Simplified universal dispersion curves for channel waveguides

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Abstract

Simplified universal dispersion curves for graded-index diffused channel waveguides have been obtained with the new normalized WKB equation using the effective index method. It is shown that, similarly to planar guides, the propagation constant of all the guided modes of a particular channel can be obtained by two parameters, called the normalized mode orders. The dispersion relations between the propagation constant and the normalized mode order of the channel have been obtained for a number of profile shapes and waveguide dimensions.

1. Introduction

Integrated optical waveguide devices that have functions of light modulation and switching require channel waveguides in which the light is transversely confined in both width and depth directions. These waveguides are formed commonly by ion-exchange and thermal diffusion which give rise to graded refractive index profiles. The calculation of waveguide dispersion parameters in these diffused channels is a central problem to the design of devices which utilize them. Diffused planar waveguides are analysed using the WKB approximate method besides other exact numerical methods [1–4]. It gives good accuracy for a wide range of profiles and can be used for inverse problems to obtain index profiles from practical effective index data [5–7]. Conventionally, the modal dispersion characteristics of a waveguide are described by $b-V$ plots, where $b$ is the normalised propagation constant and $V$ is the normalized depth or frequency [8]. Recently, Chiang [9] has proposed a new method of normalising the WKB equation for graded-index planar guides by introducing a parameter called the normalized mode order, $M$. For each profile shape, the new normalised WKB equation allows a single universal dispersion curve to be applied to all guided modes under all operating conditions and hence is more general.

In the present paper we have calculated the dispersion characteristics of a 2-D diffused channel guide with the new normalised WKB equation. Among the many numerical and approximate methods for the analysis of diffused channel waveguides [10,11] the effective-index method is one of the popular methods. Although, the effect index method overestimates the propagation constant near the cut off values, it is widely used because of its simplicity and efficiency. Also, the error is significantly less in the case of a diffused guide with small index difference ($\sim 0.01$). In this paper we have used the effective index method [12] for computing the normalized propagation constant with respect to the normalised mode order of the channel. The effective index method provides the framework for combining the results of the 1-D planar dispersion curves of Chiang [9] to obtain the effective index of the equivalent slab for the diffused channel.

Results have been obtained for different channel index profiles and dimensions. For each profile shape the new normalised WKB equation allows a single curve for all guided modes under all operating conditions and hence is more useful for design problems as compared to the $b-V$ plots.

2. Dispersion relations

The 2-D diffused channel waveguide under consideration is shown in Fig. 1. Being a diffused guide, there will not be any sharp boundaries between the waveguide and
the substrate as shown in the figure; \( W \) and \( D \) are the effective width and depth, respectively.

The refractive index profile is expressed as

\[
n_2(x, y) = n_s^2 [1 + 2 \Delta f(y/D) g(2x/W)]
\]

\[
= \begin{cases} 
0 \leq y < \infty, \\ 0 < 1 \times 1 \geq W/2, 
\end{cases}
\]

where \( \Delta \) is the relative difference between the peak index and substrate index, i.e., \( \Delta = (n_0^2 - n_s^2) / 2n_s^2 \). \( f(y/D) \) and \( g(2x/W) \) are the normalized index profile functions which have values between 0 and 1, characterizing the shape of the profile in the \( y \) and \( x \) direction, respectively.

In order to analyse the 2-D diffused guide using the effective index method an equivalent 1-D graded index guide in the \( x \)-direction has to be constructed using Ref. [12]. At each position \( x \), the following WKB characteristics equations is to be satisfied in the \( y \)-direction

\[
2k \int_{y(x)}^{y(x,y)} n_2(x, y) - n_{2\text{eff}}(x) \, dy = 2m + \pi/2 + \pi,
\]

where

\[
n_{2\text{eff}}(x) = n_s^2 + (n_0^2 - n_s^2) g(2x/W) b(x),
\]

with \( n(x,y) = n_{2\text{eff}}(x) \); the effective index of the diffused 1-D guide at \( x \); \( y_i \) is the turning point and \( m \) is the mode number in the depth direction, which takes integer values 0, 1, 2, ... for the guided modes. The phase shift at the diffused boundary in the substrate is taken to be \(-\pi/2\) (as \( n_0 = n_s \)) and at the air interface is \(-\pi\). Substituting Eqs. (1) and (3) in Eq. (2), one obtains

\[
\int_{y(x)}^{y(x,y)} [f(Y) - b(x)] \, dY = \frac{(m + 3/4) \pi}{kDn_s(2\Delta)^{1/2}},
\]

where \( b(x) \) is the normalized propagation constant of the mode at \( x \):

\[
b(x) = \frac{n_{2\text{eff}}(x) - n_s^2}{(n_0^2 - n_s^2) g(2x/W)}.
\]

From Ref. [9], the normalised WKB equation (4) can be expressed as

\[
\int_0^{Y(x)} [f(Y) - b(M(x))]^{1/2} \, dY = F_x M(x),
\]

where

\[
F_x = \int_0^\infty f(Y)^{1/2} \, dY \quad \text{with} \quad Y = y/D.
\]

\( M(x) \), the mode order at \( x \) for the diffused slab guide is expressed as

\[
M(x) = \frac{M_y}{[g(2x/W)]^{1/2}}.
\]

The 1-D planar guide mode order, \( M_y \), is given as

\[
M_y = \frac{m + 3/4}{m_c + 3/4},
\]

where \( m_c \) is the mode cutoff at which \( n_{2\text{eff}}(m_c) = n_s \) and is expressed as [9]

\[
m_c = \frac{kDn_s(2\Delta)^{1/2}}{\pi} F_x - 3/4.
\]

Thus, an equivalent 1-D guide having a graded index in the \( x \)-direction is hereby constructed.

Next, the same procedure is used again to analyse this guide. The equation to be satisfied by the mode effective index, \( n_{2\text{eff}} \) of the channel guide is

\[
2k \int_{-X}^{X} [n_{2\text{eff}}(x) - n_{2\text{eff}}^2]^{1/2} \, dx = 2p \pi + \rho,
\]

where the phase shift for total internal reflection at each diffused boundary is taken to be \(-\pi/2\). In terms of the normalised function the above can be written as [12]

\[
\int_{-X}^{X} [h(X) - b']^{1/2} \, dX = \frac{(p + 1/2) \pi}{kDn_s(2\Delta)^{1/2}},
\]

where \( X = 2x/W \), \( p \) is the mode number in the \( x \)-direction of the guide, \( b' \) is the normalised propagation constant of the equivalent channel guide and \( h(X) = h(2x/W) = g(2x/W) b(x)/b_0 \).

\( b_0 \) is the normalized propagation constant of the planar guide given as

\[
b_0 = \frac{n_{2\text{eff}} - n_s^2}{n_0^2 - n_s^2},
\]

where \( n_{2\text{eff}} \) is the effective index of a mode in the planar guide.

The WKB equation (8) can be written in the modified form in terms of the mode order \( M_x \) of the original 2D guide as

\[
\int_{0}^{X} [h(X) - b'(M_x)]^{1/2} \, dX = F_x M_x,
\]
where \( M \) = \( (p + 1/2)/(p_c + 1/2) \); \( p_c \) is the mode cutoff in the \( x \)-direction with \( n_{\text{eff}}(p_c) = n_s \) and is obtained as

\[
p_c = \frac{kW\sqrt{2(2\Delta)^{1/2}F_x}}{\pi} - \frac{1}{2},
\]

where

\[
F_x = \int_0^{\infty} k(X) dX.
\]

It is seen in Eq. (9) that the normalised \( b' \) value of the 2-D channel for a given \( k(X) \) function (which contains the \( g(X) \) and \( f(Y) \) index profile functions) depends only on the mode order \( M \). The mode order has all other details of the guide, i.e., dimension, substrate and peak index. Eq. (9) is essentially the same equation as (18) in Ref. [12], hence identical \( b' \), \( V' \), results are obtained for a given \( W/D \) and the planar guide normalised propagation constant, \( b_0 \). Dispersion relations in terms of mode order, \( M \), facilitate the design of graded-index waveguides. For example, degenerate modes in different waveguides with the same dimensions, substrate index and index profile will have the same value of \( M \) for corresponding \( M_c \). Within the approximation of the effective index method the index difference in the waveguides supporting different modes will have the ratio of

\[
\frac{p_1 + 1/2}{(\Delta n_1)^{1/2}} = \frac{p_2 + 1/2}{(\Delta n_2)^{1/2}} = \frac{p_3 + 1/2}{(\Delta n_3)^{1/2}} = \cdots
\]

where \( p_1, p_2, \ldots \) are the mode numbers and \( \Delta n_1, \Delta n_2, \ldots \) are the index step in the respective waveguide supporting the particular mode.

Also, for a channel the value of the mode order will give all possible combinations of \( p \) and cutoff values of mode number, \( p_c \). These in turn indicate the associated mode and the total number of modes supported in the guide. For example, a mode order \( (M_c) \) of 0.5 for a channel can have the following combinations:

1. \( p = 0 \) and \( p_c = 0.5 \): corresponding to \( \text{TE}_{0m} \) of a channel with single mode in width;
2. \( p = 1 \) and \( p_c = 2.5 \): \( \text{TE}_{1m} \) of a channel with 3 modes in width;
3. \( p = 2 \) and \( p_c = 4.5 \): \( \text{TE}_{2m} \) of a channel with 5 modes in width.

Thus, besides the design parameters, viz., dimension and index difference, the mode order also gives the above information.

3. Numerical results

In this section, the dispersion relations between \( b' \) and \( M \) for the channel guide are obtained by solving Eq. (9) for the planar guide parameters; \( b_0 \) and \( M_c \). The planar guide mode order \( M \) includes the details of mode number, diffusion depth, substrate index, profile shape in the depth direction and the index difference.

Fig. 2. Universal dispersion curves (between normalised propagation constant \( b' \) and normalised mode order \( M \)) of the channel. \( M_c \) is the planar guide mode order. The index profile in the width is the sum of error function as defined in (10) and in the depth direction the index profiles are in (a) complimentary error (b) Gaussian (c) exponential profiles. The dashed curve is for the channel with no sideways diffusion.
The diffused channel guides have been assumed to have an index profile in the x-direction as
\[
g(2x/W) = \frac{1}{\sqrt{\pi}} \left( \text{erf}\left[ W/2D(1 + 2x/W) \right] + \text{erf}\left[ W/2D(1 - 2x/W) \right] \right).
\]
Eq. (10) is identical to Eq. (20) in Ref. [12] with \( V' = kWn_s(2D)^{1/2} \). Similarly, all the other curves in Fig. 2 can be shown to give identical \( b' - V' \) values as in Ref. [12] as the dispersion Eq. (9) is the generalised form of Eq. (18) in Ref. [12].

4. Conclusion

The effective index method is applied to the new modified WKB equation to obtain simplified universal dispersion curves for graded index channel guides. The results are obtained for different index profiles in the depth and width direction for various dimensions of the guide. The analysis is general and can be suitably modified to include anisotropic diffusion, polarisation effects, arbitrary index profiles including buried guides.

References