Oscillatory stability limit enhancement by adaptive control rescheduling

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Abstract

This paper presents a novel concept of utilizing adaptive corrective capability of the network for enhancing oscillatory stability limit. Researchers have analyzed this stability and suggested optimization of controller parameters for mitigating the problem. When such measures are found to be inadequate during operations and instability with expected load growth is imminent, the proposed approach shows how it is possible to tackle the problem by adaptive real and reactive controls rescheduling. This is achieved through reactive and real power optimization problems for optimizing the margin of oscillatory instability, which is periodically assessed in real time. Results for a sample test system demonstrate the feasibility of the proposed approach.

Keywords: Oscillatory instability; Real and reactive rescheduling; Optimization

Nomenclature

\begin{itemize}
\item $\lambda_{BB}$: load multiplier corresponding to the oscillatory instability limit
\item NC: number of controls
\item $U_i$: $i$th control variable
\item $QG(i)$: reactive generation at $i$th PV bus
\item $QG(i)_{\text{min}}$: minimum limit on $i$th PV bus reactive generation
\item $QG(i)_{\text{max}}$: maximum limit on $i$th PV bus reactive generation
\item $V_i$: $i$th load bus voltage
\item $V_i^{\text{dmax}}$: minimum allowable $i$th load bus voltage
\item $V_i^{\text{dmax}}$: maximum allowable $i$th load bus voltage
\item $g$: set of nonlinear load flow equations
\item $\Delta V(i)$: change in $i$th bus voltage
\item $S_{i}(L_J)$: $i$th bus voltage sensitivity with respect to the $J$th control
\item $\Delta P_d(J)$: change in the $i$th power generation
\item $\Delta \delta(J)$: change in the $i$th bus angle
\item $S_{d}(L_J)$: $i$th bus angle sensitivity with respect to the $J$th control
\item $\Delta QG(i)$: change in the $i$th reactive generation
\item $S_{QG}(L_J)$: $i$th bus reactive generation sensitivity with respect to the $J$th control
\item $S_L(i)$: sensitivity of distance to oscillatory instability, in terms of load multiplier, with respect to the $i$th control
\item $\Delta U(J)$: change in the $i$th control
\item $\Delta U(J)_{\text{min}}$: minimum change in the $i$th control
\item $\Delta U(J)_{\text{max}}$: maximum change in the $i$th control
\item $\Delta P_L$: change in total real loss
\item $\Delta P_D$: change in total load demand
\item $X$: state variable
\item $Y$: control variable
\item $\omega_s, \omega$: actual and synchronous speeds in rad/s
\item $M$: machine constant proportional to machine inertia
\item $D$: damper constant
\item $K_E, T_E$: exciter gain and time constant
\item $K_v, T_v$: amplifier gain and time constant
\item $K_L, T_L$: stabilizing transformer gain and time constant
\item $I_d, I_q$: direct and quadrature axis currents
\item $R_s$: stator resistance
\item $X_d, X_q$: direct and quadrature axis reactances
\item $X_{d1}, X_{d2}$: direct and quadrature axis transient reactances
\item $E_{d1}, E_{q1}$: voltages proportional to damper winding flux linkage and field flux linkage
\item $E_d$: voltage proportional to field voltage
\item $S_{Ed}$: exciter saturation function
\item $V_R$: output of AVR amplifier
\item $V_{ref}$: terminal voltage reference setting
\item $QF$: objective function
\item $\epsilon$: tolerance
\end{itemize}
1. Introduction

One of the common limitations in operation of a power system is oscillatory instability. It has been observed that for certain topological configurations and loading conditions small perturbations such as load changes can lead to oscillations in voltages and powers in some part of the system. This is due to the lack of adequate damping under these conditions. These oscillations can disappear after some time or grow with time depending upon the degree of damping in the system.

A power system is generally expressed by nonlinear dynamical system of equations. In any nonlinear dynamical system, it is well known that a qualitative change in the behavior of the system due to change of one or more parameters is due to bifurcations. A bifurcation is a qualitative change in the phase portrait of a dynamical system that occurs when a system bifurcation parameter is quasi-statically varied. ‘Hopf’ and ‘Saddle Node’ bifurcations are mainly recognized in a variety of power system models. ‘Hopf Bifurcation’ manifests itself in oscillatory stability analysis, in the form of a complex eigenvalue which crosses the imaginary axis and enters into the right half plane. The saddle node bifurcation can occur due to effects of over excitation limiters and reverse action of OLTCs, particularly at higher system loadings. The present work, however, deals with only the oscillatory stability due to Hopf bifurcation.

De Mello and Feltes [1] have analyzed the problem of oscillatory instability caused by interaction between automatic voltage regulator and induction motor loads. This phenomenon is quite common in power systems of developing countries, or emergency power supply systems of auxiliaries. As the AVR tuned at one particular loading condition may not function satisfactorily at other loading conditions they have suggested AVR compensation as a solution.

Kim et al. [2] have proposed an algorithm to calculate the oscillatory (Hopf bifurcation) stability for large power systems. They have described a recorded event on 12 June 1992, which occurred, on the mid-western segment of the US interconnection and shown that the analysis using the developed algorithm confirms the observed phenomenon.

Stability limit. It appears logical, therefore, to explore the possibility of utilizing rescheduling capability of the network to enhance this limit. This capability has been shown to be effective in enhancing the oscillatory stability limit [2]. In this case the rescheduling of generations was carried out after the commencement of oscillations.

Samuelson and Eihasson [5] have proposed a scheme wherein utility controlled customer loads are used for damping electromechanical oscillations in a multi-machine power system. Advances in distribution automation and demand side management have opened these possibilities.

It must be noted, however, that waiting for the oscillations to develop before corrective action is fraught with danger, as there may not be sufficient time to prevent instability. A safer approach is to anticipate the danger by computing distance to instability in the direction of load increase scenario. This can be done by computing eigenvalues of the system Jacobian matrix in linearized dynamic analysis [6,7]. The change of sign from negative to positive of the real part of any complex eigenvalue signifies Hopf bifurcation or oscillatory instability [6–9].

Preventive rescheduling of controls means deviation from
the optimized schedules satisfying base case requirements and should be resorted to only when oscillatory instability is likely to occur in a time frame less than that between two successive control rescheduling steps.

Although a number of papers have appeared in the literature on oscillatory stability analysis, to the best of the authors' knowledge no efforts have been made towards optimal preventive rescheduling of such controls for enhancing such a margin. The aim of this study is to show how real and reactive power flow controls can be rescheduled in anticipation to extend oscillatory stability limit so that the next expected load step can be met. In the proposed approach the oscillatory stability margin is periodically computed in real time in the Energy Control Center. As long as the margin is greater than the next load step in a time frame which corresponds to two successive margin calculations prior to rescheduling of controls, nothing needs to be done. Otherwise sensitivities of the margin to real and reactive controls are computed. These are then used in a Linear Programming formulation to enhance the margin to the extent desired, subject to usual system constraints. As any deviation from the Economic Dispatch generation schedule has cost implications, the feasibility of utilization of only reactive controls in enhancing oscillatory stability limit sufficiently is examined first. Only when these controls are inadequate in this respect, the costlier option of real power rescheduling is considered. The flow chart in Fig. 1 explains the proposed scheme. The first part of the flow chart pertains to the monitoring stage and the latter part explains how the adaptive control rescheduling is to be carried out. The last block in the flow chart shows that if all the possible controls are exhausted, the last option like load shedding needs to be applied to save the system from collapse.

2. Mathematical modeling

When it is ascertained that the system cannot serve a predicted load until the next rescheduling of controls due to imminent oscillatory instability, measures such as reactive rescheduling, real power rescheduling, load shedding, etc. must be taken to prevent instability. Each of these measures influences the cost of operation. Hence, some optimization must be carried out to utilize these controls effectively. As these controls have to be implemented in the present operating condition and would remain unchanged until the next rescheduling step, the optimization problem must satisfy various equality and inequality constraints on system operation at these two operating points.

2.1. Reactive rescheduling for oscillatory stability enhancement

The reactive controls are voltage settings at voltage controlled (PV) buses, switched shunt capacitors/reactors and on load tap changer (OLTC) tap positions.

Whenever there is a change in control variables at base case which remain fixed until the next rescheduling step, there will be changes in bus voltage magnitudes, angles, reactive generations at the above two operating points and the distance to dynamic stability limit in terms of load multiplier $\lambda_{\text{HB}}$. These changes can be expressed as follows.

$$\Delta V(I) = \sum_{J=1}^{NC} SV(I,J) \Delta U(J),$$

(1)

$$\Delta \delta(I) = \sum_{J=1}^{NC} SD(I,J) \Delta U(J),$$

(2)

$$\Delta QG(I) = \sum_{J=1}^{NC} SQG(I,J) \Delta U(J),$$

(3)

where $SV$, $SD$ and $SQG$ are standard load flow sensitivities and are derived in Ref. [10] and

$$\Delta \lambda_{\text{HB}} = \sum_{J=1}^{NC} SL(I,J) \Delta U(J).$$

(4)

Sensitivities of the loading parameter, i.e. $SL(I,J)$ are obtained by the perturbation technique using dynamic analysis [6] as explained in the appendix.

With the abovementioned linearization, a Linear Programming problem can be set up as follows:

$$\max \lambda_{\text{HB}} = \left[ \lambda_{\text{HB}}^0 + \sum_{J=1}^{NC} SL(I,J) \Delta U(J) \right]$$

subject to

$$QG(I)_{\min} \leq QG(I) + \sum_{J=1}^{NC} SQG(I,J) \Delta U(J) \leq QG(I)_{\max}$$

$$QG(I)_{\min} \leq QG(I) + \sum_{J=1}^{NC} SQG(I,J) \Delta U(J) \leq QG(I)_{\max}$$

$$I \in \text{voltage controlled bus.}$$

(6)

$$V_L(I)_{\min} \leq V_L(I) + \sum_{J=1}^{NC} SV(I,J) \Delta U(J) \leq V_L(I)_{\max}$$

$$V_L(I)_{\min} \leq V_L(I) + \sum_{J=1}^{NC} SV(I,J) \Delta U(J) \leq V_L(I)_{\max}$$

$$I \in \text{load bus.}$$

(7)

$$\Delta U(J)_{\min} \leq \Delta U(J) \leq \Delta U(J)_{\max} \quad J \in \text{controls}$$

(8)

where, $\Delta U(I)_{\min} = U(I)_{\min} - U(I)$ and $\Delta U(I)_{\max} = U(I)_{\max} - U(I)$.

This optimization is carried out until $\lambda_{\text{HB}}$ increases to a value slightly higher than that corresponding to the next
predicted load step in a time frame corresponding to two successive control rescheduling steps.

Superscript 0 denotes the variables at the current operating point and superscript 1 denotes the variables at the oscillatory instability point.

The load increase scenario can be simulated by using a

scalar load parameter $\lambda$ to change real power generations and real and reactive loads as follows:

$$
P^1_G(I) = P^0_G(I) + \lambda_{HB} KQ(I).
$$

$$
P^1_L(I) = P^0_L(I) + \lambda_{HB} KLP(I).
$$

$$
Q^1(I) = Q^0(I) + \lambda_{HB} KLO(I).
$$

where, $P^0_G(I)$ and $P^0_L(I)$ are the base case real generation at the 4th bus and $P^0_L(I)$ are the base case real and reactive loads at the 4th bus, $KQ(I)$, $KLP(I)$ and $KLO(I)$ denote, respectively, factors at 4th bus governing real generation, real and reactive loads change scenario, respectively. $P^1_G(I), P^1_L(I)$ and $Q^1(I)$ are generation, real and reactive loads at the oscillatory instability point. $\lambda = 0$ indicates base case loading condition.

Constant values of $KQ(I)$, $KLP(I)$ and $KLO(I)$ denote the

proportional load increase scenario, i.e. all the bus load/generations increase proportionately. Although a proportional load increase scenario is adopted, any other scenario can be used if data are available. Similarly the load power factors need not remain fixed as the load changes.

As the problem is a nonlinear one, the successive linear programming technique is used to solve it. This technique was chosen because of the availability of efficient software packages and its excellent constraint handling capability. However, other mathematical programming techniques can also be used for this problem.

The equality constraint is satisfied separately by running load flow after every LP step. Although load bus voltage inequality constraints are present in the general problem formulation, it needs to be emphasized that in many cases it may be necessary to relax these since saving the system from instability is a primary concern.

2.2. Real power rescheduling for oscillatory stability enhancement

When the reactive controls are inadequate to enhance the oscillatory stability limit to the desired extent, the real power rescheduling option needs to be considered. The optimization problem for such rescheduling is formulated as follows:

$$
\max \lambda_{HB} = \left[ \lambda_{HB}^0 + \sum_{I=1}^{NG} S(I) \Delta P^0_G(I) \right]
$$

subject to

$$
QG(I)_{\text{min}} \leq QG^0(I) + \sum_{J=1}^{NG} SQQG(I,J) \Delta P^0_G(J) \leq QG(I)_{\text{max}}
$$

$$
QG(I)_{\text{min}} \leq QG^1(I) + \sum_{J=1}^{NG} SQQG(I,J) \Delta P^1_G(J) \leq QG(I)_{\text{max}}
$$

$$
I \in \text{voltage controlled bus, (13)}
$$

$$
V_L(I)_{\text{min}} \leq V_L^0(I) + \sum_{J=1}^{NG} SV^0(I,J) \Delta P^0_G(J) \leq V_L(I)_{\text{max}}
$$

$$
V_L(I)_{\text{min}} \leq V_L^1(I) + \sum_{J=1}^{NG} SV^1(I,J) \Delta P^1_G(J) \leq V_L(I)_{\text{max}}
$$

$$
I \in \text{load bus, (14)}
$$

$$
\Delta P_G(J)_{\text{min}} \leq \Delta P_G(J) \leq \Delta P_G(J)_{\text{max}} \quad J \in \text{controls, (15)}
$$

$$
\sum_{I=1}^{NG} \Delta P^0_G(I) - \Delta P^1_G - \Delta P_D^0 = 0.
$$

As the problem is nonlinear, Successive Linear Programming (SLP) is used to solve the problem. Load Flow constraints are satisfied after every LP iteration. However, as shown above, an incremental real power balance equality constraint needs to be added to the LP problem formulation, because unlike in the previous problem, the real powers are rescheduled here and the total real generation minus total real losses must remain constant to meet the load demand. The comments about voltage constraints in the previous formulation apply here as well.

Constraints (13)-(16) take care of reactive limits on generators, voltage magnitude limits on each bus, control variables and real power balance in the system, respectively.

Algorithm

1. Read Data.
2. Run Base Case Load Flow.
3. Predict load multiplier at next loading step.
4. Optimize all controls with conventional objective functions like economic load dispatch, loss minimisation etc.
5. Compute the load multiplier, $\lambda_{HB}$, at which the oscillatory instability is going to occur.
6. If $\lambda_{HB}$ is greater than $\lambda_P$ go to step 3.
7. Solve SLP with reactive controls.
   (a) Compute sensitivities, $SL$, $SQQG$, $SQQG^1$, $SV^0$, $SV^1$ w.r.t reactive controls.
   (b) Formulate and solve the Linear Programming problem.
   (c) With optimized controls, run load flow and compute $\lambda_{HB}$.
   (d) If $\lambda_{HB}$ is greater than $\lambda_P$ go to step 3.
Table 1
Real and reactive controls for 9-bus system at the current operating point

<table>
<thead>
<tr>
<th>Real power controls in p.u.</th>
<th>Reactive power controls in p.u.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1 = 1.3843$</td>
<td>$Q_1 = 1.6400$</td>
</tr>
<tr>
<td>$P_2 = 2.9340$</td>
<td>$Q_2 = 1.0250$</td>
</tr>
<tr>
<td>$P_3 = 1.5300$</td>
<td>$Q_3 = 1.0250$</td>
</tr>
</tbody>
</table>

(c) If the change in objective function is greater than $\epsilon$ go to 7(a).

8. Solve SLP with real power controls.

(a) Compute sensitivities, $Sl, SQG^0, SQG^1, SL^0, SL^1$ w.r.t real power controls.
(b) Formulate and solve the Linear Programming problem.
(c) With optimized controls, run load flow and compute $\lambda_{HE}$.
(d) If $\lambda_{HE}$ is greater than $\lambda_P$ go to step 3.
(e) If the change in objective function is greater than $\epsilon$ go to 8(a).

9. Resort to emergency measures like load shedding till $\lambda_{HE}$ is greater than $\lambda_P$ and go to step 3.

3. Implementation issues

As compared to the computational burden involved in loadability enhancement [11], the computational effort involved in the proposed oscillatory limit enhancement in real time is certainly much higher due to the sensitivity calculation of $\lambda_{HE}$ involving eigenvalue computation of the system Jacobian matrix. This is despite the fact that efficient approaches are available for the computation of the latter. In view of the above the following strategy can be adopted for making the implementation and the proposed algorithm feasible.

(a) In the morning every day, with the expected load forecast and network topology one can assess the periods of vulnerability due to oscillatory instability. The plan of action involving real and reactive control rescheduling to enhance the stability limit can also be kept ready as per the proposed algorithm. As we approach the period of concern, the deviation in load forecast and/or network topology can be taken into account to modify the action plan drawn up earlier. This essentially constitutes an offline approach.

(b) If the proposed scheme is to be implemented in real time with periodic monitoring of oscillatory instability and control rescheduling to enhance the stability limit, the frequency of this control scheme can be half an hour or even 1 hour depending on the computational resources available at the energy control center and keeping in view other constraints.

A combination of (a) and (b) would be the best strategy so that unnecessary computations involved in real time in monitoring instability can be completely eliminated if the morning evaluation has indicated vulnerability only in a limited period (generally heavy to very heavy conditions). This strategy will also allow the changes in load forecast and the topology to be taken into account to refine the already drawn up plan of adaptive control rescheduling.

More work is needed to make the proposed adaptive control strategy computationally more efficient.

It must be emphasized again that the proposed adaptive control rescheduling option for oscillatory instability enhancement is to be considered only as a last resort where system studies establish that the stabilizers and other available control measures are not effective enough and that oscillations at some operating condition are inevitable. The only option left will then be to try and damp the oscillations by generation or other control rescheduling after the oscillations have actually commenced.

4. Results

Studies for a 9-bus test system [12] have been carried out to demonstrate the potential of the proposed real and reactive rescheduling algorithms to prevent dynamic instabilities.

Although the derivations in this paper are general only constant $P, Q$ loads were assumed. The limits on various controls are as follows:

Only PV bus voltages are used as reactive controls. The limits on these bus voltages are 0.95 and 1.10 p.u. The maximum and minimum real power generations at all generator buses are taken as $P_{lim}(1) = 1.2469$ p.u., $P_{lim}(2) = 2.6406$ p.u., $P_{lim}(2) = 3.2724$ p.u., $P_{lim}(3) = 1.3770$ p.u. and $P_{lim}(3) = 1.683$ p.u.

Table 1 shows the real and reactive controls at the current operating point (i.e. at 1.8 times specified loading in Ref. [12]). The loading level is deliberately chosen to be near the dynamic instability limit $P_1$, $P_2$ and $P_3$ are the generations at buses 1, 2 and 3, respectively, and $V_1$, $V_2$ and $V_3$ are the bus voltages at buses 1, 2 and 3, respectively. at the abovementioned loading.

Table 2 shows the eigenvalues of the system state matrix.

Table 2
Eigenvalues for 9-bus system at the current operating point

<table>
<thead>
<tr>
<th>Eigenvalues at the base case</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$-4.9980 \pm 7.6790$</td>
<td></td>
</tr>
<tr>
<td>$-5.1520 \pm 7.8572$</td>
<td></td>
</tr>
<tr>
<td>$-5.1511 \pm 7.8693$</td>
<td></td>
</tr>
<tr>
<td>$-4.1519$</td>
<td></td>
</tr>
<tr>
<td>$-3.7465$</td>
<td></td>
</tr>
<tr>
<td>$-0.4127 \pm 1.3733$</td>
<td></td>
</tr>
<tr>
<td>$-0.5667 \pm 7.7900$</td>
<td></td>
</tr>
<tr>
<td>$-0.4695 \pm 0.8498$</td>
<td></td>
</tr>
<tr>
<td>$100 \times 0.0000$</td>
<td></td>
</tr>
</tbody>
</table>
for the 9-bus system at the current operating point. The negative real parts of all eigenvalues indicate that current operating point is dynamically stable.

Table 3 shows that at the next predicted load step (i.e., 1.935 times specified loading in Ref. [12]) the system has one pair of eigenvalues with a positive real part indicating oscillatory instability if controls are unchanged. It also shows that rescheduled reactive controls obtained with the proposed optimization result in oscillatory stability as all eigenvalues now have negative real parts.

Table 4 shows how the oscillatory stability at the next load step can be achieved by the real power rescheduling only (reactive controls fixed at original base case values). As mentioned earlier at the next load step the system is dynamically unstable. However, real power rescheduling with the proposed algorithm results in oscillatory stability at the next load step as all eigenvalues have negative real parts.

5. Conclusion

During power system operation if it is ascertained that the system controllers are not effective in maintaining oscillatory stability due to the Hopf bifurcation, this paper has shown how the real and reactive power rescheduling capabilities of the network can be used to stabilize the system. These controls are obtained from the real and reactive power optimization problems developed for enhancing distance to oscillatory instability. Results for a 9-bus test system have demonstrated the utility of the proposed approach.

Appendix A. Oscillatory stability margin sensitivity calculations

The sensitivities (SL) of the distance to oscillatory instability in terms of load multiplier (corresponding to the Hopf bifurcation point, \( \lambda_{HB} \)) with respect to real and reactive controls are calculated as follows:

\[
SL(J) = \frac{\delta \lambda_{HB}}{\delta U(J)} \quad J = 1, \ldots, N_C.
\]

For a particular set of controls, the load multiplier corresponding to oscillatory instability can be calculated by eigen analysis of the system Jacobian matrix. At this point at least one of the complex eigenvalues of the reduced system matrix crosses the imaginary axis and enters into the right half plane.

The reduced system matrix is formed as follows:

The nonlinear dynamical system model can be expressed by a set of differential and algebraic equations [6].

\[
\dot{X} = f(X, Y),
\]

\[
0 = g(X, Y).
\]

Differential equation (A1) include the following:

\[
\delta \dot{Y}_i = \omega_0 - \omega_i, \quad i = 1, \ldots, m.
\]

\[
M_j \frac{\partial \phi_j}{\partial t} = T_{\phi_j} - [E_{\phi_j} - X_{\phi_j}I_{\phi_j}]V_{\phi_j} - [E_{\phi_j} + X_{\phi_j}I_{\phi_j}]I_{\phi_j}
\]

\[-D_i(\omega_0 - \omega_i), \quad i = 1, \ldots, m.
\]

\[
T_{\phi_i} \frac{\partial E_{\phi_i}}{\partial t} = -E_{\phi_i} - (X_{\phi_i} - X_{\phi_j})I_{\phi_j} + E_{\phi_k}, \quad i = 1, \ldots, m,
\]

\[
T_{\phi_i} \frac{\partial E_{\phi_i}}{\partial t} = -E_{\phi_i} - (X_{\phi_i} - X_{\phi_j})I_{\phi_j}, \quad i = 1, \ldots, m,
\]

\[
T_{E_{\phi_k}} \frac{\partial E_{\phi_k}}{\partial t} = -(K_{E_{\phi_k}} + S_{E_{\phi_k}}E_{\phi_k})E_{\phi_k} + V_{t_{\phi_k}}, \quad i = 1, \ldots, m.
\]
\[ T_{\alpha\mu} \frac{dV_{\gamma}}{dt} = -V_{\beta\mu} + K_{\alpha\mu} R_{\beta\mu} - \frac{K_{\alpha\mu} K_{\Theta}}{T_{\gamma\mu}} E_{\beta\mu} + K_{\alpha\mu}(V_{\beta\mu} - V_{\gamma}) \],
\quad i = 1, \ldots, m. \tag{A8} \]

\[ T_{F\mu} \frac{dE_{\beta\mu}}{dt} = -R_{F\mu} + \frac{K_{F\mu}}{T_{F\mu}} E_{\beta\mu}, \quad i = 1, \ldots, m. \tag{A9} \]

*Stator algebraic equations*

\[ V_{i} \cos \theta_{i} + R_{ai}(U_{ai} \sin \delta_{i} + I_{qi} \cos \delta_{i}) - X_{ai}(U_{qi} \sin \delta_{i} - I_{ai} \cos \delta_{i}) - [E_{ci} \sin \delta_{i} + (X_{qi} - X'_{ci})I_{qi} \sin \delta_{i} + E_{qi} \cos \delta_{i}] = 0, \quad i = 1, \ldots, m. \tag{A10} \]

\[ V_{i} \sin \theta_{i} + R_{ai}(U_{ai} \sin \delta_{i} - I_{qi} \cos \delta_{i}) + X_{ai}(U_{qi} \sin \delta_{i} + I_{ai} \cos \delta_{i}) + I_{qi} \cos \delta_{i} - [E_{ci} \sin \delta_{i} - (X_{qi} - X'_{ci})I_{qi} \cos \delta_{i} - E_{qi} \cos \delta_{i}] = 0, \quad i = 1, \ldots, m. \tag{A11} \]

*Network equations*

\[ P_{i} = V_{i}(U_{ai} \sin(\delta_{i} - \theta_{i}) + I_{ai} \cos(\delta_{i} - \theta_{i})) + P_{li}(V_{i}), \quad i = 1, \ldots, m. \tag{A12} \]

\[ Q_{i} = V_{i}(U_{ai} \cos(\delta_{i} - \theta_{i}) + I_{ai} \sin(\delta_{i} - \theta_{i})) + Q_{li}(V_{i}), \quad i = 1, \ldots, m. \tag{A13} \]

The loads are considered to be voltage dependent and modeled as

\[ P_{i} = P_{li}(V_{i}) \quad i \in \text{load buses}, \tag{A14} \]

\[ Q_{i} = Q_{li}(V_{i}) \quad i \in \text{load buses}. \tag{A15} \]

Finally the real and reactive injections at each bus in terms of voltage magnitude, conductances and admittances are as follows:

\[ P_{i} = \sum_{k=1}^{nbus} V_{i}V_{k}(G_{ak} \cos \theta_{ak} + B_{ak} \sin \theta_{ak}) \quad i = 1, nbus, \tag{A16} \]

\[ Q_{i} = \sum_{k=1}^{nbus} V_{i}V_{k}(G_{ak} \sin \theta_{ak} - B_{ak} \cos \theta_{ak}) \quad i = 1, nbus. \tag{A17} \]

For small disturbance stability analysis, the operating point is obtained from load flow and the above complete set of equations is linearized around the operating point to give

\[
\begin{bmatrix}
\frac{d\Delta Y}{dt}
\end{bmatrix} = \begin{bmatrix}
A & B \\
C & D_{11} & D_{12} \\
D_{21} & D_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta Y
\end{bmatrix},
\]

where \( \Delta Y = [\Delta U, \Delta I, \Delta P, \Delta Q, \Delta V, \Delta \delta] \) and \( \Delta Y2 = [\Delta V, \Delta \delta] \).

From Eq. (A18)

\[ \frac{d \Delta Y}{dt} = [A - BD^{-1}C] \Delta Y = ASYS \Delta Y. \]

\( ASYS \) is the reduced dynamic system state matrix and the eigenvalues of this matrix determine the oscillatory stability of the system at the operating point.

Once the load multiplier \( \lambda_{HE}^0 \) is calculated for the set of controls at the operating point and for the set of controls at the operating point with perturbation in one of the control variables, the sensitivities \( SL \) can be calculated as

\[ SL(U) = \frac{\Delta \lambda_{HE}^0}{\Delta U(U)} = \frac{\lambda_{HE}^{op} - \lambda_{HE}^{o}}{\Delta U(U)}. \]

\( \lambda_{HE}^{op} \) is the load multiplier at the Hopf bifurcation point with a small perturbation \( \Delta U \) in control \( J \) from the original value. \( \lambda_{HE}^{o} \) is the load multiplier at the Hopf bifurcation point with original controls (without perturbation).

Padiyar and Rao [7] have explained that if it is known from operating experience that voltage instability is the major threat to power system security, it is advantageous to simplify the analysis by eliminating the generator swing equations from the problem formulation. This eliminates the electromechanical interaction, between the generating units and the grid. The dynamics of generator rotor electric circuits and voltage control, however, need to be considered.

References


