

# FORMATION TOPOLOGIES AND COOPERATIVE CONTROL OF MULTI-AGENT SYSTEMS

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# FORMATION TOPOLOGIES AND COOPERATIVE CONTROL OF MULTI-AGENT SYSTEMS

*by*

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in fulfillment of the requirements of the degree of Doctor of Philosophy

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# CERTIFICATE

This is to certify that the thesis entitled **Formation Topologies and Cooperative Control of Multi-Agent Systems** submitted by **Soumic Sarkar** to the Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy**, is a record of the bona fide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other University or Institute for the award of any degree or diploma to the best of my knowledge.

Prof. Indra Narayan Kar  
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(Supervisor)

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# ABSTRACT

There is a wide variety of research carried out on the formation control of a single group of robots to cater to various applications starting from underwater through surface to aerial, and even in space where a single robot is not sufficient to accomplish a given task like map building of an untried territory, satellite formation to extract data from space, etc. Whereas not much literature is available on the formation control of multiple groups of robots. Several interconnected groups of robots are capable of covering larger area than a single group of robots for the purpose of navigation in untried territory. Industries that build composite structures, can use multiple groups of robots to carry and bring together different segments of a composite load by means of different groups. To attain this purpose, the robots that are subjected to carry a load, should not change their respective positions when formation is achieved. Otherwise the object may fall, or may subject to damage. Hence there is a strict requirement that the robots should hold their desired positions once the desired formation is reached. It follows from this requirement that the robots are to be numbered. If the combined dynamics of all robots can be separated into several parts using some transformation, different part of the dynamics can be formulated to converge at different time, based on priority. For some specific application it is required that collecting and connecting the different segments of a composite load is more important than the tracking the centroid of the composite load to a given trajectory. This thesis addresses the solution of the problems of this kind.

The literature survey given in Chapter 1, summarizes different controller design techniques of formation control of a single group of robots adopted by researchers, and also the availability of a few works on multiple groups of robots. Based on the gap in literature, the problems which are addressed in this thesis, are formulated. In Chapter 2, a comparative study is given on the transformations used to separate the combined dynamics of all robots into shape and reference (centroid or leader), of a single group of robots. Due to such separation, separate

controllers can be designed independently for different dynamics. Using these transformations for single group of robots, the transformations for multiple groups of robots are introduced in Chapter 3. Singular perturbation technique is adopted to prove both local and global stability of the proposed formation controller. As a result, the combined dynamics, partitioned using the proposed transformations, converge to their desired value at different time as per the concept of slow and fast convergence of different subsystems employed in singular perturbation framework. The utility is also shown through simulation results. While the controllers are designed using singular perturbation technique in the Chapters 2-4, sliding mode controllers are designed in Chapter 5 both for single and multiple groups of robots. These controllers demonstrate (different) finite time convergence of different dynamics in the transformed domain using first order and non-singular terminal sliding mode controller design techniques. The Chapters 2-5 does not deal with rotation and scaling of a formation. Therefore, the *elastic formation* that captures rotation and scaling in the transformations (associated to shape spaces) for single group and for multiple groups of robots, are introduced in Chapter 6. A brief summary of the work, explored in this thesis, is documented in Chapter 7.

The study illustrated in this thesis aims to extend the formation control strategies from single group of robots to multiple groups of robots. The transformation for multiple groups of robots is able to partition a set robots into multiple groups mathematically. The shape variables of single group of robots is partitioned into intra and inter group shape variables. The intra group shape variables is also such partitioned that the shape vectors of each group can separately be identified. This partitioning gives a hierarchical level of decoupling the combined dynamics. Therefore controllers can be designed independently for different dynamics in the transformed shape spaces. The mathematical proofs of convergence of decoupled error dynamics are given and verified with simulation results. Finally future direction of research is hinted like controller decentralization, etc.

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# List of Abbreviations

VS: Virtual Structure

WMR: Wheeled Mobile Robots

SMC: Sliding Mode Control

SM: Sliding Mode

RVT: Reference Vector Based Transformations

CBT: Centroid Based Transformation

LFT: Leader-Follower Based Transformation

TSM: Terminal Sliding Mode

NTSM: Non-singular Terminal Sliding Mode



# List of Symbols

$\mathbb{R}$	Real Space
$\mathbb{R}^n$	Real Space of dimension $n$
$\mathbb{R}^{n \times n}$	Real Matrix of dimension $n \times n$
$\mathbb{C}$	Complex Space
$\mathbb{C}^n$	Complex Space of dimension $n$
$\mathbb{C}^{n \times n}$	Complex Matrix of dimension $n \times n$
$I$	Identity Matrix with appropriate dimension
$\iota$	$\sqrt{-1}$
$Q > 0 (< 0)$	Positive (negative) Definite Matrix