is the fouling factor.

Heat transfer coefficient for confined flows

For confined flow inside the tubes, Nusselt number can be calculated by the Gnielinski correlation [7]:
\[ Nu = \frac{(f/8) Pr [Re_D - 1000]}{1 + 12.7 \sqrt{(f/8)(Pr^{2/3} - 1)}} \left[ 1 + \frac{d_i}{L} \right]^{1/3} \]
(A6)

\[ Re_D = \frac{\rho U_m d_i}{\mu} \]

where \( U_m \) is the mean fluid velocity over the tube cross-section and \( d_i \) is the tube diameter. Its range of validity is

\[ 0.5 < Pr < 10^6 \]

\[ 2300 < Re_D < 5 \times 10^6 \]

The friction factor for smooth tubes is calculated by using the equation recommended by Filoneko:
\[ f = (1.82 \log_{10} Re_D - 1.64)^{-2} \]

Evaporator

In the evaporator the water is saturated at entry and only partially evaporated (typical vapour fraction \( \sim 0.2 \)) when it leaves it and re-enters the evaporator drum. The correlation of Chen [8], which has been widely recommended in the literature for such situations, has been used. This correlation suggests that the total heat transfer in boiling is contributed by two components:

\[ h_{\text{bp}} = h_c + h_{\text{nb}} \]
(A7)

where \( h_c \), the convective boiling component, can be calculated as
\[ h_c = 0.023 \left[ \frac{G (1 - x)}{\mu} \right]^{1/3} \left[ \frac{\rho_c}{k} \right]^{1/3} \left( \frac{d_i}{d} \right) \]
(A8)

where \( F \) is a function of the Martineau parameter \( (1/X_o) \),
\[ \frac{1}{X_o} = \left( \frac{x}{1 - x} \right) \left( \frac{1}{\rho_c} \right) \]
(A9)

and \( h_{\text{nb}} \), the nucleate boiling component, can be estimated as
\[ h_{\text{nb}} = 0.00122 \left[ \frac{k^2 C_p 0.4}{\rho_c 0.8} \left( \frac{\Delta T_{\text{sat}}}{\Delta T_{\text{sat}}} \Delta P_{\text{sat}} \right) \right] \]
(A10)

Chen suggests that \( S \) can be represented as a function of the local two-phase Reynolds number. Chen gives graphical relationships for estimating the values of \( F \) and \( S \) as functions of \( X_o \) and \( Re_{\text{TP}} \), respectively. The following equations have been fitted to these graphs for use in the computer:

\[ F = 1.361 + 0.7788 \left( \frac{1}{X_o} \right) \]
\[ S = 0.7194 - 0.8081 \times 10^{-7} Re_{\text{TP}} \]

Shell-and-tube condenser

To calculate the shell-side heat transfer coefficient in the condenser, the well known Nusselt equation has been used [6]:
\[ \theta_{\text{DN}} = 0.729 \left[ \frac{gr (\rho_v - \rho_s) k^2 h_s^{1/4}}{N_{ch} (T_m - T_s)d} \right] \]
(A11)
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where l and v refer to the liquid and vapour phase, N is the number of rows of tubes and $T_s$ is surface temperature. The corrected latent heat is calculated as follows:

$$h_{gs} = h_l(1 + 0.68Ja)$$

$$Ja = \frac{c_p(T_{wi} - T_j)}{h_{gs}}$$

**Gas-side heat transfer coefficient**

The hot gases flow across the tube banks formed by the various heat exchangers. To estimate the heat transfer coefficient across these bundles, the latest correlations suggested by Zukauskas [9, 10] have been used.

**Finned tubes**

$$\overline{Nu_t} = 0.192(a/b)^{0.5}(s/d_0)^{0.3}(h_l/h_d)^{-0.14}Re_l^{0.65}Pr_l^{0.25}$$

$$1.10^4 < Re < 2.10^4$$

$$\overline{Nu_t} = 0.0507(a/b)^{0.5}(s/d_0)^{0.14}(h_l/h_d)^{-0.14}Re_l^{0.65}Pr_l^{0.25}$$

$$2.10^4 < Re < 1.4 \times 10^6$$

$$1.1 < a < 4.0$$

$$1.03 < b < 2.5$$

$$0.07 < h/d < 0.715$$

$$0.06 < s/d < 0.36$$

$$\overline{Nu_t} = 0.0081(a/b)^{0.2}(s/d_0)^{0.11}(h_l/h_d)^{-0.04}Re_l^{0.95}Pr_l^{0.25}$$

$$2.10^5 < Re < 1.4 \times 10^6$$

$$2.2 < a < 4.2$$

$$1.27 < b < 2.2$$

$$0.125 < h/d < 0.6$$

$$0.125 < s/d < 0.28$$

(A10)

(Bare tubes)

$$\overline{Nu} = CRe_{D,max}^{m}Pr^{0.36}(Pr)^{1/4}$$

$$1000 < Re_{D,max} < 2 \times 10^6$$

$$0.7 < Pr < 500.$$  

The value of constant $C$ for staggered tubes is given as

$$C = 0.35(S_L/S_T)^{0.5} \quad \text{for } S_T/S_L < 2$$

$$C = 0.4 \quad m = 0.60 \quad \text{for } S_T/S_L > 2.$$  

The value of Reynolds number for the forgoing correlation is based on the maximum velocity occurring within the tube bank. For staggered configuration, the maximum velocity may occur at either transverse or diagonal plane. It will occur at diagonal plane if the rows are placed such that

$$2(S_D - d_0) < (S_T - d_0),$$

where

$$S_D = \left[ S_T^2 + \left( \frac{S_T}{2} \right)^2 \right]^{1/2} = \frac{S_T + d_0}{2}.$$  

In this case maximum velocity is given by

$$U_{max} = \frac{S_T}{2(S_D - d_0)} u.$$  

If $U_{max}$ occurs at transverse plane for staggered configuration, it may be computed as

$$U_{max} = \frac{S_T}{S_T - d_0} u.$$  

**Fin efficiency**

The fin efficiency is defined as the ratio of actual heat transfer rate to the maximum heat transfer rate that would occur with a fin of infinite thermal conductivity. The overall efficiency of a finned surface is calculated as

$$\eta_0 = 1 - \frac{A_f}{A} (1 - \eta_f),$$

where $\eta_f$ is the fin efficiency and is discussed in reference [6].
APPENDIX B

Estimation of pressure drop

The pressure drop inside the tube [11] is given as

$$(\Delta p)_{\text{in}} = (\Delta p)_{\text{fri}} + (\Delta p)_{\text{turning}}. \quad (A15)$$

Frictional pressure drop. Frictional pressure drop is calculated from the conventional Darcy equation [11]:

$$(\Delta p)_{\text{fri}} = \frac{4fL}{D_h} \left( \frac{1}{2} \rho U^2 \right). \quad (A16)$$

where $D_h$ is the hydraulic diameter and $f$ is the fanning friction factor which can be expressed as

$$f = \begin{cases} \frac{16}{Re} & \text{for laminar flow} \\ \frac{0.046}{Re^{0.25}} & \text{for turbulent flow}. \end{cases} \quad (A16)$$

Pressure drop due to flow turning. The pressure drop associated with flow turning is expressed in the form of

$$\Delta p_{\text{turning}} = k \left( \frac{1}{2} \rho U^2 \right) \quad (A17)$$

where $k$ is turning loss coefficient, which consists of two factors: $K_{90}$ and $K_{\alpha}$, $K = K_{90} \cdot K_{\alpha}$, where $K_{90}$ = loss coefficient for 90° and $K_{\alpha}$ = correction factor for turning angle; both of which are calculated from reference [11].

Pressure drop outside unfinned tubes. The most recent and reliable correlation to calculate pressure drop across a bundle of unfinned tubes, as recommended by Zhukauskas [7], is

$$\Delta p = \rho \frac{U_{\text{in}}^2}{2} \left( f \Omega \right) \quad (A18)$$

Pressure drop outside finned tubes. The total pressure drop on the fin-side across a heat exchanger core [12] can be calculated as

$$\Delta p = \frac{G^2}{2 \rho \Omega} \left[ f \Omega \frac{A_{1f} \rho_n}{A_{2f} \rho_{out}} + (1 + \sigma^2) \left( \frac{\rho_n}{\rho_{out}} - 1 \right) \right]. \quad (A19)$$

In this equation $\rho$ is the average density evaluated between inlet and outlet values

$$\frac{1}{\rho} = \frac{1}{\rho_n} + \frac{1}{\rho_{out}} \quad \text{and} \quad \frac{1}{\rho} = \frac{2 \rho_n \rho_{out}}{\rho_n + \rho_{out}}.$$  

The maximum mass velocity, $G$, is given by

$$G = \rho U_{\text{max}} = \frac{\rho U_{\text{in}}}{A_{1f} \sigma A_{2f}} = \frac{m}{\sigma A_{2f}}.$$  

In this equation $\sigma$ is the ratio of minimum free-flow area of finned passage (cross-sectional area perpendicular to flow direction), $A_{2f}$, to the frontal area, $A_{f}$, of the heat exchanger:

$$\sigma = \frac{\text{free flow area}}{\text{frontal area}} = \frac{(S_t - d_t) - (2l_t h_t)}{S_t}.$$