A class of inhomogeneous shear models for seismic analysis of landfills

Venkata Ramana Gunturi\textsuperscript{a} & Ahmed-W. M. Elgamal\textsuperscript{b}

\textsuperscript{a}Department of Civil Engineering, IIT, New Delhi, India
\textsuperscript{b}Department of Applied Mechanics and Engineering Sciences, University of California at San Diego, La Jolla, CA 92093, USA

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An analytical numerical procedure is developed to calculate the shear elasto-plastic earthquake response of hill-shaped landfills. Landfill response is evaluated on the basis of newly developed one-dimensional inhomogeneous shear beam models. In these models, the nonhomogeneity of landfill materials is taken into account by assuming a specific variation of stiffness properties along the depth. Closed-form analytical expressions are derived for natural frequencies, modal displacements, modal participation factors, absolute accelerations and maximum shear strains. Parametric results are presented in graphical and tabular forms and conclusions are drawn. Within a numerical implementation framework of the above formulation, the landfill materials may be modeled by an elasto-plastic hysteretic model following the principles of flow or incremental plasticity. The entire numerical procedure may be executed on a personal computer and consequently qualifies as a versatile tool for conducting preliminary design calculations or parametric-type investigations. Using this newly developed procedure, the seismic response of the Fresh Kills landfill in New York City is investigated.

\textit{Key words}: landfill, elasto-plastic earthquake response, seismic, earthquake, dynamic.

1 INTRODUCTION

Landfills are large three-dimensional structures composed of a mixture of highly non-homogeneous domestic trash and demolition debris. Within a framework of simplified analysis, these structures are often represented by a one-dimensional (1D) shear beam (SB) model.\textsuperscript{1–4} Historically, such a shear beam model has been quite useful for calculation of dynamic site response characteristics\textsuperscript{5–8} or for analysis of earth masses of vast lateral extent (with approximately constant lateral cross-sectional area).

For earth dams, similar useful models were developed in which the dam upstream-downstream cross-section was more appropriately represented by a shear wedge configuration.\textsuperscript{9–17} A comprehensive class of nonhomogeneous shear wedge models was developed\textsuperscript{18–20} in which: (1) the presence of a flat surface plateau along the crest was represented (in the form of a wedge truncation ratio parameter); and (2) the possible increase in material stiffness with depth was incorporated. In the study reported herein, the Dakoulas and Gazetas procedures\textsuperscript{18–20} were followed to develop a similar class of nonhomogeneous shear models for dynamic analysis of landfills. In this new class of models, the landfill is viewed as a hill or structure of conical cross-section (of arbitrary lateral cross-sectional geometry). A generalized basic formulation for this class of shear beam models was presented in Shahinpoor \textit{et al.}\textsuperscript{21–22}

In the following sections, the governing equations are presented and solved for situations of free and forced vibrations. Thereafter, the seismic response of a representative section of the Fresh Kills landfill in New York City is investigated. For this case history, the newly developed landfill model and the standard shear beam model predictions are contrasted.

2 NONHOMOGENEOUS SB MODELS: LATERAL FREE VIBRATION

The governing free-vibration equation of motion may be derived by considering the dynamic equilibrium in an
Fig. 1. Landfill cross-section and distribution of shear modulus with depth.

The infinitesimal body of arbitrary lateral cross-section (Fig. 1):

\[
\frac{\partial^2 u}{\partial y^2} = \frac{1}{y^2} \frac{\partial (y^2 \tau_{yy})}{\partial y}
\]  \hspace{1cm} (1)

with the boundary conditions \(u(H, t) = 0\) and \(\tau(h, t) = 0\). In the above, \(\rho\) is mass density of landfill materials, \(\tau\) is lateral shear stress, \(u\) is displacement in direction of \(x\)-axis, and \(H\) is untruncated height of shear model. Evidence from in situ measurements of shear wave velocities has shown that shear wave velocity increases with depth as shown in Figs 1 and 2. Herein, a simple approach was adopted in allowing for variation of material stiffness with depth (or confinement). In this approach, linear (elastic, low-strain) shear stress–strain response was expressed by \(\tau = G \sigma_{yy}\), in which \(G\) is linear modulus, \(\sigma_{yy}\) is shear stress, and

\[
G = G_0 \left(\frac{y}{H}\right)^B, \quad 0 \leq B \leq 1
\]  \hspace{1cm} (2)

The above relation, despite its limitation, may be adopted to account for many situations of practical interest.\(^{18,25}\) It is noted here that the change in \(G\) (eqn (2)) may be defined so as to account implicitly for variation of \(\rho\) (along the depth) and result in a desired shear wave velocity profile along the height. This may be important for landfills, as the density of landfill materials may increase significantly with depth.\(^{4,24}\)

Substituting the above relation (eqn (2)) into eqn (1) results in

\[
\rho \frac{\partial^2 u}{\partial t^2} - \frac{G_0}{H} \left(\frac{y}{H}\right)^B \frac{\partial^2 u}{\partial y^2} - \frac{G_0 (B-1)}{H^B} \frac{\partial u}{\partial y} - 2G_0 u \left(\frac{y}{H}\right)^B = 0
\]  \hspace{1cm} (3)

which is the differential equation governing landfill free vibration.

Let the circular frequency and mode of natural vibration be \(\omega\) and \(U(y)\) respectively. Setting

\[
u(y, t) = U(y)e^{i\omega t}
\]  \hspace{1cm} (4)

and substituting into eqn (3) leads to

\[
y^2 \frac{\partial^2 U}{\partial y^2} + (B + 2)y \frac{\partial U}{\partial y} + k^2 y^2 - B U = 0
\]  \hspace{1cm} (5)

where

\[
k^2 = \frac{\rho \omega^2 H^B}{G_0}
\]  \hspace{1cm} (6)

eqn (5) is a Bessel equation\(^{26}\) and its solution is:

\[
U(y) = y^{-\frac{B+1}{2}} \left[ C_1 J_p \left(\frac{2}{2-B} ky \frac{2-B}{2} \right) - C_2 Y_p \left(\frac{2}{2-B} ky \frac{2-B}{2} \right) \right]
\]  \hspace{1cm} (7)

where \(C_1\) and \(C_2\) are arbitrary constants, \(J_p\) and \(Y_p\) are Bessel functions of the first and second kind respectively, and of order

\[
p = \frac{B+1}{2-B}
\]

Applying the boundary requirements of zero relative displacement at the base and zero shear stress at the crest yields:

\[
H \left(\frac{B+1}{2} \right) \left[ C_1 J_p \left(\frac{2k}{2-B} H^{1-B/2} \right) - C_2 Y_p \left(\frac{2k}{2-B} H^{1-B/2} \right) \right] = 0
\]  \hspace{1cm} (8)

and

\[
- \frac{G_0 k}{H^B \sqrt{h}} \left[ C_1 J_{p+1} \left(\frac{2k}{2-B} H^{1-B/2} \right) + C_2 Y_{p+1} \left(\frac{2k}{2-B} H^{1-B/2} \right) \right] = 0
\]  \hspace{1cm} (9)

Fig. 2. Shear wave velocity of municipal solid waste (after Kavazanjian et al.\(^{25}\)).
Table 1. Fundamental characteristic root (a₁)

<table>
<thead>
<tr>
<th>λ</th>
<th>0.00</th>
<th>0.33</th>
<th>0.49</th>
<th>0.67</th>
<th>0.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>3.141</td>
<td>3.554</td>
<td>3.813</td>
<td>4.172</td>
<td>4.493</td>
</tr>
<tr>
<td>0.1</td>
<td>3.151</td>
<td>3.569</td>
<td>3.832</td>
<td>4.197</td>
<td>4.525</td>
</tr>
<tr>
<td>0.2</td>
<td>3.213</td>
<td>3.655</td>
<td>3.934</td>
<td>4.326</td>
<td>4.684</td>
</tr>
<tr>
<td>0.3</td>
<td>3.360</td>
<td>3.847</td>
<td>4.158</td>
<td>4.595</td>
<td>4.991</td>
</tr>
<tr>
<td>0.4</td>
<td>3.624</td>
<td>4.180</td>
<td>4.556</td>
<td>5.038</td>
<td>5.494</td>
</tr>
<tr>
<td>0.5</td>
<td>4.057</td>
<td>4.715</td>
<td>5.136</td>
<td>5.731</td>
<td>6.271</td>
</tr>
<tr>
<td>0.6</td>
<td>4.767</td>
<td>5.579</td>
<td>6.099</td>
<td>6.832</td>
<td>7.502</td>
</tr>
<tr>
<td>0.7</td>
<td>6.013</td>
<td>7.082</td>
<td>7.769</td>
<td>8.738</td>
<td>9.619</td>
</tr>
<tr>
<td>0.8</td>
<td>8.557</td>
<td>10.163</td>
<td>11.182</td>
<td>12.620</td>
<td>13.928</td>
</tr>
</tbody>
</table>

Using eqns (8) and (9) to eliminate C₁ and C₂, the following ‘characteristic’ relation is obtained:

\[ J_{p+1}(a\lambda^{1-B})Y_p(\xi(a\lambda^{1-B})) - Y_{p+1}(a\lambda^{1-B})J_p(\xi(a\lambda^{1-B})) = 0 \]  

in which

\[ a = \frac{2}{2-B} \sqrt{\frac{\rho o^2 H^2}{G_0}} \]

and \( \lambda = h/H \) is the truncation ratio. The characteristic equation (eqn (10)) has a discretely infinite number of roots, \( a_n = \alpha_n(\lambda) \), \( n = 1, 2, ... \), which may be computed numerically. Substituting each of the \( a_n \) values into eqn (7) leads to the following expression for displacement at the \( n \)th vibration mode:

\[ U_n(\xi) = \xi^{-(B+1)/2}N_p(\alpha_n\xi^{1-B}) \]

where \( N_p(\cdot) \) denotes the cylinder function.\(^{27}\)

\[ N_p(\cdot) = V_p(\alpha_n\xi^{1-B}) - J_p(\alpha_n\xi^{1-B}) \]

and \( \xi = y/H \).

2.1 Natural frequencies and displacement shapes

The \( n \)th natural frequency and period of the landfill, expressed in terms of shear wave velocity at the base (Fig. 1), are given by

\[ \omega_n = \frac{a_n(2-B)V_0}{2H} \]

\[ T_n = \frac{4\pi H}{a_n(2-B)V_0} \]

where \( V_0 \) is shear wave velocity at the base. The values of \( a_n \) depend on the inhomogeneity parameter \( B \), for each truncation ratio \( \lambda = h/H \). Table 1 presents the values of fundamental characteristic root for different values of truncation ratio and for five values of the inhomogeneity parameter \( B = 0.0, 0.33, 0.49, 0.67, 0.80 \); and Table 2 presents the characteristic roots for the next four modes as a ratio of the fundamental characteristic root. In the above, the value of \( B = 0.49 \) was used, as the case of \( B = 0.50 \) causes numerical difficulties in the algorithm employed (when calculating the Bessel function of second kind, \( Y_1 \)).

If desired, \( \omega_n \) and \( T_n \) can be rewritten in terms of an average shear wave velocity, \( V_s \), of the landfill.\(^{16-19}\)

Table 2. Ratio of roots (\( a_{n}, n = 2, 3, 4, 5 \)) referenced to that of fundamental root (\( a_{1} \))

<table>
<thead>
<tr>
<th>( B )</th>
<th>( a_{1} )</th>
<th>( a_{0.0} )</th>
<th>( a_{0.1} )</th>
<th>( a_{0.2} )</th>
<th>( a_{0.3} )</th>
<th>( a_{0.4} )</th>
<th>( a_{0.5} )</th>
<th>( a_{0.6} )</th>
<th>( a_{0.7} )</th>
<th>( a_{0.8} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>2.000</td>
<td>2.015</td>
<td>2.082</td>
<td>2.184</td>
<td>2.300</td>
<td>2.421</td>
<td>2.542</td>
<td>2.661</td>
<td>2.777</td>
<td></td>
</tr>
<tr>
<td>0.33</td>
<td>1.890</td>
<td>1.914</td>
<td>2.001</td>
<td>2.120</td>
<td>2.348</td>
<td>2.379</td>
<td>2.509</td>
<td>2.636</td>
<td>2.760</td>
<td></td>
</tr>
<tr>
<td>0.49</td>
<td>1.834</td>
<td>1.864</td>
<td>1.962</td>
<td>2.089</td>
<td>2.224</td>
<td>2.359</td>
<td>2.493</td>
<td>2.624</td>
<td>2.752</td>
<td></td>
</tr>
<tr>
<td>0.67</td>
<td>1.768</td>
<td>1.808</td>
<td>1.920</td>
<td>2.056</td>
<td>2.197</td>
<td>2.338</td>
<td>2.475</td>
<td>2.610</td>
<td>2.743</td>
<td></td>
</tr>
<tr>
<td>0.80</td>
<td>1.719</td>
<td>1.767</td>
<td>1.890</td>
<td>2.033</td>
<td>2.178</td>
<td>2.322</td>
<td>2.463</td>
<td>2.601</td>
<td>2.736</td>
<td></td>
</tr>
</tbody>
</table>

\[ J_{p+1}(a\lambda^{1-B})Y_p(\xi(a\lambda^{1-B})) - Y_{p+1}(a\lambda^{1-B})J_p(\xi(a\lambda^{1-B})) = 0 \]
Table 3. Analytical results for landfills with $G = G_0(y/H)^3$ and $\lambda = 0$

<table>
<thead>
<tr>
<th>Inhomogeneity factor $B$</th>
<th>$\mathcal{T}_i(V_i/H)$</th>
<th>$\omega_n(H/V_0)$</th>
<th>$n$th mode displacement shape, $U_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$ (general expression)</td>
<td>$\frac{24\pi}{a_c(2-B)(6+B)}$</td>
<td>$\frac{a_c(2-B)(6+B)}{12}$</td>
<td>$r^{-\frac{B+1}{2}} J_{\frac{B+1}{2}}(a_c r^{-\frac{B}{2}})$</td>
</tr>
<tr>
<td>0.00</td>
<td>2.000</td>
<td>$a_c(0)$</td>
<td>$r^{-\frac{B}{2}} J_{\frac{B}{2}}(a_c r^{-\frac{B}{4}})$</td>
</tr>
<tr>
<td>0.33</td>
<td>2.009</td>
<td>0.879$a_c(0.33)$</td>
<td>$r^{-\frac{5}{2}} J_{\frac{5}{2}}(a_c (0.33)r^{-\frac{5}{4}})$</td>
</tr>
<tr>
<td>0.49</td>
<td>2.028</td>
<td>0.812$a_c(0.49)$</td>
<td>$r^{-\frac{3}{2}} J_{\frac{3}{2}}(a_c (0.49)r^{-\frac{3}{4}})$</td>
</tr>
<tr>
<td>0.67</td>
<td>2.033</td>
<td>0.741$a_c(0.67)$</td>
<td>$r^{-\frac{1}{2}} J_{\frac{1}{2}}(a_c (0.67)r^{-\frac{1}{4}})$</td>
</tr>
<tr>
<td>0.80</td>
<td>2.056</td>
<td>0.680$a_c(0.80)$</td>
<td>$r^{-\frac{3}{2}} J_{\frac{3}{2}}(a_c (0.80)r^{-\frac{3}{4}})$</td>
</tr>
</tbody>
</table>

along with expressions for the modal displacement shape (for $\lambda = 0$). Specific formulae deduced from the general expressions for different values of the inhomogeneity parameter $B$ are also shown in Table 3. The effect of inhomogeneity on dynamic landfill characteristics may be observed also in Fig 3(a) which compares natural periods $T_n$ of four inhomogeneous landfills with the corresponding periods $T_{ni}$.

\[ V_i = \frac{\int_0^H V(x) x^2 dx}{\int_0^H x^2 dx} = \frac{6}{6+B} \frac{1-\lambda^{1+B/2}}{1-\lambda} V_0 \] (15)

The corresponding expressions for the fundamental period $T_1$ and the $n$th natural frequency $\omega_n$ are shown in Table 3.

Fig. 3. Effect of inhomogeneity on ratio of: (a) natural periods of the inhomogeneous and homogeneous landfills; (b) $n$th natural period to the fundamental period.

Fig. 4. Displacement modal shapes for five values of inhomogeneity parameter $B$ ($\lambda = 0$).
of a homogeneous landfill of the same height and average shear-wave velocity (eqn (15)). As can be seen from Table 3 and Fig. 3(a) ($\lambda = 0$ case), the degree of inhomogeneity has a minor effect on the fundamental period but may affect the higher resonances appreciably. For instance, for $B = 0.67$, the fourth mode natural period is about 23% higher than for the homogeneous case. As the value of $\lambda$ increases (e.g., $\lambda = 0.5$), the effect of inhomogeneity on the natural periods appears to become negligible, especially for higher modes (Fig. 3(a)). In addition, it is evident from Fig. 3(b) that, as the value of $B$ increases, the consecutive natural frequencies get closer to each other.\textsuperscript{18-19}

Modal displacement shapes for different values of $B$ are shown in Fig. 4. The differences between mode shapes become negligible as $\lambda$ increases. In all cases, the relative motion at depth is seen to decrease with the increase in inhomogeneity parameter ($B$), as was also observed earlier in the response of inhomogeneous earth dams.\textsuperscript{13-14,19}

2.2 Response to seismic base excitation

In this case, motion is represented by:

$$\frac{1}{2} \frac{\partial (\gamma y^2)}{\partial y} = \rho (\ddot{y} + \ddot{u}_b)$$

(16)

where $y$ is displacement at depth $y$ relative to the base.

and $\ddot{u}_b$ is uniform base excitation in the $x$-direction (Fig. 1). The solution, $u(y, t)$ may be obtained as a summation of mode shapes in the form:\textsuperscript{18}

$$u(y, t) = \sum_{n=1}^{\infty} P_n U_n(y) D_n(t)$$

(17)

where $P_n$ is the participation factor of the $n$th mode,

$$P_n = \frac{\int_0^l \xi^2 U_n^2(\xi) d\xi}{\int_0^l \xi^2 U_n^2(\xi) d\xi}$$

(18)

and $D_n(t)$ is the response of a single degree of freedom system having frequency $\omega_n$ and viscous damping coefficient $\beta_n$ such that:

$$D_n(t) = \frac{1}{\omega_n^2} \int_0^l \ddot{u}_b(\tau) e^{-\beta_n \omega_n (t-\tau)} \sin(\omega_n (t-\tau)) d\tau$$

(19)

in which $\omega_n = \omega_n \sqrt{1 - \beta_n^2}$.

The relative potential impact of each mode on the crest motion may be studied through an expression for crest displacement modal participation factor\textsuperscript{18-19}, defined as

$$\Phi_n = \Phi_n (h) = P_n U_n (h)$$

(20)

The variation of $\Phi_n$ at the crest of the landfill is shown in
Fig. 7. Shear strain modal participation at the base versus truncation ratio and inhomogeneity parameter.

Fig. 5. For \( \lambda = B = 0 \), the value of \( \Phi_n \) at the crest remains constant at a value of 2 (independent of \( n \)), while higher values of \( B \) lead to \( \Phi_n \) values that increase with increasing mode number \( n \). As observed earlier for earth dams, the relative contribution of higher modes to the response will increase with increasing \( B \).

2.3 Seismic shear strains

Of great interest in assessing the safety of landfills is the determination of seismic shear strains, as these strains might lead to permanent displacements. The induced seismic shear strains can be obtained by modal superposition. By differentiating the displacement with respect to \( y \), seismic shear strain may be defined as

\[
\gamma_y(t) = \sum_{n=1}^{\infty} P_n \frac{\partial U_n}{\partial y} D_n(t) \tag{21}
\]

where

\[
\frac{\partial U_n}{\partial y} = \frac{1}{H} \left[ 2 - B \right] - \frac{\alpha_n \gamma^y}{2} \left( s + \frac{1}{2} \right) N_{n+1} \left( \alpha_n \gamma^{2y} - \frac{\gamma}{2} \right) \tag{22}
\]

As in the displacement case, a shear strain modal participation factor may be defined as

\[
\Psi_y(\xi) = P \frac{\partial U_y(\xi)}{\partial \xi} = P_n H \frac{\partial U_n(\xi)}{\partial y} \tag{23}
\]

Fig. 8. Conceptual configuration of hardening in 1D stress space (after Elgamal). Fig. 6 shows the variation of \( \Psi_y \) with depth. As in the case of earth dams, the value of \( \Psi_y \) increases significantly with the increase in inhomogeneity parameter (\( B \)) within the upper half of the landfill; and \( \Psi_y \) at the base of the landfill is nearly insensitive to variations in \( B \). The variation of \( \Psi_y \) at the base of the landfill is shown in Fig. 7. At the landfill base, \( \Psi_y \) is nearly constant for different values of \( B \), with a slight increase with the increase in truncation ratio \( \lambda \).

3 ELASTO-PLASTIC CONSTITUTIVE MODEL

A model based on the flow or incremental theory of plasticity may be used to represent the one-dimensional shear response of landfill materials. In particular, the model employed herein generates a hysteresis response (Masing-type behavior) under cyclic shear loading. The constitutive equation is written in the following form:

\[
\tau = C(\varepsilon - \varepsilon_p) \tag{24}
\]

in which \( \tau \) is rate of shear stress, \( \varepsilon \) is rate of shear strain, \( \varepsilon_p \) is plastic rate of shear strain, \( C = 2G \) and \( G \) is elastic shear modulus.

The plastic rate of shear deformation is defined by

\[
\varepsilon_p = Q(L) \tag{25}
\]

where \( Q \) is a scalar, the sign of which defines the direction of plastic deformation in stress space, \( L \) is the plastic loading function, and the symbol \( Q \) denotes the Macaulay bracket \( (L) = L \) if \( L > 0 \), otherwise \( (L) = 0 \). \( L \) is defined as

\[
L = \frac{1}{H} \tau \tag{26}
\]

in which \( H' \) is plastic shear modulus. Using eqns (25) and
(26), eqn (24) may be written as

$$\tau = C\dot{\varepsilon} - \left( \frac{Q\dot{C}\dot{\varepsilon}}{H' + CQ^2} \right) CQ$$  \hspace{1cm} (27)

3.1 Yield function

In one-dimensional (1D) shear stress space, the yield function simplifies to:

$$f = 3(\tau - \alpha)^2 - k^2 = 0$$ \hspace{1cm} (28)

in which $\alpha$ and $k$ define the yield surface (function) center and size respectively. The outer normal to the yield surface ($Q = q/\partial\tau$) is normalized such that $Q^2 = 1.0$. Based on the adopted yield function (eqn (28)), eqn (27) becomes

$$\tau = H\dot{\varepsilon}$$ \hspace{1 cm} (29)

in which

$$H = \left[ \frac{1}{2C} + \frac{1}{H'} \right]^{-1}$$ \hspace{1 cm} (30)

is the elasto-plastic modulus. In general, the variation of $H$ along the spatial domain may not necessarily follow the linear shear modulus variation given by eqn (2).

3.2 Hardening rule

Kinematic (i.e. yield surface translation) and isotropic (i.e. yield surface change in size) hardening will be undergone simultaneously. Hardening, however, will always satisfy the consistency condition $f = 0$ which ensures that the stress point (in stress space) always remains on the yield surface. The consistency condition may be expressed as

$$\frac{df}{\dot{\alpha}}(\tau - \alpha) + \frac{df}{\dot{k}}k = 0$$ \hspace{1cm} (31)

or, using eqns (26) and (28),

$$H'(L) = Q\dot{\alpha} + k\dot{\varepsilon} \sqrt{3}$$ \hspace{1cm} (32)

Hardening of the yield surface $f_y$ will be such that it always remains within an outer surface $f_o$.\textsuperscript{29} The outer surface $f_o$ will, for simplicity, not be allowed to translate or change size. Centers and sizes of $f_y$ and $f_o$ will be denoted by $\alpha_y, k_y$ and $\alpha_o, k_o$ respectively. Hardening of $f_y$ will proceed as given by the following relations:

$$\dot{\alpha}_y = \dot{b}(\alpha_0 - \alpha_y) = \dot{b}(\Delta\alpha)$$ \hspace{1cm} (33)

$$\dot{k}_y = \dot{b}(k_0 - k_y) = \dot{b}(\Delta k)$$ \hspace{1cm} (34)

where $\dot{b}$ is obtained by substitution of eqns (33) and (34) into eqn (32) such that

$$\dot{b} = \frac{H'(L)}{(Q\Delta\alpha + \Delta k \sqrt{3})}$$ \hspace{1 cm} (35)

The above hardening rule leads to the yield surface $f_y$, acting as an interpolation surface between its current configuration and that of the outer surface $f_o$ thus representing conceptually an infinite number of surfaces.\textsuperscript{29} These infinite surfaces span the stress space between $f_y$ and the initial configuration of $f_y$ (see Fig. 8). As the state of stress approaches the outer surface, the inner surface will smoothly assume the location and size of the outer surface. In order to achieve more accuracy in representing small-strain soil response, $f_y$ will be shrunk to a point coincident with the stress state at the onset of loading and whenever a load reversal or reloading process is initiated. The cyclic nature of loading will lead to one of the following hardening processes:

1. Load reversal occurs prior to reaching $f_o$: The pre-reversal configuration of $f_y$ automatically becomes a temporary memory surface acting as $f_o$. A new $f_y$ is initiated inside this memory surface. The same process is repeated if another load reversal occurs prior to reaching the current $f_o$.

2. Loading proceeds till the outer surface is reached: $f_y$ will then coincide with the current $f_o$. The current $f_o$ will automatically be erased from the the materials memory and hardening proceeds, with $f_o$ being the next larger memory surface. The largest memory surface is the failure surface $f_f$. This failure surface is a permanent memory surface and, for simplicity, will neither translate nor change size ($H'$ will be zero along $f_f$, indicating perfectly plastic behavior).

3.3 Implementation of constitutive model

A backbone 1D shear stress–strain curve is necessary (Fig. 9). This backbone curve may be represented by a mathematical relation such as the hyperbolic relation (with $\gamma$ as a user defined strain) commonly used for soil:\textsuperscript{30–31}

$$\varepsilon = \frac{\gamma_0}{2(G\gamma_0 - \tau)}$$ \hspace{1cm} (36)

which, upon differentiation, yields the elasto-plastic tangent modulus ($H = \partial\varepsilon/\partial\tau$):

$$H = \frac{2}{G\gamma_0^2} (G\gamma_0 - \tau)^2$$ \hspace{1cm} (37)

The ultimate stress ($\gamma = G\gamma_0$) for the hyperbolic relation
4 COMPUTATIONAL MODEL

A Galerkin implementation of the method of weighted residuals was employed to compute seismic landfill response. In this implementation, the constitutive model presented above and the analytically derived mode shapes (eqn (7)) were used as spatial functions to describe the solution. Time integration or step-by-step solution of the nonlinear equation of motion was performed by using the Newmark family of methods.

5 FRESH KILLS LANDFILL

The Fresh Kills landfill, located in New York City, encompasses approximately 200 acres of active landfill area. This landfill was opened in 1948 and receives about 16,000 tons of domestic solid waste (paper, bottles and rubber). Unit weight of solid waste is about 6–9 kN m\(^{-3}\) (40–65 lb ft\(^{-3}\)).

The refuse being placed at Fresh Kills landfill is fairly uniform. An inspection of the landfill did not reveal any localized zones of weaker materials such as yard waste, food, shredded plastic and similar waste products.

The section to be analyzed (Figs 10 and 11) is essentially barge-fed landfill accepting about 14,000 tons of refuse per day. Compactors are used to spread and compact the refuse, while scrapers apply a 12 inch sandy loam cover to areas that have been covered with 10–12 ft of compacted refuse.

In the following section, a seismic response study of section 1/9 (Figs 10 and 11) was conducted using the newly developed shear model described earlier. The results are compared to those of the shear beam model that is widely used in professional practice. In the landfill model a truncation ratio of \(\lambda = 0.1\) was employed, and the landfill was represented at its planned maximum height of 151.5 m or 505 ft, as shown in Fig. 11.

5.1 Input earthquake motions

The Resources Conservation and Recovery Act (RCRA) Subtitle D (258) recommends the use of three time histories for response analysis of landfills, because of uncertainties in the selection of a representative earthquake time history. Additionally, for sites east of the Rocky mountains, at least one record from a western United States site should be used. One of the primary differences between earthquakes in the eastern and western United States lies in frequency content. Table 4 shows the main characteristics and peak accelerations of the three earthquake motions used in this study. These records were selected so as to offer a wide range of input motion frequency content.

5.2 Dynamic material properties

Material properties of landfills are commonly site specific, and properties at the same landfill may change with time. Differences among landfills may be attributable to a variety of factors, including differences in (1) composition of waste stream, (2) daily and intermediate cover soils, (3) operational procedures (compacted versus uncompacted waste, and recycling of leachate into waste), (4) age (older landfills tend to have greater degrees of biodegradation), and (5) climate.

On the basis of extensive geophysical testing at several landfills in southern California, Kavazanjian et al. performed
suggested a shear wave velocity profile for use in seismic analysis studies in the absence of site-specific measurements (Fig. 2). Herein, an upper bound stiff, and another lower bound soft shear wave velocity profile case will be studied. The upper bound stiff profile was defined by a shear wave velocity of 600 m s⁻¹ at the landfill base with an inhomogeneity parameter $B = 0.49$. This profile is an upper bound of the in-situ measurement results shown in

Fig. 12. Input and computed crest accelerations, Loma Prieta record.

Fig. 13. Response spectra (5% damping), Loma Prieta record (soft profile).

Fig. 14. Response spectra (5% damping), Loma Prieta record (soft profile).

Fig. 2. The lower bound soft profile, on the other hand, was defined by a shear wave velocity of 184 m s⁻¹ at the landfill base with an inhomogeneity parameter $B = 0.49$. This lower bound soft profile was based on a system identification study using seismic response records of the Operating Industries Inc. (OII) landfill in southern California during five different earthquakes. In this system identification study, the upper 30 m of OII landfill was determined to have a shear wave velocity of about 125 m s⁻¹ and the landfill seismic

Fig. 15. Input and computed crest accelerations, Northridge record.
response was found to be essentially linear (for earthquakes with base peak acceleration of up to 0.25 g). In addition, a stiffness proportional (3%) viscous damping coefficient was identified as an optimal damping mechanism, and was thus employed in the Fresh Kills landfill seismic response reported below.

6 SEISMIC ANALYSIS RESULTS

6.1 Loma Prieta input motion

The computed accelerations at the landfill crest are shown in Figs 12–14. Peak acceleration was amplified in all cases. In addition, major differences in the resonant response characteristics (Figs 13 and 14) may be observed upon comparison of the new landfill and conventional shear beam models (note that for the stiff profile \( T_1 = 0.6 \) s for the landfill model, \( T_2 = 1.21 \) s for the shear beam model; and for the soft profile \( T_1 = 1.97 \) s for the landfill model, \( T_1 = 3.66 \) s for the shear beam model). The landfill model is clearly stiffer than its corresponding shear beam, as is evident from the lower period (higher frequency) spectral responses (Figs 13 and 14). For practical purposes, the difference is quite noteworthy in the soft profile case of Figs 12 and 14, where the landfill model shows significantly larger amplification in the time and frequency domains.
6.2 Northridge input motion

In this case, a noticeable difference in response (Figs 15–17) is observed throughout between the landfill and shear beam models (crest accelerations). Primary energy in the input motion employed was below the 1 s period range, and thus was well below the fundamental shear beam period \( T_1 = 1.21 \text{ s and 3.66 s for stiff and soft profiles respectively}. \) This has apparently resulted in essentially no amplification in the shear beam crest response (stiff profile, Figs 15 and 16) and even in deamplification for the soft profile case (Figs 15, and 17). As in the Loma Prieta case, the landfill model showed significant amplification.

6.3 Saguenay input motion

This input motion (Fig. 18) is only rich in the high frequency range (< 0.5 s range, Fig. 19). Essentially, no amplification (crest acceleration) occurred in this case, and significant deamplification was observed in the soft profile case (Fig. 20).

6.4 Loma Prieta record, nonlinear response case

In view of the very large computationally predicted peak crest acceleration (soft profile case), a nonlinear computational analysis was conducted as well. In this case, a value of \( \gamma_r = 0.9\% \) was employed in order to mimic the nonlinear dependence of shear modulus on shear strain amplitude according to the analyses of Idriss et al.\(^2\). The results are shown in Figs 21 and 22. In general, nonlinear response predicted a significantly lower peak crest acceleration.

7 GENERAL COMMENTS

- Major amplification in motion might result at the Fresh Kills landfill site from input motions with rich frequency content in the range of 0.5–5 Hz.
- A shear beam model might not predict satisfactory amplification response characteristics, as the landfill geometry (hill shape) might not be appropriately represented. In such cases, the landfill will be stiffer than a representative shear beam model (with the landfill shear wave velocity profile).
- If significant amplification of input motion occurs, nonlinear response may have a noticeable impact on the Fresh Kills landfill seismic response.

8 SUMMARY AND CONCLUSIONS

A nonhomogeneous model for lateral shear vibration of hill-shaped structures was adopted for seismic analysis of landfills. An expression for free vibration response was
derived within a modal solution framework. The influence of nonhomogeneity on modal displacements and strains was studied. With the modes employed as spatial functions to represent the solution, a nonlinear hysteretic computational procedure was implemented for seismic analyses. Using this procedure, the seismic response of Fresh Kills landfill in New York City was studied under the action of three different earthquake input motions. The results, which were also compared to shear beam model predictions, revealed that: (1) significant amplification of input seismic motions might occur, and (2) in situations of hill-shaped landfills, a shear beam model with the landfill shear wave velocity profile may erroneously respond at a significantly higher fundamental period.

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