NONLINEAR COUPLLED RESPONSE OF OFFSHORE TENSION LEG PLATFORMS TO REGULAR WAVE FORCES

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Abstract—Among the compliant platforms, the tension leg platform (TLP) is a vertically moored structure with excess buoyancy. The TLP is designed to behave in the same way as any other moored structure in horizontal plane, at the same time inheriting the stiffness of a fixed platform in the vertical plane. Dynamic response analysis of a TLP to deterministic first order wave forces is presented, considering coupling between the degrees-of-freedom surge, sway, heave, roll, pitch and yaw. The analysis considers nonlinearities produced due to changes in cable tension and due to nonlinear hydrodynamic drag forces. The wave forces on the elements of the pontoon structure are calculated using Airy’s wave theory and Morison’s equation ignoring diffraction effects. The nonlinear equation of motion is solved in the time domain by Newmark’s beta integration scheme. The effects of different parameters that influence the response of the TLP are then investigated.

1. INTRODUCTION

The tension leg platform (TLP) is a kind of compliant type offshore platform which is generally used for deep water oil exploration. As reflected by its name, it is a buoyant structure anchored by pretensioned cables to the sea bed. They are designed to be more responsive to external loading than the fixed type offshore platforms. The cabling system of the platform may be vertical or splayed which restrains vertical movements, but permits some horizontal displacement. The terminal of such a platform remains virtually horizontal.

The tension cabling system consists of four or more tension legs, each leg being comprised of multiple parallel tension members terminated at the base of the structure. The composition of the tension members terminated at the base will vary depending on the mooring requirements and the type of the tension member selected.

A number of studies have been made on the dynamic behaviour of tension leg platforms under both regular and random waves (Taudin, 1978; Tan and De Boom, 1981; Denis and Heaf, 1979). The majority of these studies deal with the two dimensional behavior of the platform. Few investigations, however, considered all the six degrees-of-freedom of the platform in describing its dynamic behaviour (Morgan and Malaeb, 1983; Natvig and Pender, 1977).

Natvig and Teigen (1993) presented a review of hydrodynamic challenges in TLP design. They separated the hydrodynamics into three frequency regimes—wave, low, and high frequency ranges. Johnson et al. (1993) presented the simplified nonlinear response of the TLP in deep water using Morison’s equation investigating the effects of variation in the free surface elevation, the change in geometry due to the motions of the platforms.
and the effect of the spacing between the four cylindrical legs of the TLP. Huse and Utne (1994) presented the experimental information on the hydrodynamic springing damping of a TLP column as influenced by the presence of current and waves, and by the variation of radius of curvature at the lower edge of the column. They also compared the numerical calculations of the damping in calm water with the experiments.

Li and Kareem (1992) presented the response of a TLP to wave drift forces. The wave drift forces on the TLP are contributed to by second-order potential and viscous wave load effects, the fluctuations in wave surface elevation and the influence of platform’s displaced position on the wave excitation. Liu et al. (1993) presented the second-order double-frequency wave loads on the ISSC TLP in regular waves.

The literature survey shows the need for more parametric studies on the dynamic behaviour of such platforms incorporating various kinds of nonlinearities. In the present paper, dynamic analysis of a TLP under regular waves is performed. The analysis duly considers the coupling behaviour of various degrees-of-freedom and different nonlinearities present in the system.

Using the proposed method of analysis, some numerical studies are conducted in order to highlight the effect of few important parameters on the response.

2. THEORY

For compliant structures, the inertia forces are predominant when they are dynamically excited. For such a situation, one has to perform rigorous dynamic analysis, and there exist two possibilities. One can do the linear analysis which is cheap and easy, but the major limitation to this is that one should be confident about the system being linear or nearly linear, such that the nonlinear effects, if present, are negligible. In order to incorporate the nonlinear phenomena, a nonlinear analysis has to be performed. Nonlinear effects can not be easily included in frequency domain analysis but best handled in the time domain analysis using Newmark’s beta step-by-step numerical integration technique.

2.1. Equation of motion

The equation of motion describing the dynamic equilibrium between the inertia, damping, restoring and exciting forces can be assembled as follows:

\[ [M]\dddot{X} + [C]\ddot{X} + [K]X = \{F(X),\dot{X},\ddot{X},t\} \]

(1)

where, \([M]\) is the diagonal mass matrix for all the six degrees-of-freedom; \([C]\) is the damping matrix; \([K]\) is the nonlinear stiffness matrix; \([F]\) is the vector of forcing function; \([X]\), \([\dot{X}]\) and \([\ddot{X}]\) are the displacement, velocity and acceleration vectors, respectively.

2.2. Draft evaluation

At the original equilibrium position (Fig. 1), summation of forces in the vertical direction gives:

\[ W + T = F_b \]

(2a)

where

\[ F_b = \rho \pi g (D_i^2 D_r + D_i^2 s) \]

(2b)

From Eq. (2a), we get:
Fig. 1. Plan and elevation of the proposed TLP model.

\[ D_r = \left[ \frac{(W + T)/(\rho g) - D^2 s}{D^2_r} \right] \]  \hspace{1cm} (2c)

where, \( F_b \) is the total buoyancy force, \( W \) is the total weight of the platform in air, \( T \) is the total instantaneous tension in the tethers, \( \rho \) is the mass density of sea water, \( D \) is the diameter of TLP columns, \( D_r \) is the diameter of pontoon, \( s \) is the length of the pontoon between the inner edges of the columns and \( D_r \) is the draft.

2.3. **Restoring force**

The structure is considered as a rigid body having six degrees-of-freedom (Fig. 2) which includes three translations (surge, sway and heave) and three rotations (roll, pitch and yaw). The first column of the restoring force matrix is derived by giving the structure an arbitrary displacement \( x \) (Fig. 3) in the \( x \) direction (surge). Summation of forces in the horizontal direction yields:

\[ K_{11} = 4(T_0 + \Delta T_1) \sin \theta_x / \lambda \]  \hspace{1cm} (3a)

where, \( \Delta T_1 \) is the increase in tension in each cable for the \( x \) displacement, \( T_0 \) is the pretension in each cable and \( \theta_x \) is the angle of inclination of the cables with respect to
the vertical when under surge displacement. Through summation of the vertical forces, we get:

\[ K_{s1} = [4T_s(\cos\theta_s - 1) + 4\Delta T_1 \cos\theta_s] / x \] (3b)

Summation of moments about the y-axis gives:

\[ K_{s1} = -4(T_0 + \Delta T_1) \sin\theta_s (\bar{h} / x) \] (3c)

where, \( \bar{h} \) is the distance between the center of mass and the bottom of the platform (Fig. 3). From Eqs (3a) and (3c), we get:

\[ K_{s1} = -\bar{h}K_{s1} \] (3d)

The coefficients of the second column of the restoring force matrix are found in a similar manner by giving y displacement in the y-direction (sway):
Nonlinear coupled response to regular wave forces

\[ K_{22} = 4(T_0 + \Delta T_2) \sin \theta_y \]
\[ K_{32} = [4T_0(\cos \theta_y - 1) + 4\Delta T_2 \cos \theta_y]/y \]
\[ K_{42} = \bar{h}K_{22} \]

Where \( \Delta T_2 \) is the increase in tension due to sway and \( \theta_y \) is the angle of inclination of the cables with respect to the vertical when under sway movement. The third column is derived by giving the structure an arbitrary displacement \( z \) in the \( z \) direction (heave). The sum of the forces in the vertical direction yields:

\[ K_{33} = \rho \pi g D_2^2 + 4AE/L \]

The coefficients in the fourth column of the restoring force matrix are found by giving the structure an arbitrary rotation \( \theta_z \) about the \( x \)-axis. Summation of the moments of the resulting forces about the \( x \)-axis gives:

\[ K_{44} = \rho \pi g b^2 D_2^2 + 4\{T_0 \bar{h} (\sin \theta_z)/\theta_z\} + \{AEb^2 (\cos \theta_z)/L\} \]
\[ K_{34} = 2(\Delta T_4 + \Delta T_4')/\theta_z \]

Similarly, the fifth column of the restoring force matrix yields:

\[ K_{55} = \rho \pi g a^2 D_2^2 + 4\{T_0 \bar{h} (\sin \theta_z)/\theta_z\} + \{AEa^2 (\cos \theta_z)/L\} \]
\[ K_{35} = 2(\Delta T_5 + \Delta T_5')/\theta_z \]

The sixth column of the restoring force matrix can be obtained through rotating the structure about the \( z \)-axis. The sum of the moment about the \( z \)-axis gives:

\[ K_{66} = 4(T_0 + \Delta T_6)(a^2 + b^2)/[L^2 + \theta_z^2(a^2 + b^2)]^{0.5} \]

Finally, through summation of forces in the vertical direction one obtains:

\[ K_{36} = (4T_0/\theta_0) \{L/(L^2 + \theta_0^2(a^2 + b^2))^{0.5} \} - 1 \]
\[ + (4\Delta T_6/\theta_0) \{L/(L^2 + \theta_0^2(a^2 + b^2))^{0.5} \} \]

The other terms of the restoring force matrix which are not described above are zero. \( \Delta T_i \) is the increase in tension in each cable due to the displacement in the \( i \) direction; \( \Delta T_i' \) is for the cables on the other face (in each of the 2 cables on back face)—it only occurs when coupling of roll and pitch with the heave takes place; \( \theta_i \) is the angles made by cables with the vertical axis when the platform is displaced along its degree-of-freedom.

2.4. Structural mass

The resulting mass matrix of the entire structure is a diagonal mass matrix, having diagonal masses as: \( M_{11} \) is the mass in surge movement; \( M_{22} \) is the mass in sway movement; \( M_{33} \) is the mass in heave movement; \( M_{44} \) is the mass moment of the inertia about the \( x \)-axis, \( M_{55} \) is the mass moment of inertia about the \( y \)-axis; and \( M_{66} \) is the mass moment of inertia about the \( z \)-axis.

2.5. Structural damping

The damping matrix is assumed to be mass and restoring force proportional and can be shown in the following form:
\[
\{\phi\}^T[C]\{\phi\} = [2\zeta\omega_m]
\]  
(4)

where, \{\phi\} and \omega are the mode shapes and structure’s natural frequencies, \zeta is the structural critical damping ratio, [C] is the damping matrix and m is the corresponding element of the \{\phi\}^T [M] \{\phi\}.

2.6. Wave forces

The wave forces acting on the cylindrical members of the TLP structure are obtained by using modified Morison’s equations which takes into account the relative velocity and acceleration between the structure and fluid particles. The Morison equation is strictly valid for \((D/\lambda L) < 0.1\), where D is the diameter of the member and \(\lambda L\) is the wave length. For large diameter members, more complex theories are available in the literature which take into account the reflection or radiation of wave energy from members. For the study carried out here, the \(D/\lambda L\) ratio is considered such that it is less than 0.1. The drag, \(\delta F_d\) and inertia, \(\delta F_i\) forces on an element \(dz\) along the length of a member become:

\[
\delta F_d = 0.5\rho C_d D U_s |U_d| dz
\]

and

\[
\delta F_i = 0.25\rho \pi D^2 (C_m U - (C_n - 1) \dot{X}_s) dz
\]

where, \(U_s\) is the normal relative velocity between the member and water, and \(\dot{X}_s\) is the acceleration of the member in corresponding degree-of-freedom (either surge or heave); \(U_d\) is the undisturbed acceleration of the sea water; \(\rho\) is the mass density of the sea water; \(C_d\) and \(C_m\) are the drag and inertia coefficients, respectively. Similarly forces corresponding to all other degrees-of-freedom, caused due to wave action, are evaluated. While calculating the wave forces, the water particle kinematics for each element are determined with respect to the C.G. of the element. The effect of variable sea surface elevation on the water particle kinematics and hence, the hydrodynamic forces is incorporated by modifying Airy’s linear wave theory as proposed by Chakrabarti (1971) and Hogben et al. (1977). The inertia force due to variable added mass is computed based on the actual length of submergence of the members with the passage of waves. The fluctuating component of the tension in tethers is computed from the variable component of the buoyancy caused by the fluctuating sea surface elevation with the passage of waves. In Chakrabarti’s approach depth of water is considered up to the instantaneous sea surface, while Hogben’s approach considers the depth of water up to mean sea level. In both approaches, water particle kinematics are calculated up to the actual sea surface elevation. The integration of the elemental forces acting on the pontoons and columns is performed numerically by dividing the cylinder into small elements. The instantaneous total hydrodynamic force is determined at each time station with assigned values of structural displacements, velocities and accelerations.

3. ANALYSIS

Since the assembled equations of motion are coupled and nonlinear, a time domain analysis is employed to calculate the time history of the response. For this purpose, the Newmark’s beta integration scheme is used in a step wise manner.

In order to use the Newmark’s beta integration scheme, we write Eq. 1 as follows:
\[ [M][\dot{X}_{t+\Delta t}] + [C][\dot{X}_{t+\Delta t}] + [K][X_{t+\Delta t}] = \{F_{t+\Delta t}\} \quad (6) \]

where,
\[ \{\ddot{X}_{t+\Delta t}\} = \{\dot{X}_t\} + \{(\dot{X}_t + \dot{X}_{t+\Delta t})\} \Delta t / 2 \quad (7) \]

and
\[ \{X_{t+\Delta t}\} = \{X_t\} + \{\dot{X}_t\} \Delta t + \{(\dot{X}_t + \dot{X}_{t+\Delta t})\} \Delta t^2 / 4 \quad (8) \]

Solving Eq. 8 for \( \{\ddot{X}_{t+\Delta t}\} \) in terms of \( \{X_{t+\Delta t}\} \) and then substituting for \( \{\ddot{X}_{t+\Delta t}\} \) into Eq. 7, we obtain equations for \( \{\dot{X}_{t+\Delta t}\} \) and \( \{X_{t+\Delta t}\} \), each in terms of the unknown \( \{X_{t+\Delta t}\} \) only. These two relations for \( \{\dot{X}_{t+\Delta t}\} \) and \( \{X_{t+\Delta t}\} \) are substituted into Eq. 6 to solve for \( \{X_{t+\Delta t}\} \), after which, using Eq. 7 and 8, \( \{\dot{X}_{t+\Delta t}\} \) and \( \{X_{t+\Delta t}\} \) can also be calculated.

At each time step the restoring force matrix is updated to take into account the change in cable tension as shown in Eqs 3(a–n). Since the right hand side load vector of Eq. 1 is nonlinearly coupled, because of the presence of structural displacement, velocity and acceleration, an iterative scheme is used at each time station to obtain the right hand side load vector. The iteration starts with assumed values of the structural displacement, velocity and acceleration as those of the previous time stations. With these assumed values, the right hand side load vector is numerically calculated as explained earlier. The equation of motion is then solved for the current time station for obtaining the revised values of structural accelerations, velocities and displacements. The iteration is continued until the difference between the two successive values of structural displacement falls below a certain tolerance limit (0.5%).

4. NUMERICAL STUDY AND DISCUSSION

The TLP model being considered is square in plan as shown in Fig. 1 with four columns at each corner of diameter \( D_c \) and four horizontal cylindrical pontoons of diameter \( D \) at the bottom connecting the vertical cylinders. The structure is anchored to the seabed by vertical tethers attached at each corner. The properties of the TLP and wave data are given in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Details of TLP and wave data</th>
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<tbody>
<tr>
<td>( D_c ) = 18 m</td>
</tr>
<tr>
<td>( T_0 ) = 5500 and 2750 tons</td>
</tr>
<tr>
<td>( A ) of tether = 0.4 m²</td>
</tr>
<tr>
<td>( b ) = 45 m</td>
</tr>
<tr>
<td>( H ) = 70 m</td>
</tr>
<tr>
<td>( W ) = 50,000 tons</td>
</tr>
<tr>
<td>( \xi ) = 2% and 5%</td>
</tr>
<tr>
<td>( C_d ) = 1.0</td>
</tr>
<tr>
<td>( WH ) = 6m, 10m and 20m</td>
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</table>
The numerical studies are made in order to primarily study the responses of the TLP corresponding to soft and stiff degrees-of-freedom under different parametric combinations, namely: (i) large cable restoring force (total pretension = 22,000 tons), moderate wave height (10 m and 11 sec.) and $\zeta = 5\%$: (ii) storm condition characterised by large wave height of 20 m and 11 sec. and low structural damping, $\zeta = 2\%$ and (iii) low cable restoring force (total pretension = 11,000 tons), low wave height condition (6 m and 11 sec.) and $\zeta = 5\%$. The study also includes the evaluation of the influence of the effect of the coupling between different degrees-of-freedom on the response characteristics and the effect of variable submergence with the passage of the waves.

Table 2 shows the natural period of the TLP for two cases of total pretension i.e. 22,000 tons and 11,000 tons. It is seen that the natural periods for surge, sway and yaw are comparatively higher than those for heave, roll and pitch. It is unlikely that any resonance effect can take place in the surge, sway and yaw responses for all possible wave conditions. On the other hand, these structural periods are such that the responses corresponding to these degrees-of-freedom will have deamplification under hydrodynamic forces produced by most of the waves.

It is the heave period of the TLP which is of much concern since it lies in the vicinity of moderately large wave periods. The heave response can have resonance effect and thus the fluctuation of tension in the tethers of the TLP could be of much concern from a fatigue point of view. Moreover, if the natural periods for heave, roll and pitch are not well separated, it could lead to very serious problems from operational conditions.

The surge, sway and yaw are the responses corresponding to soft degrees-of-freedom. The steady state frequency contents of the forces and these responses are shown in Fig. 4 for a wave of 10 m and 11 sec. As expected, the surge displacement is much greater compared to sway and yaw, the last two responses being produced because of the coupling effect between surge and heave (both sway and yaw are in turn coupled with the heave motion).

The responses corresponding to stiff degrees-of-freedom are pitch, roll and heave. The frequency contents of forces and responses are shown in Fig. 5. The responses corresponding to pitch and roll degree-of-freedom are introduced due to the coupling between heave and pitch, and heave and roll, respectively. The heave motion is caused by the vertical wave forces on the pontoon as well as due to a coupling effect. Although the heave motion is small compared to surge motion, it is not insignificant. The reason for this is that the heave period of the TLP is very much closer to the wave period. The pitch and roll responses arising from the coupling effect can neither be ignored. Consequently, different tethers of the TLP are likely to be subjected to significantly different magnitudes of stress fluctuations. Thus, probabilities of fatigue damage could be different in various tethers and special attention should be given to the tethers because of their high tensile static and dynamic stresses.

<table>
<thead>
<tr>
<th>Total pretension</th>
<th>Surge</th>
<th>Sway</th>
<th>Heave</th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T = 22,000$ tons</td>
<td>39</td>
<td>39</td>
<td>12</td>
<td>27</td>
<td>27</td>
<td>51</td>
</tr>
<tr>
<td>$T = 11,000$ tons</td>
<td>62</td>
<td>62</td>
<td>13</td>
<td>27</td>
<td>27</td>
<td>76</td>
</tr>
</tbody>
</table>
Fig. 4. (a) TLP wave load amplitude, (b) TLP response amplitude.
Fig. 5. (a) TLP wave load amplitude, (b) TLP response amplitude.
In order to investigate the relative contribution of vertical wave forces and the coupling effect on the overall response of the heave motion, the TLP is analysed under two conditions, namely (i) considering coupling between various degrees-of-freedom and (ii) no-coupling between degrees-of-freedom, i.e. using only the diagonal restoring force matrix. The resulting frequency contents of heave responses are shown in Fig. 6. It is seen that the contribution of the coupling effect on heave response is considerably greater than that of the vertical wave force. Therefore, it is clear that if the coupling effect between various degrees-of-freedom is ignored in the analysis of the TLP, then the heave response will be highly underestimated.

Figure 7 shows the frequency contents of forces and the responses corresponding to soft degrees-of-freedom under storm conditions. The responses are considerably increased due to both large wave height and small structural damping, $\zeta = 2\%$, (compare Fig. 4b and Fig. 7b). However, the increase is not as large as is apparently expected. This is due to the fact that the large wave height also introduces greater hydrodynamic damping; as a result, the response does not increase in proportion to wave height (or wave force).

Figure 8 shows the frequency contents of the forces and the response corresponding to the soft degrees-of-freedom for a smaller wave height, i.e. 6 m, and low total pretension in the cable, i.e. 11,000 tons. It is interesting to note that the surge response is not very much reduced compared to that shown in Fig. 4b (i.e. for moderate wave height 10 m, and high total pretension 22,000 tons). This is due to the fact that although the wave forces due to 6 m waves are less than those for 10 m waves, the corresponding reduction in hydrodynamic damping in the former case does not allow the response to be reduced considerably. Also, the system being less stiff in the vertical direction, it provides more response in the surge direction due to heave-surge coupling effect.

Figure 9a–c shows the frequency contents of the responses for the two cases of variable

![Graph showing frequency contents of heave response amplitude.](image)

Fig. 6. TLP heave response amplitude.
Fig. 7. (a) TLP wave load amplitude, (b) TLP response amplitude.
Fig. 8. (a) TLP wave load amplitude. (b) TLP response amplitude.
Fig. 9. (a) TLP heave response amplitude, (b) TLP pitch response amplitude.
submergence and Airy's linear wave theory. The heave, pitch and surge degrees-of-freedom are predominantly affected due to inclusion of variable submergence. The inclusion of variable submergence in the analysis increases the responses between 20% to 60% depending upon the degree-of-freedom and the approach used. The higher increase is for Hogben's approach. The heave response is the most influenced and in turn causes more tension fluctuations in the tethers. Waves being unidirectional, the other degrees-of-freedom (sway, roll and yaw) are not influenced so much directly by the force but get affected due to coupling of degrees-of-freedom.

5. CONCLUSIONS

The nonlinear deterministic dynamic analysis of a TLP is presented, in which the coupling between the different degrees-of-freedom has been considered. The analysis includes all the six degrees-of-freedom of the TLP and treats them under general conditions where wave forces can act as inclination with the principal direction of the TLP. The solution is performed in the time domain using step-by-step numerical integration of the equation of motion. The numerical studies conducted with the proposed analysis method lead to the following conclusions:

(a) It is unlikely that resonance corresponding to surge, sway and yaw (soft degrees-of-freedom) can take place for any wave condition since the natural periods in surge, sway and yaw (soft degrees-of-freedom) are much higher than those for heave, roll and pitch (stiff degrees-of-freedom).

(b) Fluctuations in the tension of the tethers of the TLP could be large due to a possible resonance effect with heave frequency since the heave period of the TLP is normally close to frequently occurring wave periods.
(c) The coupling effect leads to sway and yaw motion of the TLP even when the wave force acts in the surge direction due to the coupling of the heave motion with surge, sway and yaw.

(d) Different magnitudes of stress fluctuations leading to different probabilities of fatigue damage can occur in various tethers due to the coupling effect.

(e) The heave response will be highly underestimated if the coupling effect between various degrees-of-freedom is ignored in the analysis of TLP.

(f) The change in wave height does not significantly influence the response, since the increased hydrodynamic force due to the increase in wave height is compensated by the increased hydrodynamic damping to a large extent.

(g) The effect of variable submergence introduced significant higher responses in the heave, pitch and surge degrees-of-freedom. The heave degree-of-freedom is most affected thereby causing more tension fluctuations in the tethers. Hogben’s approach leads to a greater response than that obtained by Chakrabarti’s approach.

REFERENCES


