Stimulated Raman scattering of a transverse magnetic (TM) mode in a strongly magnetised plasma

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Abstract

A strongly magnetized plasma, e.g., the one encountered in a plasma filled backward wave oscillator (BWO), supports an obliquely propagating electromagnetic (EM) mode, \( k_{\perp_1} \approx (k_{\perp_0}^2 - \omega_\perp^2/c^2)\left(\omega_p^2/\omega_\perp^2 - 1\right) \), where \( \omega_p^2 \approx \omega_c^2, \omega_\perp^2 \approx \omega_\parallel^2, \omega_p \) and \( c \) are the pump wave frequency and pump wave vector, \( \omega_p \) and \( \omega_c \) are electron plasma and cyclotron frequencies, \( c \) is the speed of light in vacuum and \( \parallel \) and \( \perp \) refer to the static magnetic field. As this mode acquires large amplitude it becomes susceptible to decay into a Trivelpiece–Gould (TG) mode and a sideband electromagnetic mode. The growth rate of the instability scales as \( \gamma \approx \frac{1}{2} v_{\text{osc}} k_{\parallel} (k_{\parallel}/k_{\perp})^{1/2} (\omega_p/k_c)^{1/2} \), where \( v_{\text{osc}} \) is the electron oscillatory velocity, \( k_{\parallel} \) and \( k_{\perp} \) are the wave vectors of the low frequency electrostatic (ES) Trivelpiece–Gould mode and EM sideband wave. For typical BWO parameters the growth rate is of the order of the \( 10^6 \text{ s}^{-1} \) range.

1. Introduction

Large amplitude waves encountered in plasma based sources of coherent radiation are susceptible to parametric instabilities. In a parametric process a large amplitude pump wave couples to a low frequency mode of the plasma to produce a sideband wave. The pump and the sideband wave exert a nonlinear force on the plasma particles to drive the low frequency mode. In this process the amplitudes of the low frequency mode and the sideband, called the daughter waves, grow with time once the pump power exceeds a threshold value, set by the linear damping or convective losses of the daughter waves. The parametric instability saturates via pump wave depletion or downward cascade of energy from the sideband wave. In plasma based sources of coherent radiation, parametric instabilities should lead to frequency broadening of the beam driven mode. Hence, an in-depth study of parametric instabilities in plasma filled devices is of considerable importance.

Parametric instabilities are known to be important in a wide variety of situations [1–8]. In laser produced plasmas two of the most significant parametric processes are stimulated Raman and Brillouin scattering (SRS and SBS) [8]. In the SRS process an intense laser beam drives a Langmuir wave and an electromagnetic sideband wave. The instability occurs at densities below quarter critical. However, the
density of the plasma should not be too small otherwise the Langmuir wave is strongly Landau damped by the electrons. The growth rate peaks when the sideband wave propagates opposite to the pump wave. In the SBS process, the laser excites an ion acoustic wave and an electromagnetic sideband wave. The instability occurs over a wide range of electron density, up to the critical layer. Nevertheless, it requires non-isothermal plasma where electron temperature is much larger than the ion temperature, otherwise the ion Landau damping will suppress the Brillouin instability.

In magnetically confined fusion devices, e.g., tokamaks, parametric instabilities are excited by the radio frequency waves employed for auxiliary heating and current drive. Four ranges of radio frequency have been used, viz., electron cyclotron range, lower hybrid range, ion cyclotron range frequency and Alfvén waves. In all the cases parametric instabilities have been encountered.

In plasma based sources of coherent radiation, e.g., a backward wave oscillator (BWO), the plasma is strongly magnetized with \( \omega_p \ll \omega_c \), where \( \omega_p \) and \( \omega_c \) are the electron plasma frequency and electron cyclotron frequency respectively. The electrostatic modes of such a plasma are strongly modified by the ambient magnetic field. The modes are characterized by the one-dimensional (only along the magnetic field line) motion of electrons. Ion motion would be one dimensional or three dimensional depending on the frequency of the mode. The electromagnetic mode of strongly magnetized plasma, for \( \omega < \omega_c \), is also very different than in an unmagnetized plasma. Thus the processes of SRS and SBS in a strongly magnetized plasma are likely to be very different than in unmagnetized or weakly magnetized plasmas. Such plasmas are encountered in backward wave oscillators [9], Q-machines [10,11] and beam plasma systems [12]. In a Q-machine a large amplitude lower hybrid wave mode has been seen to parametrically suppress drift waves and ion cyclotron waves. In a beam plasma system the large amplitude Trivelpiece–Gould (TG) mode is seen to excite modulational instability. In a BWO parametric instabilities have not been reported. Nevertheless, the level of electromagnetic waves is very high to warrant the excitation of parametric instabilities.

Carmel et al. [4] observed the generation of 8.6 GHz electromagnetic (EM) waves in a rippled wall waveguide employing a relativistic electron beam. In the absence of plasma the efficiency of the device is 5%. When a strongly magnetized plasma is introduced into the interaction region efficiency goes up to 40% at \( \omega = \omega_p/2 \), where \( \omega \) is the frequency of the driven TM mode. The electric field of the EM pump wave at such high power is \( \geq 100 \text{ eV} \) and induces an electron oscillatory velocity, \( v_{osc} > c/30 \), where \( c \) is the velocity of light. The large amplitude EM wave may excite the parametric instabilities in the plasma. Jainan et al. [13] have studied a nonlocal theory of a millimetre wave generation in a plasma filled slow wave device by means of parametric up conversion of a TM mode.

In this paper we study the stimulated Raman scattering (SRS) of a high amplitude electromagnetic wave propagating at an angle to the dc magnetic field. In a SRS process, the pump electromagnetic (EM) wave resonantly decays into a Trivelpiece–Gould (TG) mode and a sideband electromagnetic mode. The nonlinear coupling between the waves arises through the parallel motion of electrons. The electromagnetic pump wave (TM mode) provides an oscillatory velocity \( v_0 \) to plasma electrons in the direction of the dc magnetic field \( B_z \). When the backscattered EM wave is present in the system with finite electric field vector \( E_z \), the backscattered mode and the EM pump wave exert a parallel ponderomotive force on the plasma electrons driving a low frequency electrostatic (TG) mode. The density perturbations associated with the electrostatic mode couple with the oscillatory velocity due to the pump wave to produce a nonlinear current driving the backscattered sideband wave.

In Section 2 we present fluid theory of three wave parametric process in a strongly magnetized plasma ignoring boundary effects. A discussion of results is given in Section 3.

2. Instability analysis

Consider a homogeneous plasma in a strong magnetic field \( B_z \). A large amplitude pump electromagnetic (TM) mode propagates through it in the \( xz \) plane;

\[
E_0 = E_0 e^{-(x-z/k_0)^2},
\]
\[ B_0 = (c / \omega_0) k_0 \times E_0, \]  
\( \text{where } k_0 = k_{0z} \hat{z} + k_{0x} \hat{x}. \)  

The dispersion relation for the pump is
\[ k_{0z}^2 = \left( \frac{2}{\omega_0^2 / \omega_p^2} - \frac{1}{\omega_0^2 / \omega_p^2 - 1} \right), \]  
\( \text{where } \omega_p^2, \omega_0^2 < \omega_0^2. \) As \( \omega_0^2 \) has two roots in Eq. (3), the higher values correspond to \( \omega_0 > \omega_p \) and lower one to \( \omega_0 < \omega_p. \)

The velocity and the linear density perturbations of plasma electrons due to the EM pump wave can be written as
\[ v_0 = \frac{eE_{0z}}{m_1 \omega_0}, \]  
\[ \text{and} \]  
\[ n_0 = \frac{n_0^0 e k_0}{m_1 \omega_0} E_{0z}, \]  
where \( n_0^0 \) is the unperturbed density of plasma electrons. The pump TM mode decays into a low frequency Trivelpiece–Gould (TG) mode, with electrostatic potential \( \phi = \phi_0 e^{-i(\omega t - k \cdot r)} \) and a sideband electromagnetic wave,
\[ E_1 = E_0 e^{-i(\omega_1 t - k_1 \cdot r)}, \]  
\[ B_1 = (c / \omega_1) k_1 \times E_1, \]  
where \( k_1 = k_{1z} \hat{z} + k_{1x} \hat{x}, \omega_1 = \omega - \omega_0, k_1 = k - k_0. \) The dispersion relation for the sideband is the same as given for the pump except that subscript 0 is to be replaced by 1. For the TG mode it is \( \omega = \omega_p k_p / k. \)

The phase matching conditions \( \omega = \omega_0 - | \omega_1 |, k_0 = | k + k_1 | \) demand
\[ \left[ \frac{\Omega_0^2 + q_0^2}{\Omega_1^2} \right]^{1/2} = \left[ \frac{\Omega_0 - \Omega_1}{(1 - \Omega_0^2)} \right]^{1/2} = q \mp \left( \frac{\Omega_1^2 + q_1^2}{\Omega_1^2} \right)^{1/2}, \]  
\( \text{where the upper sign corresponds to the backscattering and lower one to the forward scattering respectively. } \) \( \omega_0 / \omega_p \equiv \Omega_0, \) \( 1 / \omega_0 \equiv \Omega_1 \) and \( k_{0z} c / \omega_p \equiv q. \) For \( \omega_0 \gg \omega, \omega_p \ll \omega_0, \) choosing \( q_0 \approx 2 \) and \( q = 4, \) Eq. (8) reduces to
\[ \left( \frac{\omega_0^2 + \frac{4 \Omega_0^2}{1 - \Omega_0^2}}{\Omega_1^2 + \frac{4 \Omega_1^2}{1 - \Omega_1^2}} \right)^{1/2} = 4 \frac{(\Omega_0 - \Omega_1)}{(1 - \Omega_0^2)^{1/2}}. \]  
\( \text{The response of plasma electrons to the sideband TM mode can be written as} \)
\[ v_1 = \frac{eE_{1z}}{m_1 \omega_1}, \]  
\[ n_1 = \frac{n_0^0 k_1 \cdot E_{1z}}{m_1 \omega_1}. \]  
\( \text{The nonlinear z-component of ponderomotive force due to the pump wave and the sideband is obtained as} \)
\[ F_{pz} = - \frac{1}{2} m \left( (v_0 \cdot \nabla) v_1 + (v_1 \cdot \nabla) v_0 \right)_z - (e/2 c)(v_0 \times B_0)_z - (e/2 c)(v_0 \times B_1)_z. \]  
\( \text{In presence of } F_{pz}, \) the velocity response of the electrostatic mode at \( (\omega, k) \) can be written as
\[ v_i = - \frac{ek_0 \phi}{m_0} + \frac{1}{2i \omega} \left( (v_0 \cdot \nabla) v_i + (v_i \cdot \nabla) v_0 \right). \]  
\( \text{The nonlinear density perturbation at } (\omega, k) \) is
\[ n = \frac{n_0^0 k \cdot v_i}{\omega}. \]  
\( \text{Substituting the value of } n \text{ from Eq. (14)} \) in the Poisson equation, \( \nabla^2 \phi = 4 \pi c n, \) we obtain
\[ \phi(1 + \chi_e) = \frac{e \omega_2^2 E_{0z} E_{1z} k_z^2}{2 \omega_0^2 \omega_1 \omega_1 mk_z^2}. \]  
\( \text{where} \)
\[ \chi_e = - \frac{\omega_2^2 k_z^2}{\omega_1^2 k_z^2}. \]
The nonlinear current density at \((\omega_1, k_1)\) is written as

\[
J_1 = -n_0 e\nu_v - \frac{i}{2} ne\nu_0
\]

\[
= -\frac{n_0 e^2 E_{1z}}{m\omega_1} \hat{z} + \frac{1}{2} \frac{n_0 e^2 k_1^2 \phi E_{0z}}{m^2 i \omega_0 \omega^2} \hat{z}
\]

\[
+ \frac{1}{4} \frac{n_0 e^2 k_1^2 E_{0z}^* E_{1z}^*}{m\omega_1 \omega_0 \omega^2} \hat{z},
\]

(16)

where an asterisk denotes the complex conjugate.

The wave equation for the sideband at \((\omega_1, k_1)\) is written as

\[
\nabla^2 E_i - \nabla (\nabla \cdot E_i) + \frac{\omega_i^2}{c^2} E_i = -\frac{4\pi i \omega_i}{c^2} J_i.
\]

(17)

Substituting the value of \(J_1\) from Eq. (16), using the first Maxwell equation, \(\nabla \cdot E_i = -4\pi n_i e\) and the value of \(\phi\) from Eq. (15) in (17) we obtain the nonlinear dispersion relation

\[
\left[ \frac{k_1^2}{\omega_1^2} - \frac{\omega_0^2}{\omega_1} - 1 \right] \left[ \frac{\omega^2}{\omega_p^2 k_1^2} \right] = -\frac{e^2 k_1^4 E_{0z} E_{1z}^*}{4m^3 \omega_0^2 k_1^4 \omega_1^2},
\]

(18)

where we have neglected \(1\) in comparison to the factor \(\omega_0^2 k_1^2 / \omega_1^2 k_1^2 (1 - \omega_0^2 k_1^2 / \omega_1^2 k_1^2)\).

We look for a solution around the simultaneous zeroes of the left-hand side (L.H.S.) of Eq. (18). When the term in the second parentheses on the L.H.S. of Eq. (18) is equated to zero,

\[
1 - \frac{\omega_0^2 k_1^2}{\omega_1^2 k_1^2} = 0,
\]

one obtains the low frequency electrostatic TG mode, whereas when the term in the first parentheses is equated to zero, one obtains a quadratic equation in \(\omega_1^2\) giving two roots \(\alpha_{1,2}^2\),

\[
\omega_1^2 = \frac{1}{2} \left[ \frac{\omega_p^2 + k_1^2 c^2}{\omega_0^2} \right]
\]

\[
\pm \sqrt{\left( \frac{\omega_p^2 + k_1^2 c^2}{\omega_0^2} \right)^2 - \frac{4k_1^2 c^2 \omega_p^2}{\omega_1^2}}
\]

(19)

corresponding to the electromagnetic sideband TM mode. One root \(\omega_1 = \alpha_1\) has frequency greater than \(\omega_p\) whereas the other has \(\omega_2 = \alpha_2 < \omega_p\). Taking

\[
\omega \approx \frac{\omega_p}{k} + i\gamma,
\]

(20)

\[
\omega_i \sim -\alpha_1 + i\gamma,
\]

(21)

where \(\gamma\) is the growth rate of the sideband TM mode instability, and substituting Eqs. (20) and (21) in Eq. (18) one obtains the growth as

\[
\gamma \sim \frac{\nu_{oc}}{\sqrt{\frac{k_2}{k}}} \left( \frac{\omega_p}{k} \right)^{3/2},
\]

where \(\nu_{oc} = eE_{0z} / m\omega_0\).

3. Results and discussion

For a set of typical parameters of the University of Maryland backward wave oscillator (BWO) e.g., \(n_0 = 10^{11} \text{ cm}^{-3}\); pump frequency, \(\omega_p / 2\pi \sim 8.4\) GHz, \(k_0 \approx k_{0,1} = 1 \text{ cm}^{-1}\); \(k_{1,2} \approx 1.4 \text{ cm}^{-1}\); \(v_{oc} \approx 0.05 c\), where \(c\) is the speed of light in vacuum, beam current \(I_b \approx 2 \text{ kA}\); beam voltage \(V_b \approx 625 \text{ kV}\); pulse width \(\approx 100 \text{ ns}\), the growth rate of the sideband electromagnetic (TM) mode turns out to be \(\approx 10^9\) s\(^{-1}\). Kehs et al. [9] have reported 5% conversion efficiency of beam energy into TM mode in the absence of plasma. This corresponds to TM power 50 MW. When plasma is introduced in the interaction region of the BWO the efficiency enhanced by a factor of 8, i.e. 400 MW TM mode power is produced. These powers are sufficiently high to onset sideband instability on the time scale of 10 ns.

![Fig. 1. Variation of the dimensionless pump wave (TM mode) frequency \(\Omega_p (\equiv \omega_p / \omega_{oc})\) with the dimensionless sideband wave (TM mode) frequency \(\Omega_i (\equiv \omega_i / \omega_p)\).](image)
In Fig. 1, we have plotted the variation of dimensionless sideband electromagnetic wave frequency \( \Omega_1 = |\omega_1|/\omega_p \) with the dimensionless pump frequency \( \Omega_0 = \omega_0/\omega_p \). \( \Omega_1 \) increases with \( \Omega_0 \). The variation is more than the linear. In Fig. 2, we have plotted the variation of the growth rate \( \gamma \) with the frequency of the sideband electromagnetic (EM) wave. The growth rate decreases with \( \omega_1 \). The variation is less than linear. The growth rate decreases with the plasma frequency \( \omega_p \).

Thus, high amplitude electromagnetic waves in a strongly magnetized plasma appear to be susceptible to parametric instability involving a Trivelpiece-Gould (TG) and a sideband TM mode. The parametric instability should manifest itself in the line broadening of the TM mode, yet to be confirmed in experiments, hence, it is an undesired parametric process in a BWO.

References