Post design modeling for cellular manufacturing system with cost uncertainty

Ravi Shanker, Prem Vrat*

Mechanical Engineering Department, Indian Institute of Technology, Hauz Khas, New Delhi 110 016, India

Received 9 June 1997; accepted 19 February 1998

Abstract

Post-design decisions in a cellular manufacturing system are modeled in this paper as an interval programming model in which the coefficients of the objective function are expressed in range rather than a point estimate. Uncertainties in estimations for costs related to exceptional elements and bottleneck machines have been modeled. The interval objective function involves order relations, which represents the decision makers' preference between interval costs. The objective function is minimized by converting it into a multi-objective problem using order relations. A quantitative estimate of 'risk by gain ratio' is proposed to understand the model behavior and to facilitate the selection of appropriate strategies.

Keywords: Cellular manufacturing system; Interval analysis; Multiple objective programming

1. Introduction

Cellular manufacturing system (CMS) is a manufacturing system in which parts are grouped into part families (PF) and machines are grouped into machine cells (MC). Simplicity in part design or machining operation is used to facilitate the clustering. For each part family, one dedicated machine cell is conceived so that all the processing requirements of the part family are completed in the corresponding machine cell. In practice, however, some parts are allowed to be processed in two or more cells. Similarly, some machines are required by two or more part families. These parts and machines are referred as exceptional elements (EE) and bottleneck machines (BM), respectively. To generate the effective grouping of parts and machines into PF and MC, all possible EE or BM may be temporarily removed at the clustering stage. In the post-design modeling of CMS, we evolve the strategies to deal with them. When an EE is allowed to have intercell transfer, there would be no need to duplicate the BM responsible for this intercell transfer, provided machine capacity is adequate. Otherwise, either the EE has to be subcontracted or corresponding BM has to be duplicated [1]. There are three more options for eliminating the EE: (i) re-route the EE, (ii) re-design the EE and (iii) form a remainder cell having an independent cluster of EE [2]. A survey of CMS industries in US suggests
that 20% of companies with manned cells and 14% with unmanned cells had machines shared by two or more cells. The survey also concludes that it is rare to find completely independent cells [3]. In another survey Pullen [4] reported that only 36% cells were without an EE. Therefore, the strategy selection problem in CMS is an important practical issue with many interrelated factors involving EE and BM. The crucial aspect however, is their cost estimation, in terms of the acquisition cost (AC) of BM, intercell transfer cost (IC) of EE and subcontracting cost (SC). It is observed that in real-life situations, none of these costs can be estimated accurately when the cells are designed and strategies for dealing EE and BM are decided [5]. This is mainly because of four reasons: (i) long-time gaps between design, implementation and operations, (ii) high cost in acquiring these figures with precision, (iii) lack of statistical observations and (iv) uncertainty in external factors (Table 1). Generally,

<table>
<thead>
<tr>
<th>SN</th>
<th>Cost element</th>
<th>Reasons for considering uncertainty in cost estimate</th>
<th>Internal factor</th>
<th>External factor</th>
</tr>
</thead>
</table>
| 1  | Acquisition cost of bottleneck machine | 1. Time gap between design, implementation and operation  
2. Change in management perception toward make-to-stock\capacity of the machine  
3. In-house R&D breakthroughs  
4. Impact of cost reduction due to better scheduling after machine duplication  
5. Insufficient market survey  
6. Possibilities of getting discount, if duplicate machines are ordered  
7. Installation cost not known, as layout is not finalised | 1. Change in government policies  
2. Unwillingness of suppliers to quote prices without orders in near future  
3. Inflation  
4. Change in import restrictions  
5. Change in interest rate for capital acquired to purchase machines  
6. High cost in acquiring data with precision  
7. Arrival of new technology  
8. Obsolescence of software and hardware by the time design is implemented  
9. Global exchange of technology  
10. Possibility of joint ventures and transfer of technology from partner firms | 1. Inflation  
2. Lack of statistical observations |
| 2  | Intercell transfer cost             | 1. Material handling equipment – yet undecided  
2. Likely changes in plant layout (traveling distance between each cells – yet not finalised)  
3. Number of part type travelling between cells – yet not finalised  
4. Production volume and number of time each part travels – yet not finalised  
5. Location and size of stores – yet not finalised  
6. Undecided process plan and sequence of operations | 1. Market uncertainty due to competition  
2. Unwillingness of subcontractor to quote prices without possibility of orders in near future  
3. Inflation  
4. Subcontractor finalised at this stage fails to cope up with the requirement | 1. Inflation  
2. Lack of statistical observations |
| 3  | Subcontracting cost of exceptional elements | 1. Time gap between design, implementation and operation  
2. Issues related to transfer of technology to the subcontractor – yet not finalised | 1. Market uncertainty due to competition  
2. Unwillingness of subcontractor to quote prices without possibility of orders in near future  
3. Inflation  
4. Subcontractor finalised at this stage fails to cope up with the requirement | 1. Inflation  
2. Lack of statistical observations |
decision makers tend to give the cost figures in a range estimate due to the dynamics of market forces involving competitors, subcontractors, collaborators, financial institutions, government agencies, etc. Some typical examples of range estimates are: "unit subcontracting cost of part 3 would be between $7.5 and $2.5", "acquisition cost of machine 2 would be in the range of $60,000–70,000", "cost of intercell transfer for part 8 would be $2.6 per unit with ±50% variation", etc. Exact cost estimates are more difficult particularly when the economy is volatile or if there are political uncertainties. The point estimate, normally associated with the deterministic models is thus a specific value in the set of feasible values for the corresponding range estimate. The chance that a particular value in the corresponding range set would be valid over the life cycle of a manufacturing system is quite remote. It is therefore important to capture the cost uncertainty and incorporate these while modeling the problem to decide about EE and BM.

This paper deals with the non-deterministic information of range type by constructing an interval objective function for cost minimization. This takes care of uncertainty in the cost coefficients of the model for strategy selection in CMS. In the context of CMS, this is perhaps the first attempt to capture this type of uncertainty in a mathematical programming framework.

2. Literature review

Cell formation is an area of extensive research during recent years. A large variety of approaches have been adopted for this purpose. Production flow analysis [6] has been used to form machine cells by using routing or operation sequence information. Many clustering algorithms have also been adopted for formatting the cell structure. Array-based clustering techniques such as rank-order clustering [7], direct clustering analysis [2], bond energy analysis [8], etc., use procedures to rearrange rows and columns of part-machine incidence matrix. These hierarchical clustering approaches compute similarity/dissimilarity between each pair of parts or machines. On the other hand, non-hierarchical clustering techniques, which are iterative methods to form cells, have also been used [9]. Mathematical programming approaches such as linear programming [10], integer programming [11], goal programming [12], etc., have been adopted for this problem. Comprehensive reviews of such works exist in literature [13–15].

Most of these cell formation approaches either completely ignore EE/BM from consideration, or temporarily remove them from the problem and reinstate them on the conclusion of the process [1]. This might be done to keep the problem size within computational capacity of software/hardware used. Some approaches to CMS design treat EE/BM separately, which we call in this paper as the post-design strategy selection problem. Shafer et al. [16] have proposed a mathematical programming model for such a strategy selection problem. Amirahmadi and Choobineh [17] have evolved a two-stage procedure for economically identifying the cell composition. In the first stage an orthogonal set of EE/BM is identified along with the cell formation while in the second stage strategy to deal with EE/BM is evolved. An advantage of the models used in these two papers is their small problem size, as the computational effort in integer programs used in these models depends on the cardinality of exceptional sets). Since these cardinalities are generally small, commercial optimization packages are quite adequate for many real-life problems [17]. Strategy to deal with bottleneck machine in the presence of budgetary limitation has been considered by Logendran [18] through a two-phase methodology. Seifoddini [19] compared cost of BM duplication with cost of material handling created by intercellular transfer of the EE, but ignored subcontracting option for the EE. Kern and Wei [1], however, considered subcontracting option with machine duplication for eliminating the EE. For eliminating EE, cost for each avoided intercellular transfer is calculated and this information is used in the process of strategy selection. King and Nakornchi [7], McAuley [20] and Seifoddini and Wolfe [21] used BM duplication as a strategy to eliminate intercellular transfer of an EE. However, they ignored the subcontracting option to achieve the same goal.
[22] and Kusiak and Chow [23] compared subcontracting cost with cost required for duplication of BM for evaluating EE that remains in the solution of the cell formation problem.

Very little work has been reported to deal with the uncertainty in situational parameters of CMS. Seifoddini [24] proposed a probabilistic model for selecting the cell configuration which uses different product mixes with associated probabilities to calculate the expected inter-cellular material handling cost. Some simulation-based works also exist in literature [25,26]. Shanker and Vrat [27,5] have used chance constrained and fuzzy programming approaches to deal with uncertainty.

3. Interval objective function

The objective function of a mathematical programming problem employed in this paper captures uncertainty by considering its coefficients in range interval. The form of such an objective function is

\[
\min \ Z(x) = (A_1x_1 + A_2x_2 + \cdots + A_nx_n):
\]

\[
x \in S \subseteq R^n,
\]

where \( S \) is the feasible region of \( x \) and \( A_i \) is the interval representing the uncertainty in the cost information for \( x_i \). Thus, the problem becomes that of the \textit{inexact programming type} [28]. Lowercase and uppercase letters are used in this section to represent the real-number and closed-interval, respectively. As proposed by Falk [29], the conservative strategy to get optimal solution against the worst case scenario would be a mini–max case or the right limit of \( Z(x) \) as below:

\[
\min (\max_{a \in A_i} z(x) = (a_1x_1 + a_2x_2 + \cdots + a_nx_n):
\]

\[
a_i \in A_i \& x \in S \subseteq R^n.
\]

3.1. Interval nomenclature

The closed interval as denoted by \('R'\) represents the set of all real numbers in the range. The left limit, right limit, center and width of the interval \( A \) are denoted as \( a_L, a_R, a_C \) and \( a_W \), respectively. The ordered pair \( a_L \) and \( a_R \) for an interval are defined as

\[
A = [a_L, a_R] = \{a: a_L \leq a \leq a_R: a \in R\}.
\]

Also,

\[
A = \langle a_C, a_W \rangle
\]

\[
= \{a: (a_C - a_W) \leq a \leq (a_C + a_W): a \in R\}.
\]

The center and width of the interval are calculated as follows:

\[
a_C = 0.5(a_R + a_L),
\]

\[
a_W = 0.5(a_R - a_L).
\]

Alefeld and Herzberger [31] and Ishibuchi and Tanaka [30] have discussed interval arithmetic in greater detail.

3.2. Order relation for minimization problem

The decision makers’ preferences between interval costs are represented by order relations (symbolically as \( \leq_L, \leq_R \) or \( \leq_C \)) between uncertain costs from two alternatives represented by sets of intervals \( A \) and \( B \). Ishibuchi and Tanaka [30] defined the following order relations:

\[
A \leq_L B \text{ if and only if } a_L \leq b_L \text{ and } a_R \leq b_R,
\]

\[
A \leq_R B \text{ if and only if } A \leq_L B \text{ and } A \neq B,
\]

\[
A \leq_C B \text{ if and only if } a_C \leq b_C \text{ and } a_W \leq b_W,
\]

\[
A < L B \text{ if and only if } A \leq L B \text{ and } A \neq B,
\]

\[
A \leq_C B \text{ if and only if } A \leq_C B \text{ and } A \neq B.
\]
\[ A \leq \xi B \text{ if and only if } a \leq b \text{ and } c \leq d. \] \hfill (11)

\[ A < \xi B \text{ if and only if } A \leq \xi B \text{ and } A \neq B. \] \hfill (12)

Order relation \( A \leq \xi B \) represents the decision makers' preference for alternatives with lower minimum cost and maximum cost. Order relation \( A < \xi B \) represents the decision makers' preference for the alternatives with lower expected cost and less uncertainty. Order relation \( A \leq \xi B \) represents the decision makers' preference for the alternatives with lower maximum cost and lower expected cost. In case that either \( A < \xi B \) or \( A < \xi B \) holds good, \( A \) is preferred to \( B \). Following properties of this order relation is relevant to the minimization problem:

If \( A \leq \xi B \); then \( a \leq b \). \hfill (13)

If \( a = b \)

\[ = 0; \text{ then} \]

\[ \leq \xi \text{ reduces to an inequality relation} \]

\[ \leq \text{ on a set of real numbers.} \] \hfill (14)

3.3. Minimization of the interval objective function

The interval objective function can be reformulated into a multi-objective problem with solution set defined by the preference relations between intervals. Using order relations defined earlier, which represent decision makers' preference between interval costs, solution set of (1) is defined as non-dominated solutions. The following propositions have been proved by Ishibuchi and Tanaka [30]:

\[ A \leq \xi B \text{ if and only if } A \leq \xi B \text{ or } A \leq \xi B. \] \hfill (15)

\[ A < \xi B \text{ if and only if } A < \xi B \text{ or } A < \xi B. \] \hfill (16)

The solution set of minimization problem (1) may be defined by \( x \in S \) if and only if there is no \( x' \in S', \) which satisfies \( Z(x') < \xi z(x) \) or \( Z(x') < \xi z(x). \) Using Eqs. (15) and (16), this solution set may also be defined by \( x \in S \) if and only if there is no \( x' \in S, \) which satisfies \( Z(x') < \xi z(x). \) Thus, the solution set of (1) can be obtained as the Pareto optimal solution which gives non-inferior, non-dominated solution of the following multi-objective problem:

\[ \min (z_1(x), z_2(x); \ x \in S \subseteq \mathbb{R}) \]. \hfill (17)

For \( x_i \geq 0, \) the right limit \( z_2(x) \) of the \( Z(x) \) may be calculated as

\[ z_2(x) = (a_{c1}x_1 + \cdots + a_{c2}x_2) \]
\[ + (a_{w1}x_1 + \cdots + a_{w2}x_2), \] \hfill (18)

where, \( a_{ci} \) is the center and \( a_{wi} \) is the width of coefficient \( A_i \) of interval objective function \( Z(x). \) Similarly, the center \( z_2(x) \) of \( Z(x) \) is

\[ z_2(x) = (a_{c1}x_1 + \cdots + a_{c2}x_2), \]
\[ z_2(x) = (a_{w1}x_1 + \cdots + a_{w2}x_2), \] \hfill (19)

Therefore, with interval coefficients \( A_i \) in (1), being denoted by \( \langle a_{ci}, a_{wi} \rangle, \) the right limit and center of \( Z(x) \) may be calculated from Eqs. (18) and (19).

4. Risk by gain ratio (z) in the analysis of uncertainty in the range estimate

To study the impact of uncertainty associated with the range estimate on decision variables, a concept of risk-by-gain ratio (z) is defined as follows:

\[ z = z_2(x) + \{ z_2(x) - z_2(x) \}, \] \hfill (20)

where

\[ z_2(x) = 0.5 \{ z_2(x) - z_2(x) \}. \] \hfill (21)

\( z_2(x) \) is the maximum of all \( z_2(x) \) values or budget for total cost in the objective function.

Numerator in \( z \) reflects the degree of spread or uncertainty in the range estimate. In the limiting case, when spread of an interval is zero, i.e. we have a point estimate, \( z \) would be zero. It is clear from Eq. (21) that it increases with the spread of interval. The denominator in \( z \) provides a measure of the
expected savings from a budget provision. We have
notionally taken the maximum of all $z_p(x)$ values as
the budget provision for a cost minimization prob-
lem, because the purpose of proposing this factor is
limited only to compare various alternatives.
A lower value of $\alpha$ indicates a low risk per unit gain.
Hence, from a solution set, only that decision is
preferred, which has the lowest value of $\alpha$.

5. Modeling CMS with the interval objective
function

A mathematical programming model is formu-
lated for the optimum selection of exceptional
parts to be subcontracted and bottleneck machines
to be duplicated in the CMS environment. The
objective function minimizes the total cost asso-
ciated with subcontracting EE, the intercellular
transfer cost and the acquisition cost of BM. All the
cost coefficients in the objective function are
expressed as a range of upper and lower limit of the
expected cost. The model presented here is an ex-
tension of the deterministic model of Shafer et al.
[16], incorporating the range objective function to
capture uncertainty of cost parameters.

5.1. Model formulation

Three strategies to deal with EE and BM are
considered in this model:

1. if an EE is allowed to have intercell transfer, then
the BM which is responsible for this is not dupli-
cated in the cell containing this EE;
2. if an EE is not allowed to have intercell transfer,
then the BM which is responsible for this is
duplicated in the cell containing this EE, and
3. if neither intercell transfer of EE nor duplication
of BM is possible then the EE is subcontracted.

It is assumed that the BM has adequate capacity
and adequate budget is available to implement
these strategies. Re-design and re-routing options
for EE are not considered in this model. Indexing
sets, variables and parameters used in the model
are as follows:

**Indexing sets:**

$f, i, j$ index for machine cells, parts and ma-
chines, respectively

$L, R, C, W$ index for lower, upper, center and width
of range estimate

**Decision variables:**

$X_i$ units of part $i$ to be subcontracted

$Y_{ij}$ number of machines of type $j$ to be
purchased for cell $f$

$Z_{ij}$ number of intercellular transfers re-
quired by part $i$ as a result of machine
type $j$ not being available within part's
machine cell

$M_{ij}$ number of machines of type $j$ dedicated
to production of part $i$

**Parameters:**

$[a_{jL}, a_{jR}]$ lower and upper range of annual cost
$(a_j)$ for acquiring a machine of type $j$

$[s_{jL}, s_{jR}]$ lower and upper range of incremental
cost $(s_j)$ for subcontracting unit part $i$

$[t_{jL}, t_{jR}]$ lower and upper range of incremental
cost $(t_j)$ for moving part $i$ outside a cell
as opposed to moving it within the cell

$D_t$ annual forecasted demand for part $i$

$C_j$ annual capacity of machine type $j$

$P_{ij}$ processing time to produce part $i$ on
a machine type $j$

$G_f$ set of exceptional parts of cell $f$

$H_f$ set of bottleneck machines required by
parts of cell $f$

5.2. The model

A mathematical programming model for optimal
selection of strategies in CMS with interval cost
estimate is formulated as follows:

**Objective function:**

$$\text{Min: } z(x) = \sum_f \left[ \sum_{i \in G_f} [s_{jL}, s_{jR}] X_i + \sum_{j \in H_f} [a_{jL}, a_{jR}] Y_{ij} + \sum_{k \in G_f} [t_{jL}, t_{jR}] Z_{ij} \right],$$

(22)
s.t.: 

\[ Z_{ij} + X_i + \left( C_j M_{ij} / P_{ij} \right) = D_i \quad \text{for all EE}, \quad (23) \]

\[ \sum_{j \in G_i} M_{ij} \leq Y_{jf} \quad \text{for all } j, f, \quad (24) \]

\[ X_i = 0 \quad \text{for all } i \text{ that should not be subcontracted}. \quad (25) \]

\[ X_i, Y_{jf}, Z_{ij} \text{ are all integers.} \quad (26) \]

Objective function (22) minimizes the costs associated with EE and BM, such as: subcontracting cost of EE, intercellular transfer cost of EE and discounted acquisition cost of BM. It is assumed here that only these cost informations are vague and available in the form of interval estimates.

Constraint set (23) puts restriction on each EE so that their production should conform to the corresponding part demand. This is achieved by satisfying demand of an EE by intercell part processing, subcontracting and intracell part processing. Consequently, it evaluates the number of intercellular transfers between machine type j and part i that remain in the solution. Constraint set (24) ensures that number of machines of type j purchased for cell f are the same as the number assigned to various EEs found in constraints (23). Constraint set (25) puts restrictions on parts not to be subcontracted going into solution. Constraint set (26) takes care of integer restrictions.

5.3. Conversion of interval objective function into multi-objective model

Employing concepts discussed in Section 3, the original range objective function (22) is converted

Table 2
Incidance matrix and data set of the illustrated problem

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>( a_i )</th>
<th>( C_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>2.95</td>
<td>2.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.61</td>
<td>50784</td>
<td>2000</td>
</tr>
<tr>
<td>M</td>
<td>2</td>
<td></td>
<td>2.76</td>
<td>5.18</td>
<td>1.89</td>
<td>3.89</td>
<td>5.14</td>
<td></td>
<td></td>
<td></td>
<td>67053</td>
<td>2000</td>
</tr>
<tr>
<td>A</td>
<td>3</td>
<td></td>
<td></td>
<td>5.54</td>
<td>4.29</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>43944</td>
<td>2000</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>2.91</td>
<td>1.97</td>
<td>2.59</td>
<td>4.01</td>
<td></td>
<td>2.70</td>
<td>67345</td>
<td>2000</td>
</tr>
<tr>
<td>H</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.28</td>
<td>4.51</td>
<td></td>
<td></td>
<td></td>
<td>42414</td>
<td>2000</td>
</tr>
<tr>
<td>I</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.33</td>
<td>5.52</td>
<td></td>
<td>75225</td>
<td>2000</td>
</tr>
<tr>
<td>N</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.40</td>
<td></td>
<td>1.16</td>
<td>4.72</td>
<td>2.49</td>
<td>52741</td>
<td>2000</td>
</tr>
<tr>
<td>E</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.75</td>
<td>3.85</td>
<td>63523</td>
<td>2000</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>50632</td>
<td>2000</td>
</tr>
</tbody>
</table>

\( s_i \) | 5 | 4.3 | 3.5 | 4.4 | 5.0 | 3.9 | 4.4 | 4.6 | 5.0 | 5.0 |
\( D_i \) | 32128 | 27598 | 20651 | 11340 | 18707 | 17040 | 46196 | 45384 | 16409 | 22000 |
\( t_i \) | 3.7 | 2.8 | 2.8 | 3.3 | 2.8 | 3.5 | 2.8 | 2.6 | 3.4 | 3.2 |
<table>
<thead>
<tr>
<th>Range of estimate</th>
<th>Parts sub-contracted</th>
<th>Machine duplicated</th>
<th>Inter-cell transfer</th>
<th>Objective function</th>
<th>( \alpha = z_1(x) + \frac{z_2(x) - z_3(x)}{z_4(x) - z_5(x)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>S'</td>
<td>T'</td>
<td>w</td>
<td>( Y_{12}, Y_{13}, Y_{14}, Y_{15}, Y_{61} )</td>
<td>( Z_{28} = 27.598 )</td>
</tr>
<tr>
<td>50%</td>
<td>2%</td>
<td>10%</td>
<td>0-0.07</td>
<td>0</td>
<td>( Z_{28} = 27.598 )</td>
</tr>
<tr>
<td>0.08</td>
<td>0</td>
<td>( Y_{12}, Y_{13}, Y_{14}, Y_{15}, Y_{61} )</td>
<td>( Z_{28} = 27.598 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.09</td>
<td>0</td>
<td>( Y_{12}, Y_{13}, Y_{14}, Y_{15}, Y_{61} )</td>
<td>( Z_{28} = 27.598 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td>10%</td>
<td>2%</td>
<td>0-1</td>
<td>0</td>
<td>( Z_{28} = 27.598 )</td>
</tr>
<tr>
<td>0.4-0.6</td>
<td>( X_1 = 32128 )</td>
<td>( Y_{12}, Y_{13}, Y_{61} )</td>
<td>( Z_{28} = 27.598 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7-0.8</td>
<td>( X_1 = 32128 )</td>
<td>( Y_{12}, Y_{13}, Y_{61} )</td>
<td>( Z_{28} = 27.598 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9-1</td>
<td>( X_1 = 32128 )</td>
<td>( Y_{13} )</td>
<td>( Z_{28} = 27.598 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10%</td>
<td>2%</td>
<td>50%</td>
<td>0</td>
<td>0</td>
<td>(Y_{1,2}, Y_{2,2})</td>
</tr>
<tr>
<td>-----</td>
<td>----</td>
<td>-----</td>
<td>--</td>
<td>---</td>
<td>-------------------</td>
</tr>
<tr>
<td>0.1-0.2</td>
<td>0</td>
<td>(Y_{1,2}, Y_{2,2})</td>
<td>(Y_{11}, Y_{11})</td>
<td>(Y_{12}, Y_{12})</td>
<td>(Z_{28} = 27.598)</td>
</tr>
<tr>
<td>0.3-1</td>
<td>(X_6 = 2276)</td>
<td>(Y_{1,3}, Y_{2,2})</td>
<td>(Y_{11}, Y_{11})</td>
<td>(Y_{12}, Y_{12})</td>
<td>(Z_{28} = 50.50)</td>
</tr>
</tbody>
</table>

| 2% | 50% | 10% | 0-0.2 | 0 | \(Y_{1,3}, Y_{2,2}\) | \(Y_{11}, Y_{11}\) | \(Y_{12}, Y_{12}\) | \(Z_{28} = 27.598\) | \([2.900, 6.457]\) | \<(4.68, 1.78)\> | 0.761 |
|-----|----|-----|------|---|-------------------|-----------------|----------------|-----------------|----------------|-----------------|-----------------|--------|
| 0.3-0.8 | \(X_4 = 32128\) | \(Y_{1,3}, Y_{2,2}\) | \(Y_{11}, Y_{11}\) | \(Y_{12}, Y_{12}\) | \(Z_{28} = 27.598\) | \([3.76, 5.96]\) | \<(4.86, 1.09)\> | 0.509 |
| 0.9-1.0 | \(X_4 = 32128\) | \(Y_{1,3}, Y_{4,3}\) | \(Y_{11}, Y_{11}\) | \(Y_{12}, Y_{12}\) | \(Z_{28} = 27.598\) | \([4.22, 5.93]\) | \<(5.08, 0.85)\> | 0.438 |

| 2% | 10% | 50% | 0 | 0 | \(Y_{1,3}, Y_{2,2}\) | \(Y_{11}, Y_{11}\) | \(Y_{12}, Y_{12}\) | \(Z_{28} = 27.598\) | \([3.65, 5.70]\) | \<(4.68, 1.03)\> | 0.44 |
|-----|----|-----|--|---|-------------------|-----------------|----------------|-----------------|----------------|----------------|----------------|--------|
| 0.1-0.2 | 0 | \(Y_{1,3}, Y_{2,2}\) | \(Y_{11}, Y_{11}\) | \(Y_{12}, Y_{12}\) | \(Z_{28} = 50.50\) | \([4.119, 5.25]\) | \<(4.68, 0.57)\> | 0.243 |
| 0.3-1 | \(X_6 = 2276\) | \(Y_{1,3}, Y_{2,2}\) | \(Y_{11}, Y_{11}\) | \(Y_{12}, Y_{12}\) | \(Z_{28} = 50.50\) | \([4.167, 5.22]\) | \<(4.69, 0.58)\> | 0.227 |
into a multi-objective mixed integer programming [MOMIP1] problem, as follows:

\[ \text{Min} [z_k(x), z_c(x)], \quad \text{s.t. (23)-(26)].} \quad (27) \]

Using the weighing method, we can obtain the \textit{Pareto-optimal solution} of [MOMIP1] from the following mixed integer linear programming problem [MILP1];

\[ \text{Min} [wz_k(x) + (1-w)z_c(x)], \quad \text{s.t. (23)-(26).} \quad (28) \]

The \textit{Pareto-optimal solution} of [MOMIP1] is obtained from [MILP1] by gradually increasing \( w \) in (28) from 0 to 1 and observing changes in decision variables and objective function at every stage.

6. Illustrative example

To illustrate the model proposed here, an example with marginal modification in data set is adopted from literature [16]. All costs are expressed as upper and lower value of interval.

For comparing results with the deterministic model of Shafer et al. [16], the average of upper and lower limit of each cost interval is maintained at a value which is the same as its deterministic counterpart. Table 2 gives the part-machine incidence matrix along with the central value for various cost intervals. The entries at machine-part intersection in the incidence matrix represent the processing time in minutes. For example, part 4 needs 3.89 min on machine 2. Last two columns are machine acquisition cost and machine capacity in hour per year, respectively. Last three rows in this table contain the unit subcontracting cost, annual demand of parts and unit intercellular transfer cost, respectively. The left and right values of these cost intervals are varied upto \( \pm 50\% \) around the central value. For example, subcontracting cost of part 1, \( s_1 \) is expressed as \( [7.5, 2.5] \) or \( \langle 5, 2.5 \rangle \) for a \( \pm 50\% \) variation around the central value 5.

7. Results and discussions

[MILP1] is employed to the problem defined in the model (22)-(26). For a range of weight factor \( w \), number of Pareto optimal solutions are obtained using LINGO package on a personal computer. These are given in Table 3.

Here, a \( \pm 2\% \) variation around the central value is treated as a nearly deterministic cost estimate while \( \pm 50\% \) variation is taken as quite uncertain cost information. Thus, a sequence of (2, 2, 2) would represent a variation of \( \pm 2\% \) in all the three cost estimates and is \textit{nearly deterministic} situation, whereas (50, 50, 50) would represent \textit{nearly uncertain} system parameters. In between, various combinations of these are considered and results are shown in Table 3. Table 4 shows the results for various situations involving same degree of uncertainty in all the three parameters.

It can be seen from Table 3 that for a combination of (50, 2, 10), the expected cost minimization criterion would prefer solution 1, while a worst-case minimization criterion would prefer solution 3. A \textit{risk-by-gain ratio} minimization criterion will also prefer solution 3.

In case of multiple solutions, the decision maker may go for a solution with lowest \textit{risk-by-gain ratio}. For example, in Table 3, in case of range (50, 2, 10), out of the three alternatives, the solution at weight 1.0 provides the same set of strategies to deal with EE and BM for lowest \textit{risk-by-gain ratio}, lowest \( z_k(x) \) and highest \( z_c(x) \). Neither of the three solutions in this set satisfies both objectives of worst case (i.e. \( z_k(x) \)) minimization and expected cost (i.e. \( z_c(x) \)) minimization. Ishibuchi and Tanaka [31] do not provide any way out to resolve the decision makers’ dilemma at this stage. However, lowest \textit{risk-by-gain ratio} has the potential to capture the combined effect of both objectives. Therefore, under this set of cost uncertainty, the prudent strategies would be the solution set with minimum \( \alpha \).

For this, no part is subcontracted (as all \( X_i \) are zero). Machines 4, 6 and 8 are duplicated in cell 1; machine 2 is duplicated in cell 2 and machines 1 and 4 are duplicated in cell 3, respectively, as \( Y_{12}, Y_{61}, Y_{81}, Y_{22}, Y_{13} \) and \( Y_{43} \) are equal to 1. Other sets of results in Table 3 may be similarly interpreted.
Table 4
"Risk-by-gain ratio" \(\phi\) for various scenarios of informational uncertainty

<table>
<thead>
<tr>
<th>SN</th>
<th>Range of estimate (in % of (z_0) the center of interval)</th>
<th>Nature of informational uncertainty in the model</th>
<th>Objective function (00000)</th>
<th>(\sigma = z_0(\phi) + \frac{z_0(\phi)}{z_0(\phi) - z_0(\phi)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0% 0% 0%</td>
<td>Deterministic</td>
<td>[4.678, 4.678]</td>
<td>(\langle z_0(\phi), z_0(\phi)\rangle)</td>
</tr>
<tr>
<td>2</td>
<td>2% 2% 2%</td>
<td>Nearly deterministic</td>
<td>[4.58, 4.77]</td>
<td>(\langle 4.678, 0.0935\rangle)</td>
</tr>
<tr>
<td>3</td>
<td>10% 10% 10%</td>
<td>Fairly deterministic</td>
<td>[4.21, 5.14]</td>
<td>(\langle 4.678, 0.4678\rangle)</td>
</tr>
<tr>
<td>4</td>
<td>20% 20% 20%</td>
<td>Moderately deterministic</td>
<td>[3.74, 5.61]</td>
<td>(\langle 4.678, 0.934\rangle)</td>
</tr>
<tr>
<td>5</td>
<td>30% 30% 30%</td>
<td>Moderately uncertain</td>
<td>[3.27, 6.08]</td>
<td>(\langle 4.678, 1.40\rangle)</td>
</tr>
<tr>
<td>6</td>
<td>40% 40% 40%</td>
<td>Fairly uncertain</td>
<td>[3.8, 6.55]</td>
<td>(\langle 4.678, 1.87\rangle)</td>
</tr>
<tr>
<td>7</td>
<td>50% 50% 50%</td>
<td>Nearly uncertain</td>
<td>[2.339, 7.017]</td>
<td>(\langle 4.678, 2.339\rangle)</td>
</tr>
</tbody>
</table>

Note: For all the calculations of \(\phi\) in Tables 3 and 4, we have taken \(z_0(\phi)\) as \(z_0(\phi)\) in SN 7 of Table 4, i.e. 7.017. It is the maximum among all options.

The model is tested for validation and the results are shown in Table 4. For the present data set, when all estimates have zero range, \((0, 0, 0), i.e.\) a deterministic case, the results are the same as those obtained from the deterministic model of Shafer et al. [15]. It is observed that as the total system uncertainty increases, the risk-by-gain ratio also increases, which is quite logical even intuitively.

The system behavior is also observed with respect to the level of informational uncertainty in machine acquisition cost (AC) (Fig. 1), inter-cell transfer cost (IC) (Fig. 2) and subcontracting cost of EE (SC) (Fig. 3). It can be seen from Figs. 1–3 that at a given level of cost uncertainty of one kind, the situation aggravates (resulting in increased value of risk-by-gain ratio) when the other two informational uncertainties increase. This effect is even more demonstrative at a higher level of cost uncertainty. It can be seen from Fig. 1 that the impact of uncertainty of machine acquisition cost is the most significant. This is because of its major contribution to the objective function. The impact of informational uncertainty in subcontracting cost is relatively insignificant, particularly when the other two uncertainties are in the narrower range (Fig. 3).

The interval objective function models need to be compared with its deterministic counterpart. It is clear that when there is informational vagueness in the design parameters of the objective function, the model proposed in this paper is very insightful as compared to the deterministic model. Alternative solutions may be analyzed and risk associated with any set of decisions may be assessed. Management may look into various options which they may like to adopt. Some of these options are: (i) solution set with low expected cost but high risk (for risk

![Figure 1](https://via.placeholder.com/150)

Fig. 1. Effect of uncertainty in machine acquisition cost due to the range estimate.
Fig. 2. Effect of uncertainty in intercell transfer cost due to the range estimate.

Fig. 3. Effect of uncertainty in subcontracting cost due to the range estimate.

takers), (ii) solution set with low risk but high expected cost (for risk avoiders) and (iii) solution set with low risk per unit gain (for risk neutrals). Such flexibility of alternative scenarios may not be available in conventional LP formulations. Moreover, deterministic model provides only one of the solutions generated by the model discussed here. Impact of some information, for example, acquisition cost in our model, is more significant as compared to the rest. Therefore, decision maker should focus on getting more reliable data for these parameters through intense market research and sophisticated forecasting techniques. For the remaining informational inputs no major effort except, expert opinion about the interval estimate may be required. This can be exploited to prioritize the time, effort and cost required in generating a reliable data base.

Computational effort for each run is nearly the same in deterministic and the proposed model, because the number of variables and problem size is nearly the same. However, more runs are required in interval model due to large enumeration involved in Pareto optimal solutions with two objective functions.

8. Conclusions

This paper has attempted to model a more realistic situation for the post design strategy selection problem in the CMS environment. Informational uncertainty associated with cost estimates pertaining to EE and BM has been considered. In the dynamic business situation, exact estimates of costs are unlikely to be available at the stage of such decision making. Hence, the range objective function proposed here is a very useful way to capture these uncertainties. Concept of proposed risk-by-gain ratio is insightful to the decision makers in weighing the risk and gain associated with a feasible set of solutions. Eventually, the model is solved by the conventional LP package and hence no special purpose software is needed to operationalize the proposed model.
Acknowledgements

The authors gratefully acknowledge the two anonymous referees of this paper for their valuable suggestions for the improvement of this paper.

References