APPROXIMATE METHOD FOR COMPUTATION OF GLASS COVER TEMPERATURE AND TOP HEAT-LOSS COEFFICIENT OF SOLAR COLLECTORS WITH SINGLE GLAZING

N. AKHTAR and S. C. MULLICK†
Centre for Energy Studies, Indian Institute of Technology, New Delhi 110016, India

Received 19 May 1998; revised version accepted 12 March 1999

Communicated by BRIAN NORTON

Abstract—An improved equation form for computing the glass cover temperature of flat-plate solar collectors with single glazing is developed. A semi-analytical correlation for the factor \( f \)—the ratio of inner to outer heat-transfer coefficients—as a function of collector parameters and atmospheric variables is obtained by regression analysis. This relation readily provides the glass cover temperature \( T_c \). The results are compared with those obtained by numerical solution of heat balance equations. Computational errors in \( T_c \) and hence in the top heat loss coefficient \( (U_t) \) are reduced by a factor of five or more. With such low errors in computation of \( T_c \) and \( U_t \), a numerical solution of heat balance equations is not required. The method is applicable over an extensive range of variables: the error in the computation of \( U_t \) is within 2% with the range of air gap spacing 3 mm to 90 mm and the range of ambient temperature 0°C to 45°C. In this extended range of variables, the errors due to simplified method based on empirical relations for \( U_t \) are substantially higher.

1. INTRODUCTION

The calculation of heat loss from the collector to the surroundings is required for the design or simulation of performance of solar collectors. The major heat loss is from the top through the glass cover. The heat losses through the bottom and the sides are very small in comparison to the top heat loss and are easier to calculate. The top heat loss coefficient \( (U_t) \) is evaluated by considering convection and radiation losses from the absorber plate in the upward direction. There are two approaches for calculating \( U_t \): (i) by numerical/iterative procedure/spread sheet; and (ii) approximate method (Klein, 1975; Agarwal and Larson, 1981; Malhotra et al., 1981; Bhowmick and Mullick, 1985; Klein's new equation in Duffie and Beckman (1991). The popularity of the approximate, non-iterative method is obvious from the literature (Garg and Datta, 1984; Nagar et al., 1984; El-Issy and Clark, 1988; Beinor et al., 1988; Altfield et al., 1988; Duffie and Beckman, 1991; El-Sebaii, 1997; Chaudhuri, 1998).

The original approximate method consists of using an empirical relation for \( U_t \), e.g. Klein's equation (Duffie and Beckman, 1991):

\[
U_t = \left[ \frac{N}{\bar{C} \left( \frac{T_s - T_a}{T_s + f} \right)^{N-1}} + \frac{1}{h_{in}} \right]^{-1} + \frac{\sigma(T_p + T_e^4)}{\varepsilon_\lambda^{N+1} + \left[ \frac{(N + f - 1 + 0.133\varepsilon_\lambda)}{\varepsilon_\lambda} \right]^{N}}
\]

where

\[
f = (1 + 0.089h_{in} - 0.1166h_{in}\varepsilon_\lambda)(1 + 0.07866N)
\]

\[
\varepsilon = 0.43 \left( 1 - \frac{100}{T_p} \right)
\]

\[
C = 520(1 - 0.000051\beta^2) \text{ for } 0 < \beta < 70.
\]

For \( 70 < \beta < 90 \), use \( \beta = 70 \).

In the empirical relation for \( U_t \), the convective and radiative terms are regrouped and approximated since the glass cover temperature, \( T_c \), is unknown. The regrouping causes large errors. The method was slightly modified (Mullick and Sandurshi, 1988), proposing the use of analytical equation for \( U_t \), i.e. without altering the convective and radiative terms (made possible by use of an empirical relation for estimating \( T_c \)):

\[
U_t = \left[ \left( h_{in} + h_{e,x}\beta \right)^{-1} + \left( h_{in} + h_{e,y}\beta \right)^{-1} + \frac{L}{k_x} \right]^{-1}
\]
This modification considerably lowers the computational errors in \( U_i \). Moreover, the user can see the values of the different heat transfer coefficients and has a better appreciation of the heat transfer processes. The method employs the following empirical relation\(^1\) to estimate the glass cover temperature (\( T_g \))

\[
T_g = T_a + h_w \times \beta \left[ 0.567 \varepsilon_p - 0.403 + T_p / 429 \right] (T_p - T_a) \tag{3}
\]

However, this empirical relation for \( T_g \) is approximate. In the present work an improved equation form for glass cover temperature is developed leading to a reduction in computational errors in \( T_g \) and hence in \( U_i \). The relation proposed for \( T_g \) is based on information on the basic heat transfer processes in a flat plate solar collector.

The heat balance and individual heat transfer coefficients are given in Appendix A. The estimated values of \( T_g \) can now be used in Eqs. (A3), (A4) and (A6) for calculation of the individual heat transfer coefficients of Eq. (2). Thus, \( U_i \) is known.

The objective of the present work is to minimize the errors resulting from the approximations made in the method employed for calculation of \( U_i \) of flat-plate solar collectors. The wind heat transfer coefficient, \( h_w \), has been taken as an independent variable and the choice of using an appropriate value or correlation for \( h_w \) is left to the user.\(^2\) It may be noted that the overall accuracy of \( U_i \) would depend also on the reliability of estimation of \( h_w \), regardless of the method chosen for computation of \( U_i \). It is found by analysis that 10% error in the value of \( h_w \) leads to 1–2% error in the value of \( U_i \).

2. PROPOSED METHOD FOR PREDICTION OF GLASS COVER TEMPERATURE

For a flat-plate solar collector, the top heat losses and the glass cover temperature depend on variables: \( \varepsilon_p \), \( L \), \( T_p \), \( T_\infty \), \( h_w \) and \( \beta \). A preliminary study showed that very simple correlations can not predict the glass cover temperature accurately. Hence a semi-analytical equation for \( T_g \) is developed as follows:

Assuming one-dimensional heat flow, neglecting thermal capacity and temperature drop across the glass cover, the ratio of outer to inner thermal resistance, \( f \), can be written as

\[
f = \frac{T_a - T_g}{T_p - T_g} = \left( \frac{h_{reg} + h_w}{[h_{reg} + h_w + h_{cog}]^{0.5}} \right)^{1.5} \tag{4a}
\]

The heat transfer coefficients \( h_{reg} \) and \( h_{cog} \) are given by Eqs. (A3) and (A6), and \( h_{reg} \) by the non-dimensional correlation, Eq. (A4). Approximating these (the radiation coefficients are known functions of emittance and cubes of mean absolute temperatures, whereas \( h_{reg} \) is a function of \( L \), \( \cos \beta \) and temperature difference), the following semi-analytical correlation for the factor \( f \) is developed by regression analysis:

\[
f = \frac{1}{\left( \frac{15 \times 10^{-4} + 0.2T_g^3 + L\alpha}{5 \times 10^{-4} (\varepsilon_p + 0.033) (T_p + 0.5T_g)^2 + 0.02^2 (T_p - T_\infty)^2 \cos \beta} \right)^{0.5}} \tag{4b}
\]

where, \( 0.3L \) accounts for a suitable fraction of thermal resistance of glass cover. Hence the glass cover temperature can be calculated as:

\[
T_g = \frac{(fT_p + T_a)}{(1 + f)} \tag{4c}
\]

Knowing the glass cover temperature, \( U_i \) can be evaluated by Eq. (2). A user's guide to the method is given in Appendix B. The computational errors of the method are so small that numerical solution is not required.

The method enables application over a broad range of variables, e.g. air gap spacing 8 mm to 90 mm and ambient temperature -5°C to 45°C, as given in Table 1. Glass cover thickness of 2 mm to 5 mm is considered.

Another major advantage of this method using Eq. (B3), over the approximate method using empirical relations for \( U_i \), is that the former would be applicable also when the relations for the

\begin{table}[h]
\centering
\begin{tabular}{|l|l|}
\hline
Variables & Range \tabularnewline \hline
Ambient temperature, \( T_a \) & 253–318 K \tabularnewline
Air gap spacing, \( L \) & 8–90 mm \tabularnewline
Absorber plate temperature, \( T_p \) & 323–423 K \tabularnewline
Absorber plate emittance, \( \varepsilon_p \) & 0.1–0.95 \tabularnewline
Wind heat transfer coefficient, \( h_w \) & 5–45 W m\(^{-2}\) K\(^{-1}\) \tabularnewline
Collector tilt angle, \( \beta \) & 0–70° \tabularnewline
\hline
\end{tabular}
\caption{Range of variables}
\end{table}
individual heat transfer coefficients change. Only the empirical relation for $f$ would change. However, this factor affects $U_i$ only to a limited extent. What is more, the relations for radiation coefficients—Eqs. (A3) and (A6)—are not likely to change. The wind heat transfer coefficient, $h_w$, has been used as an independent variable. Only the non-dimensional correlation for convective heat-transfer coefficient, $h_{tpg}$, may change.

A detailed study indicates that even in the case where the convective heat transfer coefficient changes by ±10% and if the relation for $f$ is not corrected to account for this, the maximum error in $U_i$ may go up by 0.2% (i.e. from 2% to 2.2%), which is marginal. The user could apply a very approximate correction for this if so desired. However, this correction is not required in practice.

3. RESULTS AND DISCUSSION

For any combination of the variables, $T_g$ is obtained by Eq. (B2). The values of glass cover temperature obtained are compared with those obtained by numerical solution as well as by the previous work (Mullick and Sandarshi, 1988) in Fig. 1. It can be observed that the values of $T_g$ calculated by using the present method—Eq. (B2)—are very close to the values obtained by numerical solution, whereas the values estimated by the previous relation—Eq (3)—differ from numerical solution by a few degrees. A comparison of absolute error in $T_g$ using the previous and the proposed relations is shown in Fig. 2. In addition, it is found by analysis that when $T_g$ is low and at the same time $T_p$, $L$ and $\beta$ are high, the absolute error in computation of $T_g$ will be

![Fig. 1. Variation of glass cover temperature with absorber plate temperature.](image1)

higher than what is seen from these two figures. Therefore, a comparison of the maximum errors in $T_g$ and $U_i$, considering all possible combinations of the variables is required. Such an analysis shows that the maximum computational error of 11° in $T_g$ using relation of the previous study is reduced to just 1 or 2° using the proposed relation. As a result, the maximum computational error in $U_i$ is reduced from 2.4% to 0.4%.

A major advantage of the proposed method is its applicability over a wider range of variables: 8 mm to 90 mm in air gap spacing and 273 K to 318 K in ambient temperature, as given in Table 1. So, the proposed equations are analyzed in the extended range of variables given in Table 1. A comparison of percentage error in the calculation of $U_i$ obtained by the proposed relation with respect to numerical solutions is shown in Fig. 3. Also shown are the computational errors for the approximate method based on use of an empirical equation for $U_i$. It can be seen from Fig. 3 that the absolute errors of proposed Eq. (2) are negligible as compared to empirical Eq. (1) over the entire range of plate temperature. The empirical relation has been derived for a limited range of variables,
temperature 0°C to 45°C. The maximum error in the calculation of the top heat loss coefficient by using the present method in comparison to that obtained by numerical solution of heat balance equations is about 2%.

APPENDIX A. THE HEAT BALANCE

Under steady-state conditions the rate of heat loss per unit area from the absorber plate at an average temperature $T_a$ to the glass cover at an average temperature $T_g$, equals that from the glass cover to the ambient air at $T_a$. Upward heat loss from the absorber to the glass cover is given by

$$Q_a^u = (h_{rkg} + h_{cpg})(T_a - T_g)$$

(A1)

and from the glass cover to the atmosphere by

$$Q_g^u = (h_{rga} + h_{rfa})(T_g - T_a)$$

(A2)

Solar radiation absorbed in the glass cover has been neglected. This energy influx should be accounted for in the overall energy balance by increasing the energy absorption term (through an artificial enhancement in the optical efficiency, since the magnitude of this energy influx is proportional to the insolation) rather than by decreasing the heat loss term (Duffie and Beckman, 1991). It is desirable to have an expression for $U_i$ for any absorber temperature, independent of the insolation level (Hottel and Woertz, 1942; Kreith and Kreider, 1978; Duffie and Beckman, 1991).

The glass cover temperature, $T_g$, has to be found by an iterative, or numerical solution of Eqs. (A1) and (A2) since the radiative and convective heat transfer coefficients are non-linear functions of $T_g$.

The individual heat-transfer coefficients

The heat transfer coefficient between the absorber plate and the glass cover is the sum of the radiative heat transfer coefficient, $h_{rkg}$, and the convective heat transfer coefficient, $h_{cpg}$. The radiative heat transfer coefficient is

$$h_{rkg} = \sigma(T_a^4 + T_g^4)(T_a + T_g)$$

$$\left(\frac{1}{\varepsilon_a} + \frac{1}{\varepsilon_g} - 1\right).$$

(A3)

The convective heat transfer coefficient can be calculated by using the following well known correlation for Nusselt number provided by Hollands et al. (1976):

Table 2. Maximum errors in $U_i$ with reference to numerical solution for the range of variables given in Table 1

<table>
<thead>
<tr>
<th>Glass cover thickness, $L_g$ (mm)</th>
<th>Maximum percentage error in $U_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.87</td>
</tr>
<tr>
<td>3</td>
<td>1.84</td>
</tr>
<tr>
<td>4</td>
<td>1.65</td>
</tr>
<tr>
<td>5</td>
<td>1.97</td>
</tr>
</tbody>
</table>
\[ Nu = 1 + 1.44[1 - 1708/Ra']^{1/4} \times [1 - 1708(\sin 1.8\beta)^{1.5}/Ra'] + [Ra'/5830]^{1/5} \]

where the + exponent implies that only positive values of the terms in the square brackets should be used (and ignored when they become negative).

Alternatively one may use the following three-region correlation for Nusselt number by Buchberg et al. (1976):

\[ Nu = 1 + 1.446(1 - 1708/Ra') \quad \text{for} \quad 1708 < Ra' < 5900; \quad Nu = 0.229(Ra')^{0.255} \quad \text{for} \quad 5900 < Ra' < 9.23 \times 10^4; \quad Nu = 0.157(Ra')^{0.255} \quad \text{for} \quad 9.23 \times 10^4 < Ra' < 10^6 \quad (A5) \]

where \( Ra' = Ra \cos \beta \).

The radiative heat transfer coefficient between the glass cover at temperature \( T_g \) to the ambient temperature at \( T_a \) is

\[ h_{rga} = \sigma e_g (T_g^4 + T_a^4)/(T_g + T_a) \quad (A6) \]

The properies of air: thermal conductivity, viscosity, volumetric coefficient of expansion and Prandtl number are considered to be functions of temperature.

**APPENDIX B. USERS' GUIDE FOR THE PROPOSED APPROXIMATE METHOD OF \( U_T \)**

The top heat loss coefficient can be computed using the analytical equation (without regrouping or approximating individual heat-transfer coefficients):

\[ U_T = \left[ \frac{\sigma(T_p^4 + T_a^4)(T_p + T_a)}{(1/\varepsilon_p + 1/\varepsilon_g - 1) + k Nu/L} \right]^{-1} + \left[ \frac{\sigma e_g (T_g^4 + T_a^4)(T_g + T_a) + h_{rga}}{1 + L/\varepsilon_g} \right]^{-1} \]

where

\[ f = \frac{(T_p + T_a)}{(1 + f_0)} \]

\[ f_0 = \left[ \frac{1 + 1.44[1 - 1708/Ra']^{1/4} \times [1 - 1708(\sin 1.8\beta)^{1.5}/Ra']}{[(Ra/5830)^{1.5}]^{1/5}} \right]^{-1} \]

**REFERENCES**


