Propagation characteristics of novel gain-guided segmented planar waveguides

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Abstract

We propose a new class of segmented waveguides, namely gain-guided periodic segmented waveguides (GG-PSW), in which the guidance is brought about by a periodic linear array of gain guiding regions embedded in a homogeneous medium. Corresponding to a planar GG-PSW with segments having parabolic gain variation in the transverse direction, we have obtained the fundamental guided mode parameters using the complex ABCD matrix formulation for Gaussian beam propagation. The dependence of the effective index, spot size and gain coefficient of the mode on the duty cycle of segmentation and the gain parameter of the segments is presented for practical values corresponding to InGaAsP. Such waveguides are expected to find applications in the design of novel waveguide devices in semiconducting substrates.

1. Introduction

Segmented waveguides consisting of a periodic linear array of high refractive index regions embedded in a lower refractive index surrounding have recently attracted considerable attention for applications in both nonlinear [1,2] and linear [3,4] devices. These waveguides differ from the conventional waveguides in having a periodic linear array of high refractive index regions along the propagation direction which are responsible for waveguidance (see Fig. 1). In these structures, domain inversion and waveguidance can be achieved simultaneously, making them attractive for nonlinear interactions employing quasi phase matched (QPM) schemes. Segmented waveguides are also interesting for applications to linear waveguide devices, since the effective index and spot size of the guided mode can be tailored by appropriately choosing the duty cycle of segmentation; this can be used in the efficient design of waveguide devices such as mode expanders, polarization converters and Bragg reflectors [3]. There have recently been some studies on the modelling of linear characteristics of step index [4–6] and parabolic index [7] segmented waveguides, and some experimental studies on the characterization of segmented waveguides in LiNbO₃ [8–10] and KTP [11].

It is well known that waveguidance in a medium can be achieved by either having a suitable transverse refractive index profile, or by having a transverse variation in gain or loss (gain decreasing, or loss increasing away from the waveguide axis). Gain guidance is known to be very important from the point of view of semiconductor lasers [12] or even in other solid state lasers [13]. Segmented waveguides studied till now are index guided structures, and may be referred to as index-guided periodic segmented waveguides (IG-PSW). In this paper we propose a new class of segmented waveguides namely gain-guided periodic segmented waveguides (GG-PSW), in which wave-
guidance is brought about by a periodic linear array of gain guiding regions embedded in a homogeneous medium. By considering the specific example of a GG-PSW comprising of segments with parabolically varying gain in the transverse (x) direction (see Fig. 2). A is the period of segmentation, and the non-gain segments are of length \( A - d \). The complex refractive index profile of such a waveguide may be written in the form

\[
n(x, z) = n_1(1 - \alpha^2 x^2/2), \quad 0 < z < d,
\]

\[
= n_2, \quad d < z < A,
\]

where

\[
n_1 = n_1 + in_{11}, \quad \alpha^2 = 2in_{11}/(n_1a^2),
\]

\[
n(x, z + A) = n(x, z).
\]

In Eq. (2), \( n_1 \) and \( n_{11} \) are the real and imaginary parts of refractive index on the axis \((x = 0)\) and \( a \) is a constant characterizing the rate of decrease of gain away from the axis. When \( n_2 = n_1 + n_{11} \), the above refractive index profile corresponds to an infinite medium having periodic gain segments along the z-direction. Waveguidance in this case would then be purely due to gain-guiding mechanism.

Since segmented waveguide is not uniform along the propagation direction, the conventional definition of a mode may not be applied. We may however define the mode of a segmented waveguide as that transverse field distribution which repeats after every period of segmentation; the reduction in amplitude of the field will correspond to the propagation loss, while the phase change with propagation determines the effective index.
of the mode. Such a definition is very similar to the
definition of a mode in a lens waveguide [14]. Since a
Gaussian beam forms a mode of the gain guided uni-
form (nonsegmented) waveguide with a parabolic gain
variation in the transverse direction [14], and since a
Gaussian beam propagates as a Gaussian in a homo-
genous medium, we expect the fundamental mode of
a GG-PSW to be a Gaussian of the form

\[ f(x, z) = P \exp \left( -i \frac{k_0 x^2}{2q(z)} \right), \]

where \( P \) is an arbitrary constant, \( k_0 \) is the free space
propagation constant and

\[ \frac{1}{q(z)} = \frac{n(z)}{q(z)} = \frac{1}{\hat{R}(z)} = -i \frac{\lambda_0}{\pi w^2(z)}. \]

Here \( q(z) \) is the complex beam parameter and \( \hat{q}(z) \) is
the reduced complex beam parameter (see, e.g. [15]),
\( w(z) \) represents the spot size and \( \hat{R}(z) \) is the reduced
radius of curvature of the Gaussian beam at any \( z \). The
quantity \( R(z) = n(z) \hat{R}(z) \) can be interpreted as the real
radius of curvature of the phase front where \( n(z) \) is
the real part of the refractive index at any \( z \) [15].
Now, to be a mode of the structure in Fig. 2, the field dis-
bution \( f(x, z) \) should reproduce itself after propagating
through one period of segmentation \( A \) [7]. Thus for a
mode, we must have

\[ q(z + A) = q(z). \]

In order to obtain the modal characteristics, we use
the \( ABCD \) matrix formulation for Gaussian beam prop-
gation, according to which if an optical system is rep-
resented by a \( 2 \times 2 \) transfer matrix known as \( ABCD \)
matrix, then the output and the input complex beam
parameters \( q_o \) and \( q_i \), respectively, are related through
(see, e.g. [14])

\[ q_o = \frac{Aq_i + B}{Cq_i + D} \]

and the corresponding reduced complex beam param-
eters are given by

\[ \hat{q}_o = \frac{\hat{q}_i q_o}{n_o}, \quad \hat{q}_i = \frac{\hat{q}_i}{n_i}. \]

For a medium with gain, the \( ABCD \) matrix is complex
but the known \( ABCD \) rules remain valid [15]. If \( q_i \) and
\( q_o \) represent the complex beam parameters at \( z = 0 \) and
at \( z = A \) in a GG-PSW represented by Eq. (1) (see Fig.
2), then the corresponding complex \( ABCD \) matrix is
given by

\[ \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & n_2/n_1 \end{pmatrix} \begin{pmatrix} 1 & A-d \\ 0 & 1 \end{pmatrix} \]

\[ \times \begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix} \begin{pmatrix} \cos \alpha d & (\sin \alpha d)/\alpha \\ -\alpha \sin \alpha d & \cos \alpha d \end{pmatrix} \]

\[ = \begin{pmatrix} \cos \alpha d - \frac{n_1}{n_2} \alpha (A-d) \sin \alpha d & \sin \alpha d + \frac{n_1}{n_2} (A-d) \cos \alpha d \\ \alpha \sin \alpha d & \cos \alpha d \end{pmatrix}. \]

In the above equation, the individual matrices corre-
spond (in reverse order) to propagation in a parabolic
gain medium, refraction at the interface (between the
guiding and nonguiding segments), propagation
through a homogeneous medium of thickness \( (A-d) \),
and refraction at the interface between the segments.

For a mode \( q_o = q_i \); using this condition in Eq. (7)
and solving for \( q_i \), we obtain

\[ \frac{1}{q_i} = -\frac{A-D}{2B} + \frac{1}{B} \sqrt{\left(\frac{(A+D)^2}{4} - 1\right)}, \]

where we have used the condition \( AD-BC=1 \). In
writing Eq. (10) we have chosen only the positive
square root so as to obtain a real value of \( w \).

For a given optical system (in this case GG-PSW),
one can evaluate \( A, B, C \) and \( D \), and thus obtain the
complex beam parameter \( q \) of the fundamental Gaus-
sian mode. Knowing \( q \), one can obtain the radius of
curvature \( R(z) \) and the spot size \( w(z) \) from the follow-
ing equations:

\[ R(z) = n(z)(1/\hat{q}_i)^{-1}, \]

\[ w(z) = \left( -\frac{\lambda_0}{\pi \text{Im}(1/\hat{q}_i)} \right)^{1/2}. \]

From Eq. (12) we see that if \( \text{Im}(1/\hat{q}_i) > 0 \), \( w \)
becomes imaginary, and hence in such a case the seg-
mented structure does not support any confined mode.
Thus, like in the case of IG-PSW [7], GG-PSW would
also show regions of instability depending on various
waveguide parameters such as period and the gain coef-
ficient of the gain segments.
3. Calculation of effective index and gain

The above formulation gives the complex beam parameter of the fundamental mode. In order to obtain the effective index and the gain, we need to actually propagate the modal field distribution over one period and then obtain the change in amplitude and phase of the Gaussian beam. To do this, we let \( f(x) \) be the field on the plane \( z = 0 \) (see Fig. 2):

\[
f(x) = \exp\left(-i\frac{k_0 x^2}{2\hat{q}_1}\right),
\]

(13)

where we arbitrarily assume a unit amplitude and \( \hat{q}_1 \) is the reduced complex beam parameter of the fundamental mode.

For an input field given by Eq. (13) on the plane \( z = 0 \), the field after propagation through one period can be written as (see Appendix A)

\[
u(x) = u_0 \exp\left(-i\frac{k_0 x^2}{2\hat{q}_5}\right),
\]

where \( u_0 \) and \( \hat{q}_5 \) are given by Eq. (A.12). Since the input Gaussian beam was assumed to have unit amplitude, we obtain the amplitude after propagation through one period as

\[
\Gamma = \text{Abs}[u_0]
\]

(14)

and phase change in one period as

\[
\phi = -\tan^{-1}\left(\frac{\text{Im}[u_0]}{\text{Re}[u_0]}\right).
\]

(15)

If \( n_{\text{eff}} \) represents the effective index of the mode, then

\[
n_{\text{eff}} = \frac{\phi}{k_0/\Lambda}.
\]

(16)

Similarly, if \( \sigma \) represents the power gain coefficient of the mode, then

\[
\sigma = \frac{2}{\Lambda} \ln \Gamma.
\]

(17)

4. Equivalent waveguide model

An exact analysis of segmented waveguides is complicated due to sharp transitions in the refractive index along the direction of propagation. The study of segmented waveguides becomes simpler by using the equivalent waveguide model. Equivalent \( z \)-invariant waveguides have been defined earlier for step index [5] as well as parabolic index [16] segmented waveguides. In a similar fashion, for a given GG-PSW we may define an equivalent \( z \)-invariant waveguide by the following equation:

\[
n_{\text{eq}}(x) = n(x) + n_2(1 - \gamma),
\]

(18)

where \( n_{\text{eq}} \) is the refractive index profile of the equivalent waveguide and \( \gamma \) is the duty cycle of segmentation \((\gamma = d/\Lambda)\). Eq. (18) represents weighted average of the refractive indices along the \( z \)-direction, for a given value of \( x \). For the case of a GG-PSW defined by Eq. (1), we get

\[
n_{\text{eq}}(x) = n_{\text{eq},1}(1 - \frac{1}{2} \alpha_{\text{eq}}^2 x^2),
\]

(19)

where

\[
n_{\text{eq},1} = n_1 \gamma + n_2(1 - \gamma), \quad \alpha_{\text{eq}}^2 = \frac{n_1 \gamma \alpha^2}{n_{\text{eq},1}}.
\]

(20)

The complex beam parameter \( q_{\text{eq}} \) of the Gaussian mode of a gain-guided waveguide with a profile given by Eq. (19) is [14]

\[
\frac{1}{q_{\text{eq}}} = i\alpha_{\text{eq}}, \quad \frac{1}{\hat{q}_{\text{eq}}} = \text{Re}[1/n_{\text{eq},1}],
\]

(21)

from which one can obtain the spot size and the radius of curvature of the phase fronts:

\[
w_{\text{eq}} = \sqrt{-\frac{\lambda_0}{\pi \text{Im}[1/\hat{q}_{\text{eq}}]}}, \quad R = n_{\text{eq},1} \text{Re}[1/\hat{q}_{\text{eq}}]^{-1},
\]

(22)

where \( n_{\text{eq},1} \) is the real part of \( n_{\text{eq},1} \). The corresponding complex effective index is given by (see, e.g. [17])

\[
n_{\text{eff},\text{eq}} = n_{\text{eq},1}\left(1 - \frac{\alpha_{\text{eq}}}{k_0 n_{\text{eq},1}}\right)^{1/2}.
\]

(23)

The real part of which gives the effective index of the mode and the imaginary part gives the gain coefficient.

5. Results and discussion

We consider a GG-PSW with the following values of various parameters
Fig. 3. Variation of $n_{\text{eff}}$ and gain coefficient of the fundamental mode with duty cycle.

Fig. 4. Variation of the beam spot size and the radius of curvature of the phase front of the Gaussian mode as a function of the propagation distance $z$ for a duty cycle of 0.3. We see that the spot size of the mode decreases as the mode propagates in the gain medium, the phase front always remains convex, representing the flow of power radially outwards away from the axis, and the radius of curvature in the gain medium first increases and then decreases after attaining a maximum value. When the mode reaches the homogeneous medium (with no gain), since the phase front is convex, the beam continues to diverge, and the spot size increases. Fig. 3 also shows the variation of gain with duty cycle. The behavior shown in Fig. 3 follows from the fact that as the duty cycle of segmentation is increased the mode has to travel a greater distance in the gain medium and hence experiences more gain.

We thus see that the spot size and the effective index of the mode can be controlled by choosing an appropriate duty cycle. Once the waveguide is made, the duty

$\lambda_0 = 1.55 \ \mu\text{m}, \ A = 5 \ \mu\text{m},$

$n_1 = 3.38 + 0.005i, \ n_2 = 3.38, \ a = 15 \ \mu\text{m}. \quad (24)$

The above parameters correspond to typical values for gain guiding structures with current injection in InGaAsP substrates. Fig. 3 shows the variation of the effective index of the fundamental mode as a function of the duty cycle of segmentation. We note that unlike the case of IG-PSW, in the case of GG-PSW, the effective index of the mode decreases with increasing duty cycle. As the duty cycle of segmentation is increased the mode sees more gain medium and the confinement of the beam increases, which results in the decrease of spot size as shown in Fig. 5. If the spot size of the beam is smaller, the phase change in a given length is also smaller which leads to a decrease in the effective index. This is also consistent with the fact that in the equivalent waveguide model, $\alpha_{\text{eq}}$ increases with the increase in duty cycle, leading to a decrease in the effective index of the mode. Fig. 4 shows the variation of the spot size and the radius of curvature of the phase front of the Gaussian mode as a function of the propagation distance $z$ for a duty cycle of 0.3. We see that the spot size of the mode decreases as the mode propagates in the gain medium, the phase front always remains convex, representing the flow of power radially outwards away from the axis, and the radius of curvature in the gain medium first increases and then decreases after attaining a maximum value. When the mode reaches the homogeneous medium (with no gain), since the phase front is convex, the beam continues to diverge, and the spot size increases. Fig. 3 also shows the variation of gain with duty cycle. The behavior shown in Fig. 3 follows from the fact that as the duty cycle of segmentation is increased the mode has to travel a greater distance in the gain medium and hence experiences more gain.

We thus see that the spot size and the effective index of the mode can be controlled by choosing an appropriate duty cycle. Once the waveguide is made, the duty

Fig. 6. Variation of $n_{\text{eff}}$ and gain coefficient of the fundamental mode with $n_{1}$. 
cycle of segmentation cannot be changed; however the value of $n_{1i}$ can be altered by just varying the injection current. Figs. 6 and 7 show the variation of effective index and spot size of the mode as a function of $n_{1i}$. Increase in the value of $n_{1i}$ leads to more gain and hence more confinement of the beam. Due to a greater confinement of the beam the spot size decreases as the value of $n_{1i}$ is increased, which leads to a decrease in the effective index of the mode.

Table 1 shows a comparison of effective index, gain and spot size of the Gaussian mode of a GG-PSW with that obtained from the equivalent waveguide considerations discussed above for various duty cycles. It is evident from the table that the equivalent waveguide model works very well for the case of GG-PSW just as for step-index and parabolic-index segmented waveguides [5,16].

As mentioned earlier, GG-PSW also exhibit regions of instability [7]. For example, we have verified that for a GG-PSW with values given by Eq. (24), for a duty cycle of 0.8, the guide becomes unstable for periods lying approximately between 700 and 2100 μm. Increasing the value of $n_{1i}$ results in unstable regions even for smaller periods.

Gain-guided segmented waveguides may be fabricated in semiconductor materials such as GaAs and InGaAsP by using, for example, a finger electrode pattern for current injection to create periodically varying gain regions. By appropriately choosing the period and the duty cycle of segmentation, various parameters of the waveguide mode such as effective index, gain and spot size can be easily varied. The dependence of spot size on duty cycle of segmentation can be used for the realization of tapered transitions in such materials by simply designing an appropriate electrode pattern. We can also envisage using such waveguides in the design and fabrication of waveguide devices with gain by a single step diffusion or ion-exchange process in dielectric substrates such as glass and LiNbO₃.

6. Conclusions

In this paper we have proposed a new class of periodic segmented waveguides based on gain guiding. We have shown that for a GG-PSW with parabolically varying gain segments, the fundamental mode has a Gaussian field distribution. Dependence of various modal parameters such as effective index, spot size, gain and radius of curvature on waveguide parameters have been obtained. We have also observed the existence of unstable regions of operation in such waveguides. Such GG-PSW are expected to find applications in the design and fabrication of novel waveguide devices in semiconductor substrates. Although a segmented waveguide with infinitely extended segments in the transverse direction is not a realistic structure, the simple analysis provides a good physical insight of propagation in segmented waveguides, and may be used to study, with little modification, propagation through buried graded index segmented waveguides.

### Table 1

<table>
<thead>
<tr>
<th>Duty cycle</th>
<th>Segmented waveguide</th>
<th>Equivalent waveguide</th>
</tr>
</thead>
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<tr>
<td></td>
<td>$w_{0s} (\mu m)$</td>
<td>$n_{eff}$</td>
</tr>
<tr>
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<td>3.379700</td>
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<tr>
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<tr>
<td>0.1</td>
<td>13.442</td>
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</tr>
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</table>
Acknowledgements

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Appendix A

For an input field given by Eq. (13) on the plane \( z = 0 \), under paraxial approximation the field at \( z = d \) is given by

\[
g(x) = \frac{\delta}{\sqrt{-2i \sin \alpha d}} \int_{-\infty}^{\infty} \exp \left( -i \frac{k_0 x^2}{2q_1} \right) \exp(-ik_1d) \times \exp \left[ i \frac{\delta^2}{\sin \alpha d} xx' - i \frac{\delta^2}{2} (x^2 + x'^2) \cot \alpha d \right] dx'
\]

\[
= \frac{\delta}{\sqrt{-2i \sin \alpha d}} \exp(-ik_1d) 
\times \exp \left( -i \frac{\delta^2}{2} x^2 \cot \alpha d \right) I_1,
\]

where \( \delta = k_1 \pi \alpha \).

\[
I_1 = \int_{-\infty}^{\infty} \exp \left[ -x^2 \left( \frac{ik_1}{2q_1} + i \frac{\delta^2}{2} \cot \alpha d \right) \right]
+ x' \left( - \frac{i \delta^2 x'}{\sin \alpha d} \right) dx'
\]

\[
= \left( \frac{\pi}{ik_1/2q_1 + i(\delta^2/2) \cot \alpha d} \right)^{1/2} \times \exp \left( - \frac{\delta^4 x'^2}{4 \sin^2 \alpha d(ik_1/2q_1 + i(\delta^2/2) \cot \alpha d)} \right).
\]

(A.2)

Thus

\[
g(x) = g_0 \exp \left( -i \frac{k_0 x^2}{2q_2} \right),
\]

(A.3)

where

\[
g_0 = \frac{\delta}{\sqrt{-2i \sin \alpha d}} \left( \frac{\pi}{ik_1/2q_1 + i(\delta^2/2) \cot \alpha d} \right)^{1/2} 
\times \exp(-ik_1d),
\]

(A.4)

\[
\frac{1}{q_2} = \frac{2}{k_0} \left( \frac{\delta^2}{2} \cot \alpha d - \frac{\delta^4/4 \sin^2 \alpha d}{k_1/2q_1 + (\delta^2/2) \cot \alpha d} \right).
\]

(A.5)

The field after refraction at the interface at \( z = d \) is given by

\[
h(x) = n_0 \exp \left( -i \frac{k_0 x^2}{2q_3} \right),
\]

where

\[
q_3 = q_2.
\]

(A.6)

From \( z = d \) to \( z = A \), the field undergoes diffraction. Using the well known diffraction formula for Gaussian beam, we obtain the field at \( z = A \) as

\[
u(x) = \nu_0 \exp \left( -i \frac{k_0 x^2}{2q_4} \right),
\]

where

\[
\nu_0 = \frac{\delta_0}{\sqrt{\lambda_2(A-d)}} \left( \frac{i}{2} \left( \frac{1}{A-d} + \frac{1}{q_3} \right) \right)^{1/2} \times \exp \left[ -ik_2(A-d) \right].
\]

(A.7)

\[
\frac{1}{q_4} = \frac{1}{n_2} \left( \frac{1}{q_3 \left( A-d \right)} \right) \left( \frac{1}{A-d} \right)^{-1}.
\]

(A.8)

Hence the field after one complete period is

\[
u(x) = \nu_0 \exp \left( -i \frac{k_0 x^2}{2q_5} \right),
\]

where

\[
u_0 = \nu_0, \quad q_5 = q_4.
\]

(A.9)

References