PERFORMANCE OF AMPLITUDE MODULATION SCHEMES FOR MOLECULAR COMMUNICATION OVER A FLUID MEDIUM

AMIT SINGHAL

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY DELHI
APRIL 2016
PERFORMANCE OF AMPLITUDE MODULATION SCHEMES FOR MOLECULAR COMMUNICATION OVER A FLUID MEDIUM

by

AMIT SINGHAL

DEPARTMENT OF ELECTRICAL ENGINEERING

Submitted

in fulfillment of the requirements of the degree of Doctor of Philosophy

to the

INDIAN INSTITUTE OF TECHNOLOGY DELHI

APRIL 2016
Certificate

This is to certify that the thesis entitled “Performance of Amplitude Modulation Schemes for Molecular Communication Over a Fluid Medium” being submitted by Mr. Amit Singhal to the Department of Electrical Engineering, Indian Institute of Technology Delhi, for the award of the degree of Doctor of Philosophy is the record of the bona-fide research work carried out by him under my supervision. In my opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other university or institute for the award of any degree or diploma.

(Dr. Brejesh Lall)  
Associate Professor  
Department of Electrical Engineering  
Indian Institute of Technology Delhi

(Dr. Ranjan K. Mallik)  
Professor  
Department of Electrical Engineering  
Indian Institute of Technology Delhi

Date:  
New Delhi
Acknowledgements

First and foremost, I would like to gratefully acknowledge my advisors Dr. Brejesh Lall and Prof. Ranjan K. Mallik for their immense support and guidance of my research with true enthusiasm. Their profound technical knowledge and passion for work has always inspired me to work harder. I deem myself extremely fortunate to have the opportunity to work in guidance of top caliber faculty.

I am thankful to my student research committee members Prof. S. D. Joshi, Dr. Manav Bhatnagar, and Prof. Monika Aggarwal for many useful interactions and for their comments and suggestions on my work. I am also grateful to all the professors at IIT Delhi from whom I have learned a lot during the courses.

I would also like to thank all my friends and members of the communication group, especially Dr. Abhinav Kumar Sharma for his continuous support and encouragement. I feel lucky to have so many incredible friends around. I thank each and every one of them for their trust and friendship.

Most importantly, my heartfelt gratitude is for my parents Mr. V. K. Singhal and Mrs. Vipan Singhal who have always supported me in my academic pursuits and encouraged me to do my best. I must acknowledge my wife Mrs. Manika Singhal for the continuous motivation throughout my thesis work. I would also like to thank my sister Mrs. Aastha Aggarwal for her unceasing encouragement and loving support.
I also place on record, my sense of gratitude to one and all who, directly or indirectly, have bestowed their helping hand in this imperil.

Amit Singhal
Molecular communication involves the use of nanomachines to perform the role of transmitters and receivers. The signals are carried in the form of molecules propagating through the medium connecting the nanomachines. It serves as a mechanism to carry information at nano-scales. The information can be encoded in the count, concentration, type or the release time of the various molecules. The quantity-based amplitude modulation schemes are found to offer a promising approach to set up molecular communication over diffusion-based channels. Before the practical deployment of such schemes, appropriate channel models and their performance potential need to be analyzed.

As a first contribution of this thesis, we consider molecular communication between two nanomachines placed in a fluid medium for three different amplitude modulation schemes. The number of molecules transmitted represents the amplitude levels for these schemes. We consider a time-slotted channel, where the information in every slot is corrupted by stray molecules from the previous slots. The capacity of such a molecular communication channel is investigated for all the three modulation schemes. Analytical expressions for the end-to-end symbol error probability are derived, considering maximum likelihood detection at the receiver. Numerical results indicate that arbitrarily
low probabilities of error can be achieved for high drift velocities. An increase in the
slot length further improves the performance, albeit at the cost of data rate. The results
also demonstrate the improvements offered by the amplitude modulation schemes over
the previously proposed time modulation schemes.

Next, we explore the effect of molecular noise in diffusion-based molecular commu-
nication. As a molecule travels through a fluid medium, its continuous collision with
the molecules of the fluid can be considered as a continuous random force acting on the
molecule. This force is the source of uncertainty in the position of the molecule and
can be treated as a noise. We propose a mathematical model for such noise. The role
of finite life expectancy of molecules and inter symbol interference is also considered.
Closed-form expressions for the probability of error and the channel capacity are de-

erived considering a binary communication scheme. The numerical results illustrate the
impact of these parameters on the performance of molecular communication techniques.

Thereafter, we extend the performance analysis for the multiple molecule case as
opposed to single molecules for defining the various amplitude levels. We consider an
\( M \)-ary modulation scheme and also propose an extended scheme, which is a variation
of a binary modulation scheme. The received symbol is corrupted by interference from
the previous symbols as well as the noise sources present in the medium. We propose an
end-to-end model addressing all the relevant noise sources affecting such communication
systems. Considering maximum likelihood detection at the receiver, we derive analyt-
ical expressions for the end-to-end symbol error probability and the capacity for these
modulation schemes. Numerical results bring out the impact of various parameters on
the performance of the system.
# Table of Contents

Certificate i

Acknowledgements ii

Abstract iv

List of Contents vi

List of Figures x

1 Introduction 1

1.1 Brief Overview ............................................. 1

1.2 Key Features of Molecular Communication ........................ 2

1.3 Molecular Propagation ........................................ 4

1.3.1 Diffusion Process ......................................... 5

1.3.2 Diffusion with Drift ....................................... 6

1.4 Inter Symbol Interference ..................................... 8

1.5 Related Work .................................................. 9

1.6 Motivation ..................................................... 11
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7</td>
<td>Key Contributions</td>
<td>12</td>
</tr>
<tr>
<td>1.8</td>
<td>System Model for a Time-slotted Molecular Channel</td>
<td>14</td>
</tr>
<tr>
<td>1.8.1</td>
<td>System Model-I</td>
<td>14</td>
</tr>
<tr>
<td>1.8.2</td>
<td>System Model-II</td>
<td>17</td>
</tr>
<tr>
<td>1.9</td>
<td>Organization of Thesis</td>
<td>19</td>
</tr>
<tr>
<td>2</td>
<td>Single Molecule Amplitude Modulation Schemes</td>
<td>22</td>
</tr>
<tr>
<td>2.1</td>
<td>Introduction</td>
<td>22</td>
</tr>
<tr>
<td>2.2</td>
<td>Amplitude Modulation Schemes</td>
<td>23</td>
</tr>
<tr>
<td>2.2.1</td>
<td>Binary Modulation Scheme</td>
<td>23</td>
</tr>
<tr>
<td>2.2.2</td>
<td>4-ary Modulation Scheme</td>
<td>24</td>
</tr>
<tr>
<td>2.2.3</td>
<td>Third Modulation Scheme</td>
<td>25</td>
</tr>
<tr>
<td>2.3</td>
<td>Capacity Analysis</td>
<td>27</td>
</tr>
<tr>
<td>2.3.1</td>
<td>Capacity for Binary Modulation Scheme</td>
<td>28</td>
</tr>
<tr>
<td>2.3.2</td>
<td>Capacity for 4-ary Modulation Scheme</td>
<td>31</td>
</tr>
<tr>
<td>2.3.3</td>
<td>Capacity for Third Modulation Scheme</td>
<td>33</td>
</tr>
<tr>
<td>2.4</td>
<td>SEP Analysis</td>
<td>34</td>
</tr>
<tr>
<td>2.4.1</td>
<td>SEP for Binary Modulation Scheme</td>
<td>35</td>
</tr>
<tr>
<td>2.4.2</td>
<td>SEP for 4-ary Modulation Scheme</td>
<td>38</td>
</tr>
<tr>
<td>2.4.3</td>
<td>SEP for Third Modulation Scheme</td>
<td>41</td>
</tr>
<tr>
<td>2.4.4</td>
<td>Numerical Results</td>
<td>43</td>
</tr>
<tr>
<td>2.5</td>
<td>Summary</td>
<td>45</td>
</tr>
<tr>
<td>3</td>
<td>Role of Molecular Noise</td>
<td>54</td>
</tr>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>54</td>
</tr>
</tbody>
</table>
3.2 Langevin Model ........................................ 55
  3.2.1 Mean Square Displacement ................. 56
  3.2.2 Molecular Displacement .................. 56
3.3 Communication Scheme ............................... 58
3.4 Noise Model ........................................ 60
3.5 Performance Analysis ............................... 61
  3.5.1 Probability of Error ....................... 62
  3.5.2 Channel Capacity .......................... 63
  3.5.3 Numerical Results ...................... 64
3.6 Summary ........................................... 65

4 Multiple Molecule Amplitude Modulation Schemes 68
  4.1 Introduction ..................................... 68
  4.2 Amplitude Modulation Schemes ................. 69
    4.2.1 M-ary Modulation Scheme ................. 69
    4.2.2 Extended Modulation Scheme .............. 73
  4.3 Capacity Analysis ................................ 78
    4.3.1 Capacity for M-ary Modulation Scheme .. 79
    4.3.2 Capacity for Extended Modulation Scheme 81
  4.4 Imperfect Transmitter ........................... 85
  4.5 Numerical Results ................................ 86
  4.6 Summary ......................................... 90

5 Conclusions and Future Work .......................... 98
  5.1 Contributions of this Research ................ 98
List of Figures

1.1 Communication model of a basic time-slotted channel for a total of $k$ slots. 15
1.2 Communication model of a time-slotted channel for $k$th slot. 18

2.1 Mutual information versus probability $a_1$ for binary scheme. 47
2.2 Mutual information versus probability $a_0$ for 4-ary scheme. 48
2.3 Variation of SEP with drift velocity $v$ for all three schemes when $D = 1 \times 10^{-10} \text{ m}^2/\text{s}$, $T = 20 \text{ ns}$, $\alpha = 0.1$. 49
2.4 Variation of SEP for binary scheme with drift velocity $v$ for different values of degradation parameter $\alpha$ when $D = 1 \times 10^{-10} \text{ m}^2/\text{s}$, $T = 20 \text{ ns}$. 50
2.5 Variation of SEP with diffusion coefficient $D$ for all three schemes when $v = 0.25 \text{ m/s}$, $T = 50 \text{ ns}$, $\alpha = 0.1$. 51
2.6 Variation of SEP with slot length $T$ for all three schemes when $v = 0.5 \text{ m/s}$, $D = 1 \times 10^{-10} \text{ m}^2/\text{s}$, $\alpha = 0.1$. 52
2.7 Comparison of amplitude modulation schemes with time modulation schemes, considering variation of SEP with drift velocity $v$, when $D = 1 \times 10^{-10} \text{ m}^2/\text{s}$, $T = 20 \text{ ns}$, $\alpha = 0.1$. 53

3.1 Probability of error $P_e$ versus average SINR $\bar{\Gamma}$. 66
3.2 Channel capacity $C_k$ versus average SINR $\bar{\Gamma}$.  

4.1 Variation of SEP with drift velocity $v$ when $D = 1 \times 10^{-10} \text{m}^2/\text{s}$, $\alpha = 0.1$.  

4.2 Variation of SEP with diffusion coefficient $D$ when $v = 10 \mu\text{m}/\text{s}$, $\alpha = 0.1$.  

4.3 SEP versus noise variance $\sigma^2_{N_o}$ when $v = 14 \mu\text{m}/\text{s}$, $D = 1 \times 10^{-10} \text{m}^2/\text{s}$, $\alpha = 0.1$.  

4.4 SEP versus transmitter variance $\sigma^2_t$ when $v = 14 \mu\text{m}/\text{s}$, $D = 1 \times 10^{-10} \text{m}^2/\text{s}$, $\alpha = 0.1$.  

4.5 SEP versus number of molecules $\beta_1$ when $v = 10 \mu\text{m}/\text{s}$, $D = 1 \times 10^{-10} \text{m}^2/\text{s}$.  

4.6 Mutual information versus a priori probability $a_1$ for binary scheme when $\alpha = 0.1$.  

4.7 Capacity versus noise variance $\sigma^2_{N_o}$ when $v = 14 \mu\text{m}/\text{s}$, $D = 1 \times 10^{-10} \text{m}^2/\text{s}$, $\alpha = 0.1$.  
