IDENTITIES INVOLVING GENERALIZED DERIVATIONS IN PRIME AND SEMIPRIME RINGS

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NEW DELHI, HAUZ KHAS, INDIA

OCTOBER 2016

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by

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Submitted

in fulfillment of the requirements of the degree of Doctor of Philosophy

to the

Indian Institute of Technology Delhi

October 2016

Dedicated to

My Parents

Certificate

This is to certify that the thesis entitled **“Identities Involving Generalized Derivations in Prime and Semiprime Rings”** submitted by **Mr. Shailesh Kumar Tiwari** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy**, is a record of the original bona fide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

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Acknowledgements

The journey of completing a PhD thesis is culminations of never-ending efforts and in pursue of the same, one requires the involvement of multiple individuals. I take this opportunity to express my thanks to all those who have guided, encouraged, remained patient and showed faith in me to conclude my academic pilgrimage.

First and foremost, I would like to express my deepest regards to my thesis supervisor Prof. R. K. Sharma. I am extremely fortunate to have supervisor who gave me the freedom to explore on my own, and at the same time the guidance to recover whenever my steps faltered. At every crucial juncture, I was amazed at Prof. Sharma's innate ability to anticipate my concerns and to skillfully guide me to the right direction. Their enthusiasm, knowledge, support and patience throughout the program kept me on track and encouraged me when the going got tougher.

I would like to give my special thanks to my SRC (Student Research Committee) members Dr. A. Priyadarshi, Dr. R. Sarma and Prof. S. K. Gupta for their valuable time and suggestions. I am greatly indebted to all faculty members of Department of Mathematics IIT Delhi, for their co-operation and support. Many thanks to CSIR-UGC and IIT Delhi authorities for providing me the research fellowship and necessary facilities all through the PhD program.

I would like to show my gratitude and give my special thanks to Dr. B. Dhara (Belda college West Bengal) and Dr. B. Prajapati for providing me valuable encouragement and support in various ways. Most of the time I had fruitful discussions with them of the subject.

The co-operation, moral support and constant encouragements, I have always received from my *friends and juniors can not be expressed in words and I feel lucky to be blessed with such wonderful peoples. I immensely express my heartiest thanks to my all close friends for all their lively discussions and support.*

I appreciate all my seniors for making me comfortable in the department and motivating in the initial days of my research. Continuation of this, I would also like to give my heartily thanks to Dr. Alok Mishra, Dr. Sunil Kumar Prajapati, Amit Kumar Sarma, Dr. Sarvesh Kumar Dubey, Dr. Sudhakar Chaudhary and Dr. Ratikant Behra in overcoming the initial hurdles of research orientation are highly appreciated. I am highly grateful to them for being there, when I needed them the most.

I am immeasurably grateful to my family who gently offered counseling and gave unconditional support at each and every point of my academic journey. It was the patience and silent sacrifice of my father and mother, which led me to complete this sojourn. My achievements are outcome of their dedication and their belief in my potentials. It is their blessings that made me reach this milestone. I would like to thank my uncle and aunt for their support and care. Without them, it would not have been possible. Thanks to all brothers, sisters, nephews and nieces for their love and kindness through this long process. Last but not the least, I acknowledge my deep gratitude to the memory of my beloved grandfather, grandmother who would have been proud of the biggest accomplishment of their grandson and I thank them for their everlasting blessings which they are still bestowing on me.

I would also like to thank all my friends Bijaya, Abhay Chaturvedi, Meenu, Anju, Vishal, Chirag, Yogesh, Anuj, Puneet, Chiranjeet, Abhilash and all my juniors who generously shared time at different stages of my work, and the enjoyable moments spent with them will always be memorable.

Most importantly, I thank almighty God for his blessings in helping me reach this landmark.

New Delhi *Shailesh Kumar Tiwari*

Abstract

In ring theory, additive mappings play a vital role. Derivations are additive mappings. Many authors generalized ‘derivation’ in various directions such as generalized derivation, Jordan derivation, (α, β)-derivation, multiplicative derivation, multiplicative generalized derivation,$\cdots$ etc. In this thesis, we study generalized derivations on prime rings, and prove that, if it satisfies some identity, then it has some particular form. In the definition of generalized derivation if we remove the additivity of F and drop restrictions on d (d need not be additive and need not be a derivation) it is called multiplicative (generalized)-derivation. This was introduced by Dhara and Shakir. In the context of multiplicative (generalized)-derivation, we also study some identity in semiprime rings in this thesis.

Let R be a ring and $d : R \to R$ an additive mapping on R. Then d is said to be a reverse derivation on R, if $d(xy) = d(y)x + yd(x)$ holds for all $x, y \in R$. If R is commutative, then reverse derivation coincide with derivation. The reverse derivation introduced by Herstein in 1957. Further, we introduce the concept of a multiplicative (generalized)-reverse derivation and study the structure of rings, and the behavior of mappings.

In 1957, Posner proved that if d is a derivation on prime ring R and $a \in R$ such that $ad(x) = 0$ for all $x \in R$, then either $d = 0$ or $a = 0$. Several authors have studied left annihilator of identity. In this direction, we study the relationship between the behavior of a suitable additive mapping F, defined on a ring R of characteristic different from 2, and the structure of R. More precisely, the case when F is a generalized derivation of a prime ring R has been considered. We also study the left annihilator of identity involving generalized derivation and multilinear polynomials in prime rings. Extension of the well known Posner's theorem regarding centralizing derivations on prime rings has been studied. Next, we also study the structure of the ring as well as the nature of mapping when derivation vanishing on commutator identities. Interesting results have been obtained when derivation vanishes on commutators with generalized derivations of order 2 in prime rings with characteristic different from 2. Further, we also study the identity, when generalized derivations vanishes on commutators with generalized derivation in prime rings. Finally, we study left annihilator of identity on Banach algebra and obtained significant results.

Contents

List of Symbols and Notations

$\mathbb{N}$	the set of natural numbers
$\mathbb{Z}$	the set of integers
$\mathbb{Q}$	the set of rational numbers
$\mathbb{R}$	the set of real numbers
$\mathbb{C}$	the set of complex numbers
$=$	equal to
$\neq$	not equal to
id	identity mapping
S_n	symmetric group of n symbol
$Z(R)$	center of a ring R
$char(R)$	characteristic of R
U	Utumi quotient ring
C	extended centroid of a prime ring R
$S = RC$	central closure of a prime ring R
d	derivation, multiplicative derivation
F	generalized derivation, multiplicative (generalized)-derivation
$\forall x$	for all x
$x \in R$	x is a member of R
$x \notin R$	x is not a member of R

$A \subseteq R$	A is a subset of R
$A \subset R$	A is a proper subset of R
$A \nsubseteq R$	A is not a proper subset of R
$[x, y]$	Lie product $xy - yx$ of x and y
$x \circ y$	Jordan product $xy + yx$ of x and y
$ann(R)$	annihilator of R
$\cup, \cap$	union, intersection
$A \times B$	A cartesian product B
$A \bigoplus B$	A direct sum B
$A \cong B$	A is isomorphic to B
$M_m(K)$	$m \times m$ matrices over K
e_{ij}	matrix with ij^{th} entry is 1 and rest entries are zero
e	idempotent element of R
GPI	generalized polynomial identity
$soc(R)$	socle of R
$ad_q(x) = [q, x]$	inner derivation induced by q